Relaxation Dynamics in Two Quantum Dots Coupled to Environment: the Role of Coupling Asymmetry

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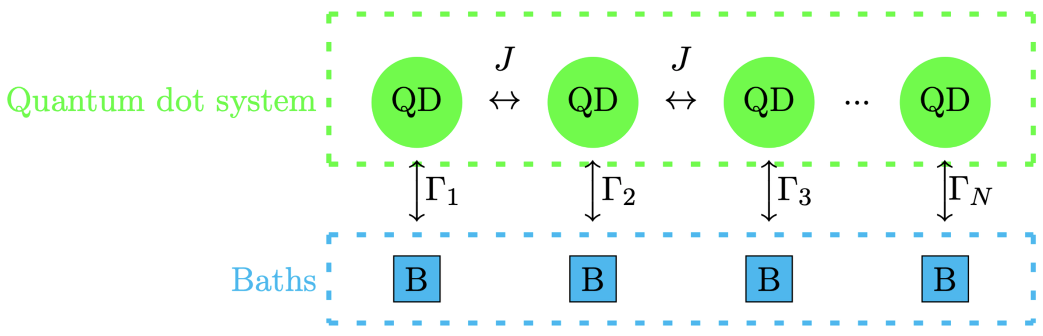
1. Motivation

- Individual QDs which form a quantum dot system and are coupled with environment
- Local coupling between the individual QDs and baths



- Popular master equations describe the coupling in respect to the energy levels and eigenstates of the total quantum dot system
  → Possibly a conflict with the local coupling B $\overset{\Gamma_m}{\longleftrightarrow}$ QD

⇒ **Understand relaxation effects of local couplings with global energy levels of the total quantum dot system**

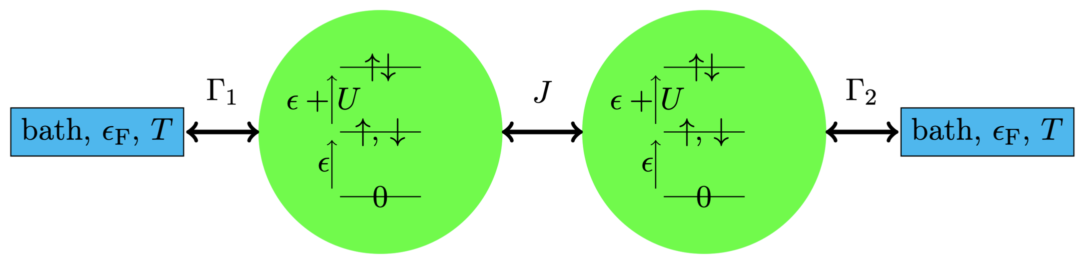
In particular:
The influence of various $\Gamma_1 \neq \Gamma_2 \neq \ldots \neq \Gamma_N$ on the relaxation of the quantum dot system

Parameter regimes we focus on:

- Temperature $T \approx 0$ is small compared to all relevant energy scales
- Strong Coulomb interaction regime $J \ll U < \infty$
  - nevertheless we look at processes which are suppressed with U
- Chemical potential near the particle hole symmetric point $\epsilon < \epsilon_{\mathrm{F}} \approx \epsilon + \frac{U}{2} < \epsilon + U$

2. Model

- $N = 2$ is the smallest QD system which consists of individual QDs



QD system: Fermi-Hubbard model

$$H_{\mathrm{S}} = -J\sum_{\sigma}(c^{\dagger}_{1,\sigma}c_{2,\sigma} + c^{\dagger}_{2,\sigma}c_{1,\sigma}) + U\sum_{m} n_{m,\uparrow}n_{m,\downarrow} + \epsilon\sum_{m,\sigma} n_{m,\sigma}$$

→ Simplest system which includes Coulomb interaction U

Baths: non-interacting electrons

$$H_{\mathrm{B}} = \sum_{k,\sigma,m}\epsilon_k a^{\dagger}_{m,\sigma,k}a_{m,\sigma,k}$$

→ Fermi function $f(\epsilon_k, T)$ (thermodynamic equilibrium)

Local coupling between bath m and QD m:

$$H_{\mathrm{C}} = \sum_{k,\sigma,m} t_{m,k}c^{\dagger}_{m,\sigma}a_{m,\sigma,k} + \text{h.c.}$$

with $\Gamma_m(\omega) = 2\pi\sum_k |t_{m,k}|^2\rho(\omega - \epsilon_k) \approx \Gamma_m$ in the wide band limit

3. Methods

- No exact solution known for $0 < U < \infty$
  - Benchmark: exact solution for $U = 0$ (cf. 8)
  - Master equations of Redfield-I type lead to Lindblad equations

$$\dot{\rho} = -i[\tilde{H}_{\mathrm{S}}, \rho] + \sum_{\nu}\Gamma_{m_\nu}\left[L_\nu\rho L^{\dagger}_{\nu} - \frac{1}{2}\left\{L^{\dagger}_{\nu}L_\nu, \rho\right\}\right]$$

  with static and time dependent Lindblad operators L_μ

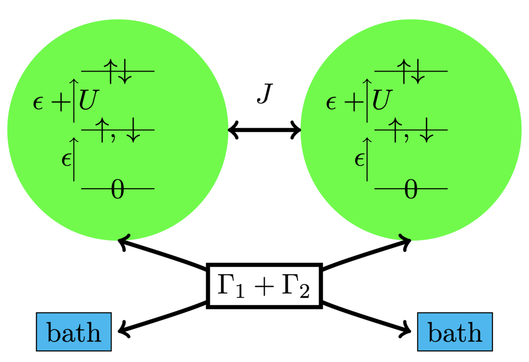
- Time dependency using an effective time-dependent temperature $T_{\mathrm{eff}}(t)$ instead of T [1]

Lindblad operators result from secular approximation:

$$L_{m,\sigma,\alpha}(\Delta E) = \sqrt{f_\alpha(\Delta E, T)}\sum_{k,l}\delta_{E_l - E_k,\alpha\Delta E}\,|E_k\rangle\langle E_k|\,c^{\alpha}_{m,\sigma}\,|E_l\rangle\langle E_l|$$

→ No difference between $L_{1,\sigma,\alpha}(\Delta E)$ and $L_{2,\sigma,\alpha}(\Delta E)$

⇒ Secular Lindblads induces total global coupling



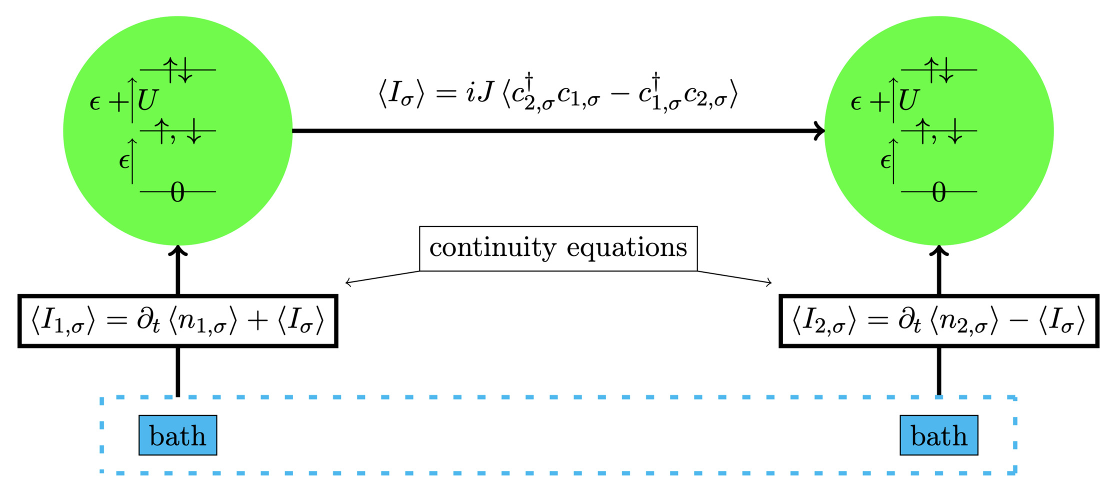
⇒ **Impossible to study influence of $\Gamma_1 \neq \Gamma_2$ with secular Lindblads**

Lindblad operators result from coherent approximation [2]:

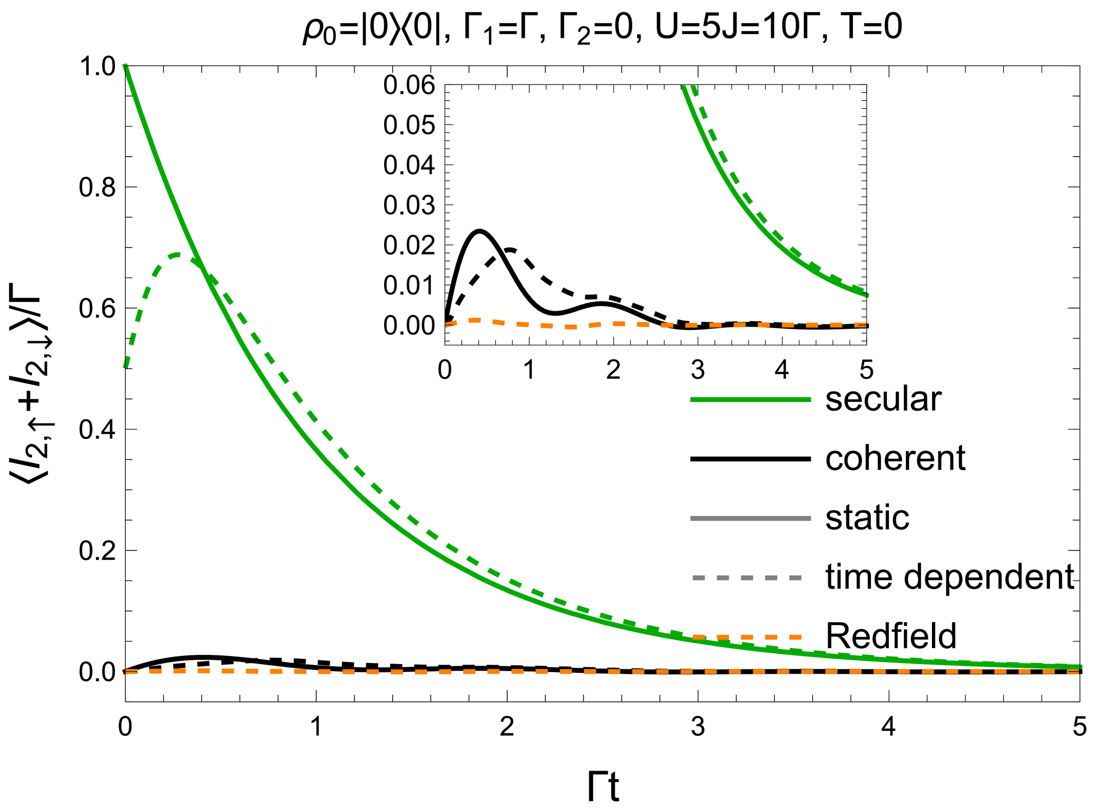
$$L_{m,\sigma,\alpha} = \sum_{k,l}\sqrt{f_\alpha(\alpha(E_l - E_k), T)}\,|E_k\rangle\langle E_k|\,c^{\alpha}_{m,\sigma}\,|E_l\rangle\langle E_l|$$

→ $L_{m,\sigma,\alpha}$ become local if $\frac{U}{J} \to \infty$ or mostly local if $U > 2J$
→ Time-dependent $L_{m,\sigma,\alpha}$ for early times always becomes local

4. Currents

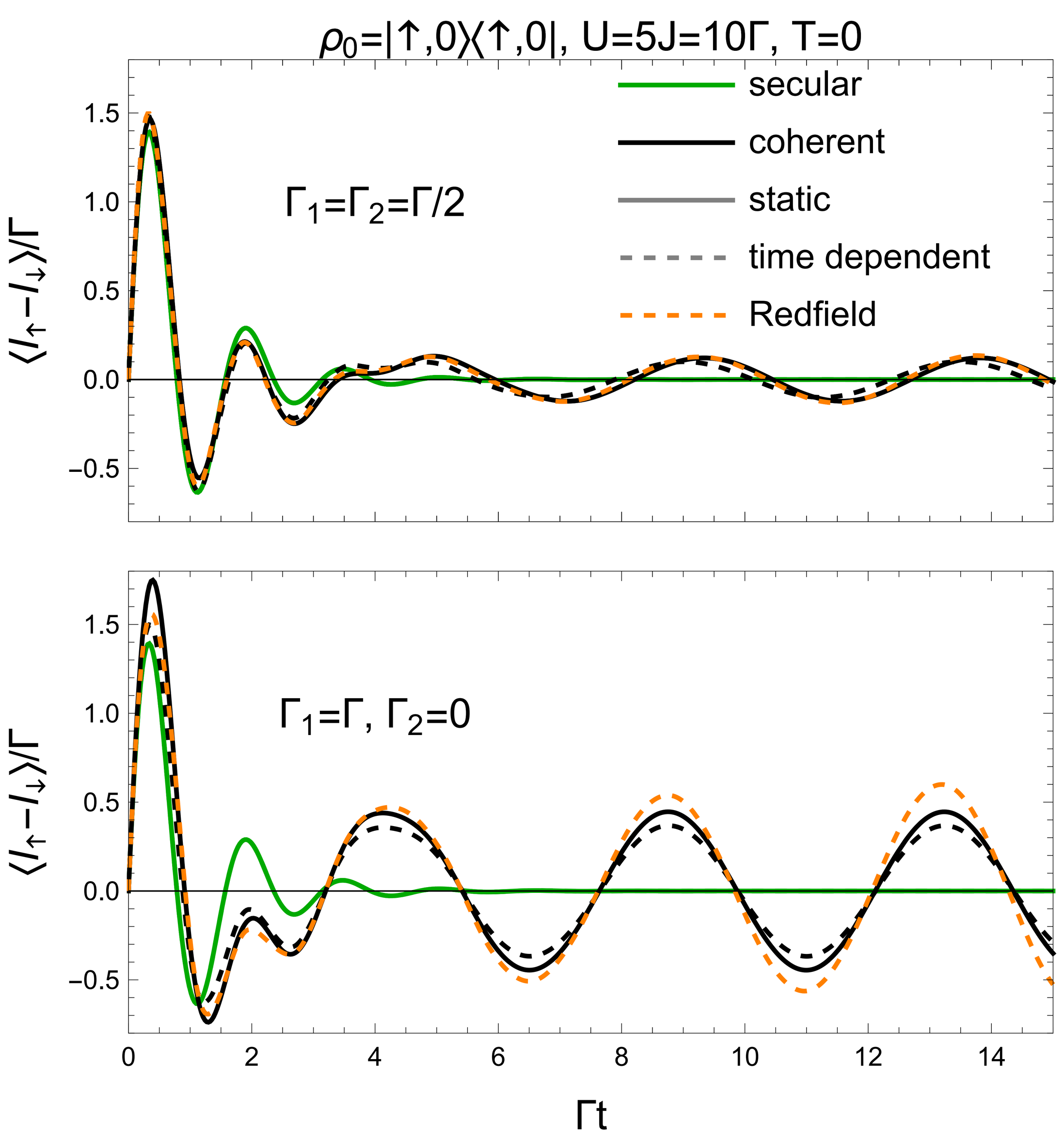


- If $\Gamma_1 = \Gamma$, $\Gamma_2 = 0$ then $\langle I_{2,\sigma}\rangle = 0$ should hold
  - Nevertheless, due to a global coupling, the master equations i.g. yield $\langle I_{2,\sigma}\rangle \neq 0$
- $\langle I_{2,\sigma}\rangle$ is a quantity which shows how far the influence of $\Gamma_1 \neq \Gamma_2$ can be studied with a master equation
  - $\langle I_{2,\sigma}\rangle \approx 0$ local coupling → may be possible to study $\Gamma_1 \neq \Gamma_2$
  - $\langle I_{2,\sigma}\rangle \gg 0$ global coupling → impossible to study $\Gamma_1 \neq \Gamma_2$



5. Long-time oscillation

- Spin current difference $I_\uparrow - I_\downarrow$ for initial state $\rho_0 = |\uparrow, 0\rangle\langle\uparrow, 0|$
  - Relaxation into long-time oscillations for coherent Lindbalds and Redfield eq.



- $\Gamma_1 = \Gamma$, $\Gamma_2 = 0$ leads to a higher oscillation amplitude than $\Gamma_1 = \Gamma_2 = \frac{\Gamma}{2}$
- Oscillations given by coherences between $|T_0\rangle$ and $|\tilde{\mathrm{S}}\rangle$ describing mainly oscillations between

$$|\downarrow,\uparrow\rangle = \frac{1}{\sqrt{2}}\left(|T_0\rangle - |\tilde{\mathrm{S}}\rangle\right) + 2\frac{J}{U}\,|\mathrm{DH}\rangle + \mathcal{O}(J^2/U^2),$$

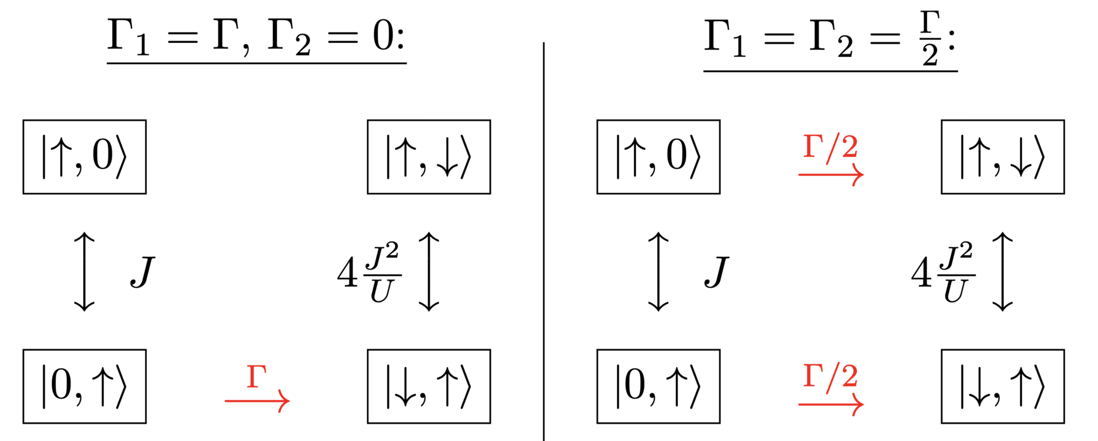
$$|\uparrow,\downarrow\rangle = \frac{1}{\sqrt{2}}\left(|T_0\rangle + |\tilde{\mathrm{S}}\rangle\right) - 2\frac{J}{U}\,|\mathrm{DH}\rangle + \mathcal{O}(J^2/U^2)$$

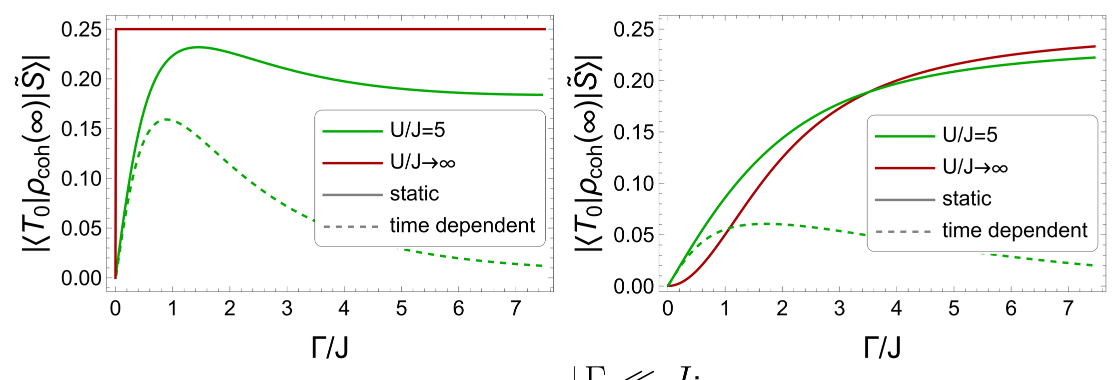
with frequency $\omega_{\mathrm{S,T}} = 4J^2/U + \mathcal{O}\left(J^2/U^2\right)$ involving

$$|\uparrow,\downarrow\rangle \xrightarrow{J} |\uparrow\downarrow, 0\rangle \xrightarrow{J} |\downarrow,\uparrow\rangle\,.$$

- **Only occupation of $|\uparrow,\downarrow\rangle$ or $|\downarrow,\uparrow\rangle$ ⇒ maximal value of $|T_0\rangle\langle\tilde{S}|$**
  - regime $\Gamma_1 = \Gamma$, $\Gamma_2 = 0$
- **Equal occupation of $|\uparrow,\downarrow\rangle$ and $|\downarrow,\uparrow\rangle$ ⇒ no $|T_0\rangle\langle\tilde{S}|$ coherences**
  - regime $\Gamma_1 = \Gamma_2 = \frac{\Gamma}{2}$

6. Dependence on J, U, Γ_1, Γ_2





- $4\frac{J^2}{U}$ - hopping equalises $|\uparrow,\downarrow\rangle$, $|\downarrow,\uparrow\rangle$ occupation
  → Destroying coherences
- **Strong Coulomb interaction**
  ⇓
  Strong coherence

$\Gamma \ll J$:

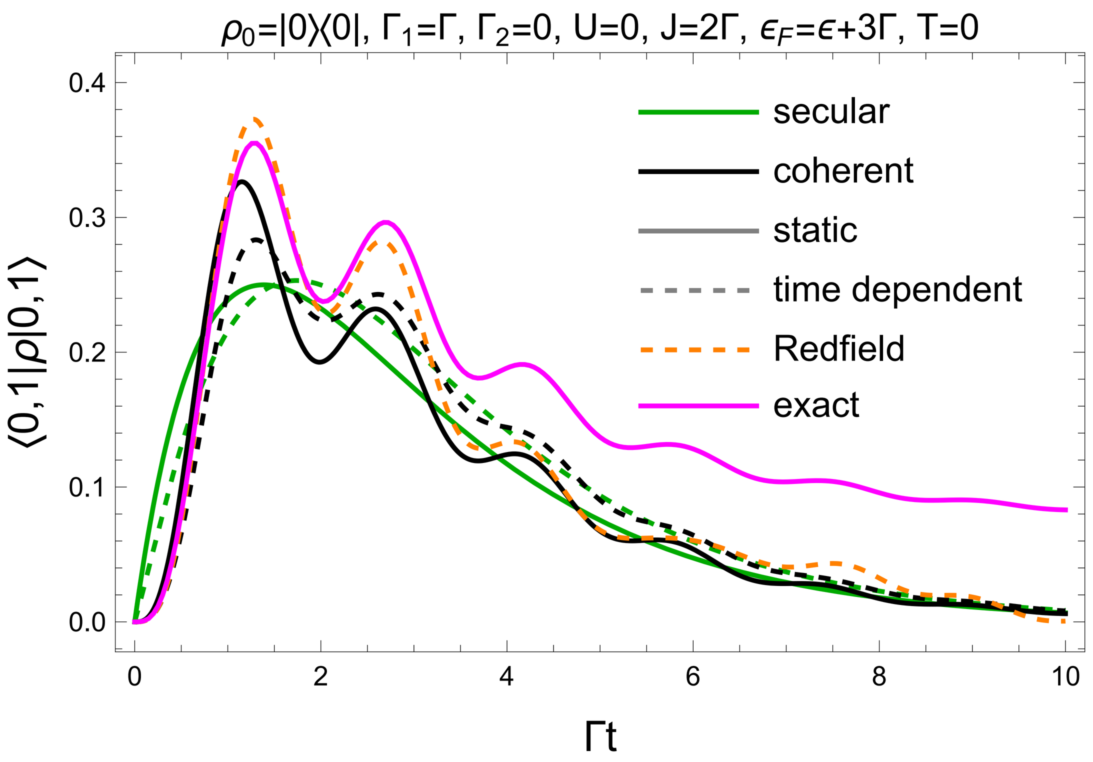
- if $4\frac{J^2}{U}$ far away from J → occupation becomes equal
- if $4\frac{J^2}{U}$ next to J → $|\uparrow,\downarrow\rangle$, $|\downarrow,\uparrow\rangle$ occupation becomes more unequal
- **Strong Coulomb interaction**
  ⇓
  Weak coherence

$\Gamma \gg J$: Same behaviour as in the $\Gamma_1 = \Gamma$, $\Gamma_2 = 0$ case

- Values for the time-dependent approximation vanish in the limit of $\Gamma \to \infty$ because of relaxation during high effective temperature

8. Benchmark for $U = 0$

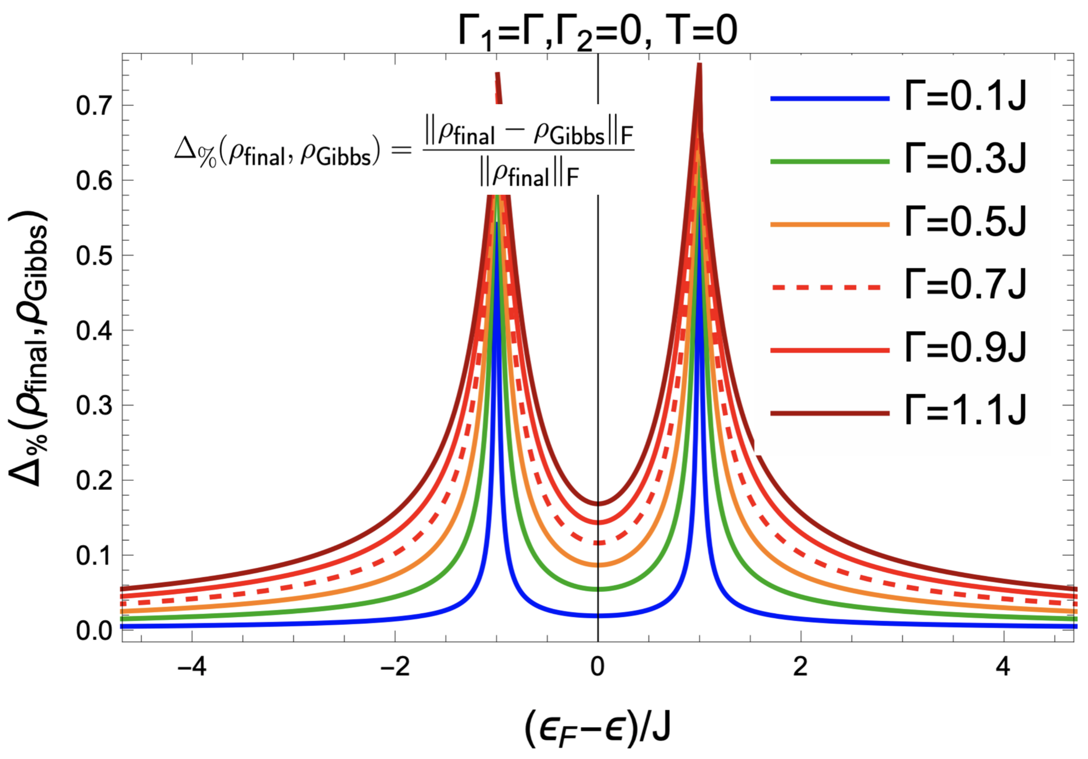
- System with $U = 0$ → effective: a system without spin
- Exact solution via Laplace transformed equation of motion



- Redfield and time-dependent coherent approximation agree with the exact solution for small and intermediate times
- Static coherent approximation agrees with the exact solution for some parameter regimes (local coupling)
- Secular approximation fails for $\Gamma_1 \neq \Gamma_2$

Final state $t \to \infty$:

- Long term deviations due to higher order in Γ



- Long term deviations vanishes if $|\Delta E - \epsilon_{\mathrm{F}}| \gg \Gamma_1 + \Gamma_2$

Summary

- Secular approximation fails to describe influence of $\Gamma_1 \neq \Gamma_2$
  - coherent approximation is a practical alternative and is mostly closest to the exact solution
- We looked on effects which are suppressed with U
  - coherent long time oscillations
- Value of energy coherences depends strongly on coupling asymmetry

References

-L. Litzba, N. Szpak, J. König (in preparation)
[1] N. Szpak *et. al.*, *DPG Frühjahrstagung 2022* **DY** 35.9-Talk
[2] E. Kleinherbers, N. Szpak, J. König, R. Schützhold. *PRB* **101**, 125131 (2020)