Coherent Charge Oscillations in Coupled Double Quantum Dots

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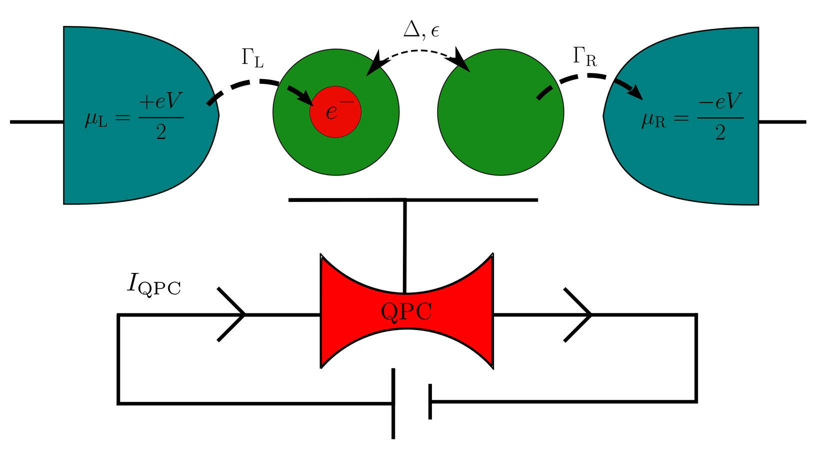
Double Quantum Dot (DQD)



- Hamiltonian of system & environment

$$\mathcal{H}_{\text{dots}} = \sum_{\alpha=\text{L,R}} E_\alpha c_\alpha^\dagger c_\alpha + U n_\text{L} n_\text{R} + \frac{\Delta}{2}\left(c_\text{L}^\dagger c_\text{R} + c_\text{R}^\dagger c_\text{L}\right)$$

$$\mathcal{H}_{\text{leads}} = \sum_{\alpha\vec{k}} \varepsilon_{\vec{k}} a_{\alpha\vec{k}}^\dagger a_{\alpha\vec{k}}$$

basis: $|\chi\rangle \in \{|0\rangle, |\text{L}\rangle, |\text{R}\rangle, |\text{d}\rangle\}$

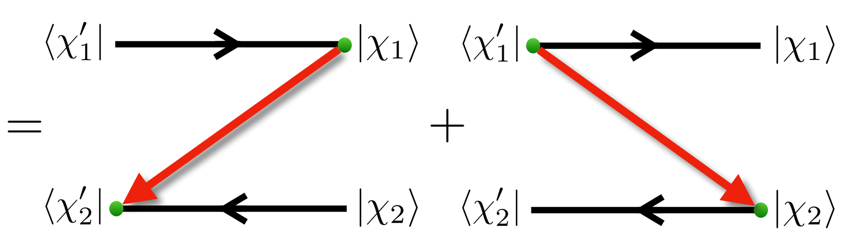
$$\mathcal{H}_{\text{tun}} = \sum_{\alpha\vec{k}} t_\alpha c_\alpha^\dagger a_{\alpha\vec{k}} + \text{h.c.}$$

- *real-time diagrammatic technique*: perform perturbative expansion in $\mathcal{H}_{\text{tun}}$ and integrate out lead degrees of freedom to obtain a generalized master equation for the *reduced density matrix*:

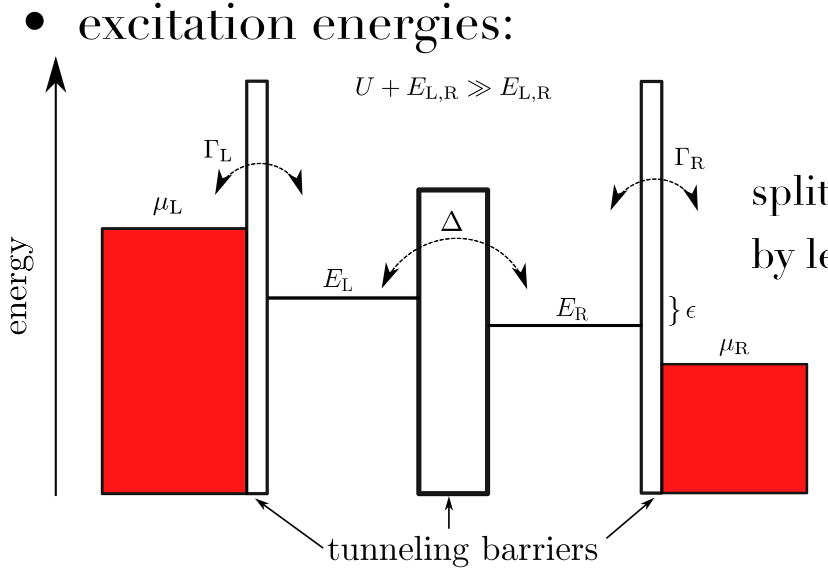
$$\dot{\rho}(t) = -\frac{i}{\hbar}[\mathcal{H}_{\text{dots}}, \rho(t)] + \mathcal{W}\rho(t)$$

here: *weak-coupling limit*, i.e. only leading order in $\Gamma_\alpha \propto |t_\alpha|^2$

$\mathcal{W}^{\chi_1'\chi_1}_{\chi_2'\chi_2} =$



- excitation energies:



splitting not resolved by leads since $\Gamma_\alpha \lesssim \Delta, \epsilon$

- *coherent charge oscillations* between left and right quantum dot conveniently described with isospin $\mathbf{I} = \boldsymbol{\sigma}/2$:

$$\dot{\mathbf{I}} = \mathbf{I} \times \mathbf{B} + \left(\frac{\partial \mathbf{I}}{\partial t}\right)_{\text{acc}} + \left(\frac{\partial \mathbf{I}}{\partial t}\right)_{\text{rel}}$$

with $\mathbf{B} = (\Delta, 0, \epsilon_{\text{ren}})$ and *renormalized* detuning

- Oscillations detectable in a transport measurement without destroying coherences? **Yes,** by measuring the charge state of the system with an electrometer (e.g. QPC) that is insensitive to the individual dots!
- quantities able to measure oscillations: finite frequency noise, generalized factorial cumulants [1], **waiting-time distribution** [2]

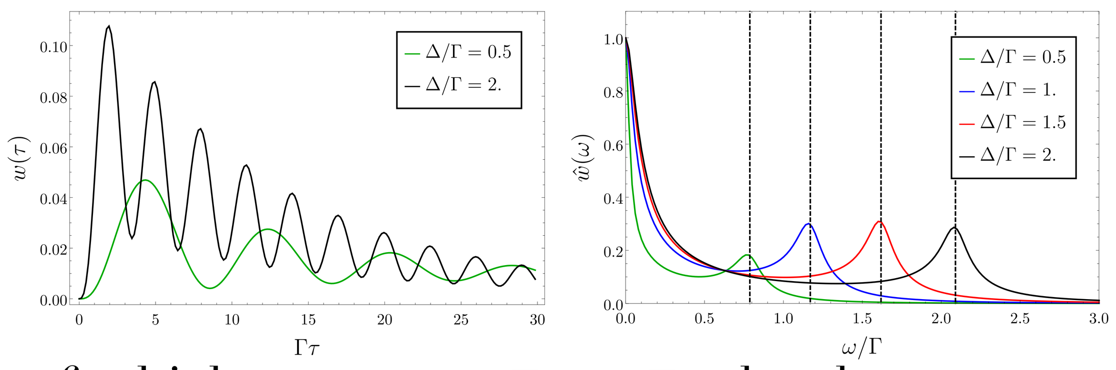
Waiting-time distribution

waiting-time distribution: probability of two consecutive tunneling events with time span τ

$$w^{\beta\beta'}(\tau) = \frac{\text{tr}\mathcal{J}^\beta e^{\mathcal{W}_0\tau}\mathcal{J}^{\beta'}\rho_0}{\text{tr}\mathcal{J}^{\beta'}\rho_0}$$

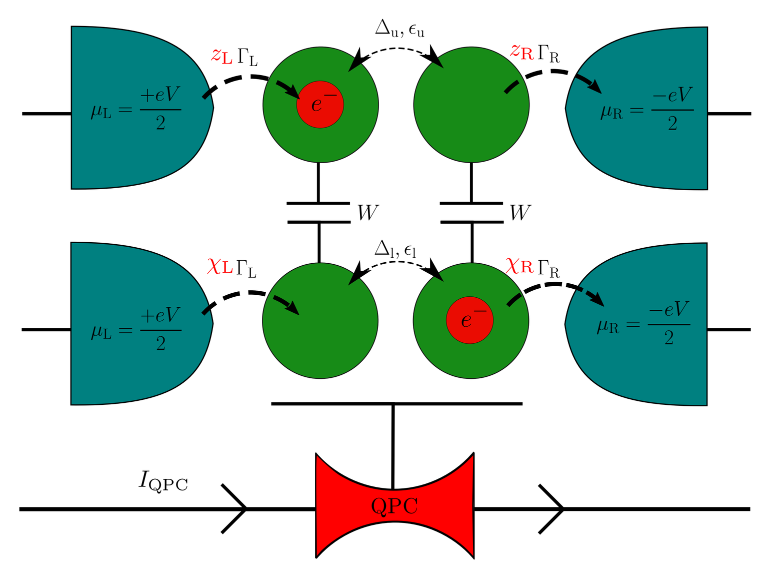
with jumps $\mathcal{J}^\beta$ and "free" propagation $e^{\mathcal{W}_0\tau}$

- DQD: consider consecutive outgoing electrons



- for high asymmetry $\Gamma_\text{L} \gg \Gamma_\text{R}$ the electron stays long enough in the DQD and coherent oscillations can be nicely observed

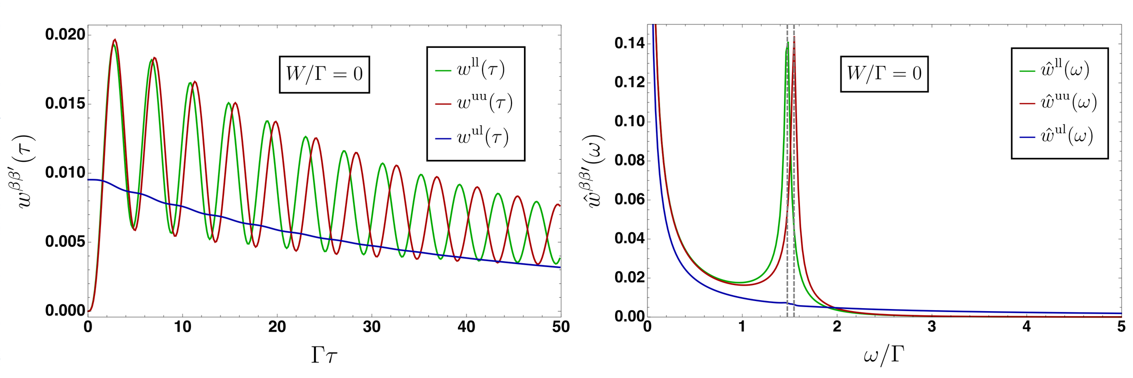
Coupled DQD's



- duplicate system and add Coulomb term:

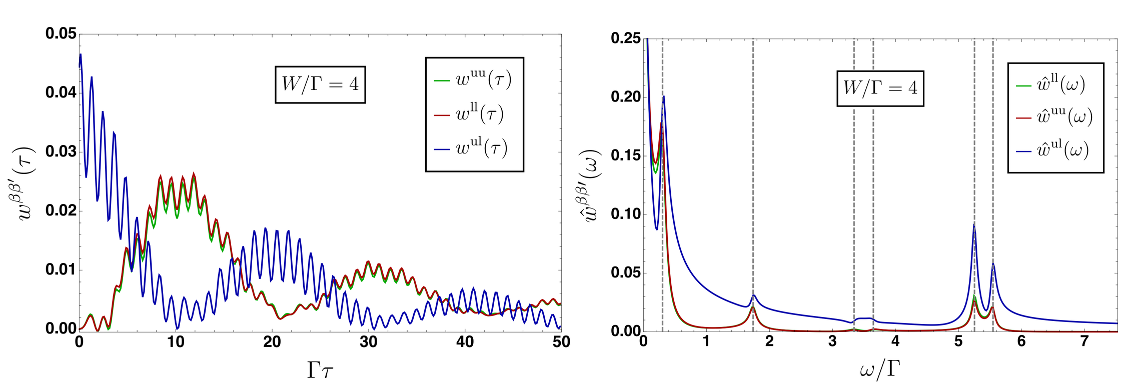
$$\mathcal{H} = \mathcal{H}^{\text{u}}_{\text{dots}} + \mathcal{H}^{\text{l}}_{\text{dots}} + W\left(n_{\text{Lu}}n_{\text{Ll}} + n_{\text{Ru}}n_{\text{Rl}}\right)$$

- study waiting times:
  - $w^{\text{uu}}(\tau), w^{\text{ll}}(\tau)$ for each subsystem separately
  - $w^{\text{ul}}(\tau)$ cross-correlated waiting time
- without interaction $W = 0$:



two distinct frequencies $\omega_\beta = |\mathbf{B}_\beta|$, no cross-correlation

- with interaction $W > 0$:



complex cross-correlated behavior, six oscillation frequencies

How to explain the spectrum of the coupled oscillators?

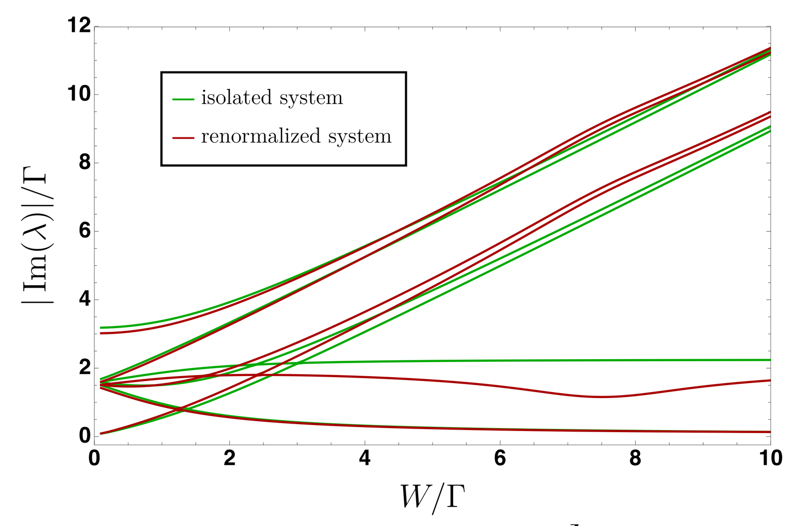
Analyzing the Spectrum

- consider subspace with one electron in each DQD: $|\chi^\text{u}, \chi^\text{l}\rangle \in \{|\text{L,L}\rangle, |\text{L,R}\rangle, |\text{R,L}\rangle, |\text{R,R}\rangle\}$
- use isospins $\mathbf{I}^\text{u}, \mathbf{I}^\text{l}$

$$\mathcal{H}_{\text{red}} = \mathbf{B}_\text{u} \cdot \mathbf{I}^\text{u} + \mathbf{B}_\text{l} \cdot \mathbf{I}^\text{l} + 2W I_z^\text{u} I_z^\text{l}$$

Coulomb term corresponds to antiferromagnetic Ising interaction

- eigenvalues λ of $-\frac{i}{\hbar}[\mathcal{H}_{\text{red}}, \ldots]$ give rise to spectrum ($|\operatorname{Im}\lambda| \equiv \omega$):

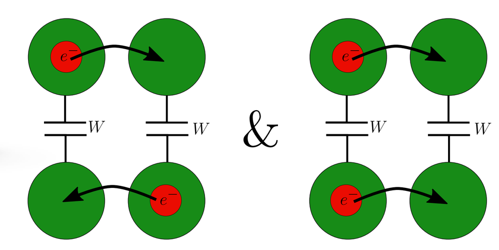


- tunneling induces *renormalization*:

$$\tilde{\epsilon}_\beta = \epsilon_\beta + \delta\epsilon \quad \text{and} \quad W_\alpha = W + \delta W_\alpha$$

$(\alpha \in \{\text{L,R}\}, \beta \in \{\text{u,l}\})$

- small interaction $W \ll \Gamma$:
  - four frequencies $\omega_\text{u}, \omega_\text{l}$ and $|\omega_\text{u} \pm \omega_\text{l}|$
- strong interaction $W \gg \Gamma$:
  - four W-dependent frequencies (oscillation in only one DQD)
    - $|\text{L,L}\rangle\langle\text{R,L}| : \omega \approx W + \epsilon_\text{u}$
    - $|\text{L,L}\rangle\langle\text{L,R}| : \omega \approx W + \epsilon_\text{l}$
    - $|\text{R,R}\rangle\langle\text{L,R}| : \omega \approx W - \epsilon_\text{u}$
    - $|\text{R,R}\rangle\langle\text{R,L}| : \omega \approx W - \epsilon_\text{l}$
  - two W-independent frequencies
    - $|\text{L,L}\rangle\langle\text{R,R}| : \omega \approx \epsilon_\text{u} + \epsilon_\text{l}$ in phase
    - $|\text{L,R}\rangle\langle\text{R,L}| : \omega \approx |\epsilon_\text{u} - \epsilon_\text{l}|$ in antiphase



Collective coherences induced by Coulomb repulsion!

Summary & Outlook

- By performing a counting experiment on only one DQD all six oscillation frequencies can be measured and conveniently analyzed via a waiting-time distribution.
- For strong interaction the envelope function of $w^{\text{ul}}(\tau)$ suggests a coherent antiphase oscillation of the electrons to avoid each others company.
- *Outlook*: attractive electron-electron interaction [3], analogue system with coupled spin valves [1]

list of references:

[1] P. Stegmann, J. König, and S. Weiss, *Phys. Rev. B.* **98**, 035409 (2018)

[2] T. Brandes, *Ann. Phys.* (Berlin) **17**, 477 (2008)

[3] E. Kleinherbers, P. Stegmann, and J. König, *New. J. Phys.* **20**, 073023 (2018)

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