Pushing the limits in real-time measurements of quantum dynamics

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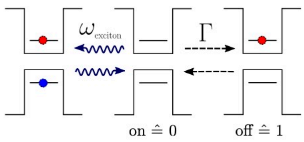
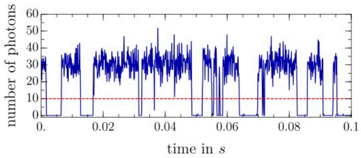
Measurement

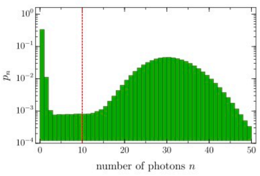
- electron statistics of a quantum dot measured optically via driving an exciton with a laser (AG Lorke, **A01**)



- measure single photons as function of time and sample the data (error of limited time resolution, here: $\Delta t = 100\ \mu s$)

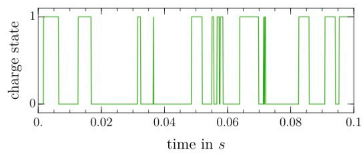


- introduce threshold n_c to distinguish "on" ($n > n_c$) from "off" ($n < n_c$)



- noise on the "on"-state (0) state leads in every time step with probability $p(1|0)$ to a false state "off" (1)
- no noise on the "off"-state (1), $p(0|1) = 0$

- final binary signal with finite measurement time T



sources of errors:

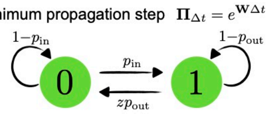
A. limited time resolution Δt
B. noise on charge states $p(1|0)$
C. finite length of time trace T

Limited Time Resolution

- introduce coarse graining in time:
  - from rate equation to difference equation (Markov chain)

$$\dot{\rho} = \mathbf{W}\rho \quad \rightarrow \quad \rho_{t+\Delta t} = \mathbf{\Pi}_{\Delta t}\rho_t$$

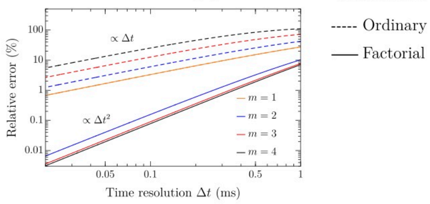
  - minimum propagation step $\mathbf{\Pi}_{\Delta t} = e^{\mathbf{W}\Delta t}$



- $p_{\text{in}}/p_{\text{out}}$ is the probability for a physical transition where an electron enters/leaves the quantum dot
- generating function is a finite polynomial in z with $K = t/\Delta t$
  - meaning: in a time step Δt at most one tunneling event will be counted

$$\mathcal{M}(z,t) = \text{tr}(\mathbf{\Pi}^K_{\Delta t}\rho_{\text{st}}) = \sum_{N=0}^{K} z^N P_N$$

- factorial cumulants $C_{\text{F},m} = [\partial_z^m \log \mathcal{M}(z,t)]_{z=1}$
- ordinary cumulants $\kappa_m = \sum_{k=1}^{m} s_{m,k} C_{\text{F},k}$ — $s_{m,k}$: Stirling numbers of second kind



- cumulant-generating function:

$$\log \mathcal{M} = \log \mathcal{M}_0 - \alpha \Delta t(z-1) + \mathcal{O}(\Delta t^2)$$

Noise on Charge Levels

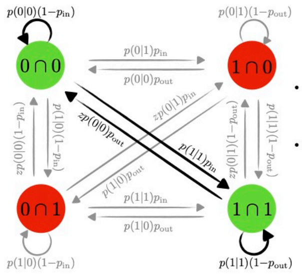
- noise: measurement outcome m_a may differ from eigenvalue m_b of the actual state ρ_t with probability $p(a|b)$
- physical state ρ_t and measurement outcome m_a appear in all combinations:

$$\rho_t^{\text{noise}} = (\rho_t^1, ..., \rho_t^M) \qquad M \times D \text{ components}$$

(# of measurement outcomes; # of density-matrix elements)

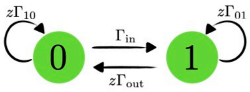
- propagator $\Pi_z^{\text{noise}} = \mathbf{N}(Z \otimes \Pi_{\Delta t})$

$$\Pi_z^{\text{noise}} = \begin{pmatrix} p(0|0) & 0 & 0 & 0 \\ 0 & p(0|1) & 0 & 0 \\ 0 & 0 & p(1|0) & 0 \\ 0 & 0 & 0 & p(1|1) \end{pmatrix} \left[\begin{pmatrix} 1 & z \\ 1 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1-p_{\text{in}} & p_{\text{out}} \\ p_{\text{in}} & 1-p_{\text{out}} \end{pmatrix}\right]$$



- green/red indicates true /false
- gray indicates noise-related transition

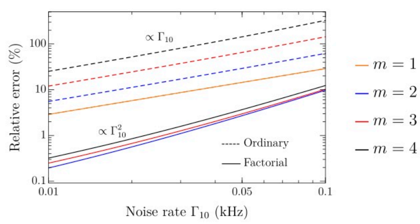
- in the limit $\Delta t \to 0$:



noise rates:

$$\Gamma_{ab} = \lim_{\Delta t \to 0} \frac{p(a|b)}{\Delta t}$$

- ordinary vs factorial cumulants ($\Gamma_{\text{in}} \approx \Gamma_{\text{out}}$)



- cumulant-generating function:

$$\log \mathcal{M} = \log \mathcal{M}_0 + \Gamma_{\text{noise}} t(z-1)$$

- symmetric noise on charge levels leads to Poisson statistics on top of the actual electron statistics!

Finite Measurement Time

- uncertainty in probability distribution $P_N^\eta \to P_N + \eta_N$ modeled by independent stochastic variables

$$\langle \eta_N \eta'_{N'} \rangle = \delta_{NN'} P_N (1-P_N) t/T \qquad \langle \eta_N \rangle = 0$$

- with $P_N^\eta = \dfrac{P_N + \eta_N}{1 + \sum_{N'} \eta_{N'}}$ we find for long times

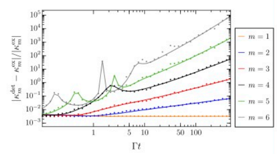
$$\Delta\kappa_m \approx \Delta C_{\text{F},m} \approx \sqrt{\kappa_2^m m! t/T} \propto t^{(m+1)/2}$$

$$\Delta N^m \approx \Delta N^{(m)} \approx m\kappa_1^{m-1}\sqrt{\kappa_2 t/T} \propto t^m$$

equal absolute errors!

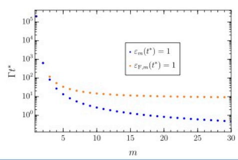
$$\Delta\kappa_m/\kappa_m \propto t^{(m-1)/2}$$

$$\Delta C_{\text{F},m}/C_{\text{F},m} \propto t^{(m-1)/2}$$



- for relative errors $\varepsilon_m := \Delta\kappa_m/\kappa_m \qquad \varepsilon_{\text{F},m} := \Delta C_{\text{F},m}/C_{\text{F},m}$

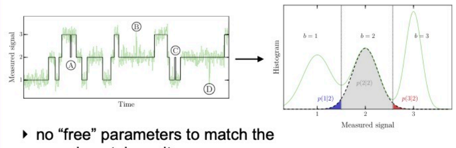
we find $\varepsilon_m/\varepsilon_{\text{F},m} = (2m-3)!! > 1$ for $m > 1$



the error of factorial cumulants "explodes" later

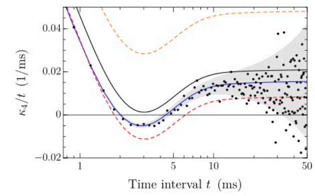
Comparison with Experiment

- How to extract noise probabilities?

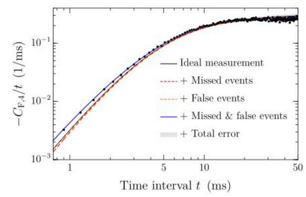


  - no "free" parameters to match the experimental results

- How well are the errors described for an actual experiment?
  - here: 95% of all photons deleted to increase noise

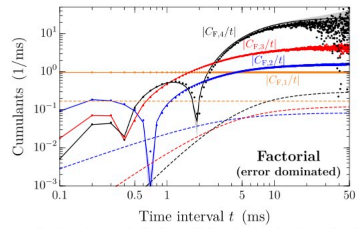


- noise alone overshoots
- finite resolution alone undershoots



- factorial cumulants less sensitive to all three errors

- theoretical description works even for extreme situations:
  - neglected errors (dashed) vs. fully accounted errors (solid) for a noise-dominated telegraph signal

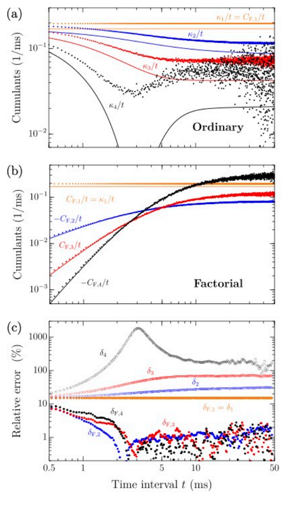


noise leads to violation of the sign criterion $(-1)^{m-1}C_{\text{F},m} \geq 0$ falsely indicating correlations in the statistics

- main results summed up:

A. Our proposed theories to model the three errors: i) limited time resolution, ii) noise, and iii) finite data work extremely well!

B. Use factorial cumulants, because they are less sensitive to errors than ordinary cumulants!



Outlook

- "stochastic resonance": improve the signal by adding noise!
- influence of errors on coherent oscillations in a DQD (BA Moritz Winterott)
- preprint on arXiv: 2106.12502