Pair-amplitude dynamics in strongly-coupled superconductor-quantum dot hybrids

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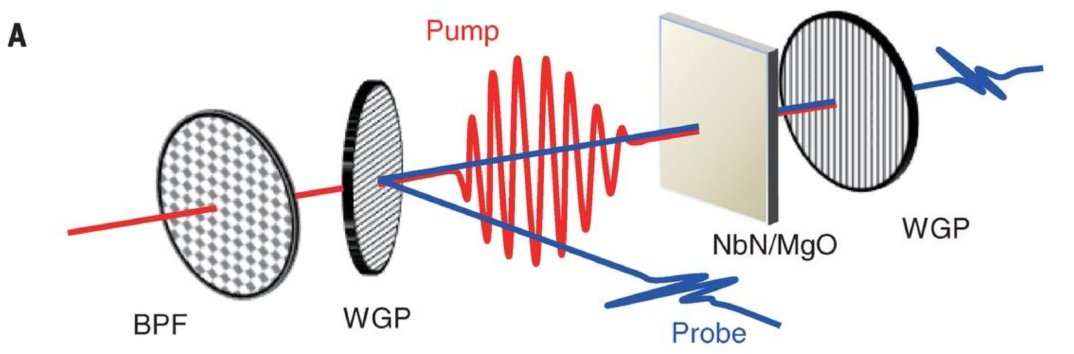
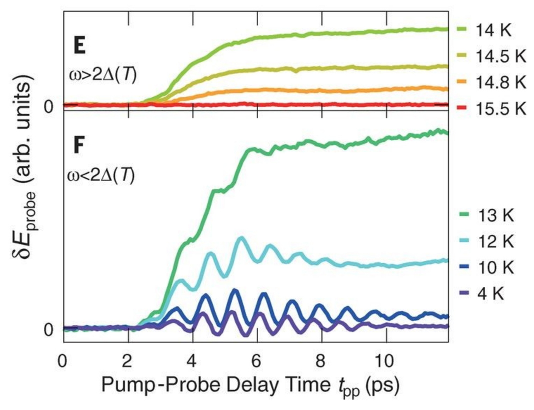
Higgs mode in superconductor



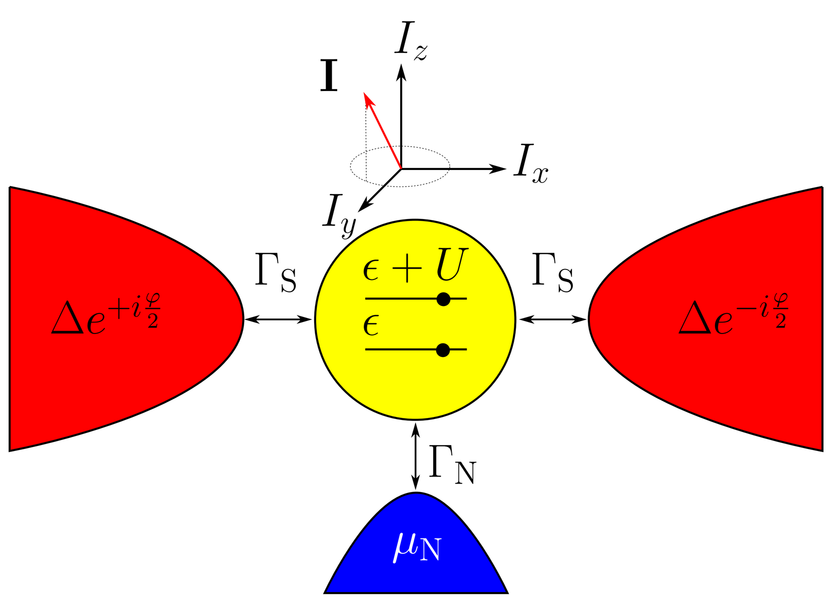
- THz-Pump-Probe experiment with superconductor [1]
- Measured oscillating order parameter



Motivation

- S-QD hybrids: controllability, tunability and strong interactions
- Interplay of superconductivity, nonequilibrium physics and strong Coulomb interaction
- Spontanous symmetry breaking: Pair-amplitude dynamics

System



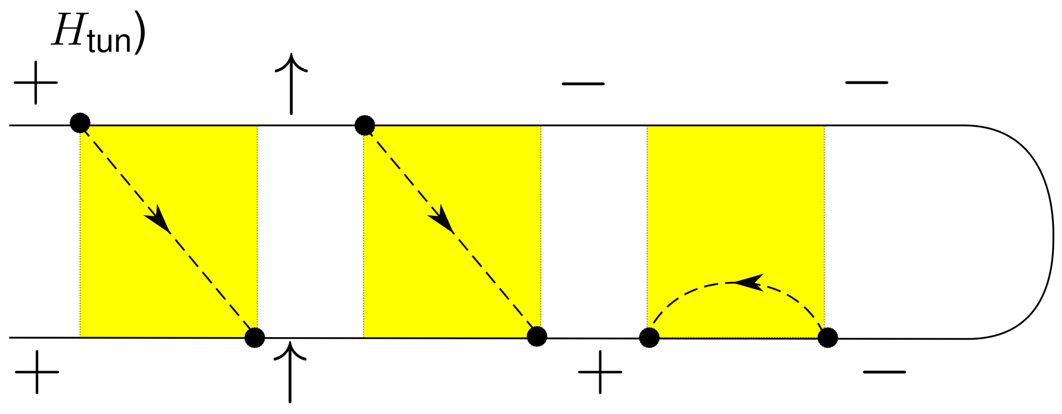
- Full Hamiltonian: $H = \sum_\eta H_\eta + H_{\text{dot}} + H_{\text{tun}}$
- $H_{\text{dot}} = \sum_\sigma \epsilon c^\dagger_\sigma c_\sigma + U c^\dagger_\uparrow c_\uparrow c^\dagger_\downarrow c_\downarrow$
- $H_\eta = \sum_{\mathbf{k}\sigma} \epsilon_{\eta\mathbf{k}} a^\dagger_{\eta\mathbf{k}\sigma} a_{\eta\mathbf{k}\sigma} + \sum_{\mathbf{k}} \left(\Delta_\eta e^{i\varphi_\eta} a_{\eta\mathbf{k}\uparrow} a_{\eta-\mathbf{k}\downarrow} + \text{H.c.}\right)$
- $H_{\text{tun}} = \sum_{\eta\sigma\mathbf{k}} t_\eta a^\dagger_{\eta\sigma\mathbf{k}} c_\sigma + \text{H.c.}$

Infinite-gap limit

- Pair-amplitude dynamics governed by subgap physics
- Choose infinite-gap limit for system description
- Allows for arbitrarily strong coupling between QD and superconductors
- Possibility to study strong proximity effects on the QD
- Described in the eigenbasis of $H_{\text{eff}} = H_{\text{dot}} - H_{\text{p}}$
- $H_{\text{p}} = \chi^* d_\downarrow d_\uparrow + \chi d^\dagger_\uparrow d^\dagger_\downarrow$, $\chi = \frac{1}{2}\sum_\eta \Gamma_\eta e^{i\phi_\eta}$

Real-time diagrammatics [2]

- Integrate out noninteracting leads
- Describe quantum dot by reduced density matrix ρ_{red}
- Generalized master equation for density matrix elements $P^\chi_{\chi'} = \langle\chi|\rho_{\text{red}}|\chi'\rangle$
- Transition rates given by irreducible diagrams on the Keldysh contour (systematic expansion in H_{tun})



$$\frac{dP^{\chi_1}_{\chi_2}(t)}{dt} = -i(E_{\chi_1} - E_{\chi_2})P^{\chi_1}_{\chi_2}(t) + \sum_{\chi'_1,\chi'_2} \int_{-\infty}^{t} dt' W^{\chi_1\chi'_1}_{\chi_2\chi'_2}(t,t') P^{\chi'_1}_{\chi'_2}(t')$$

Master equation in pseudospin representation

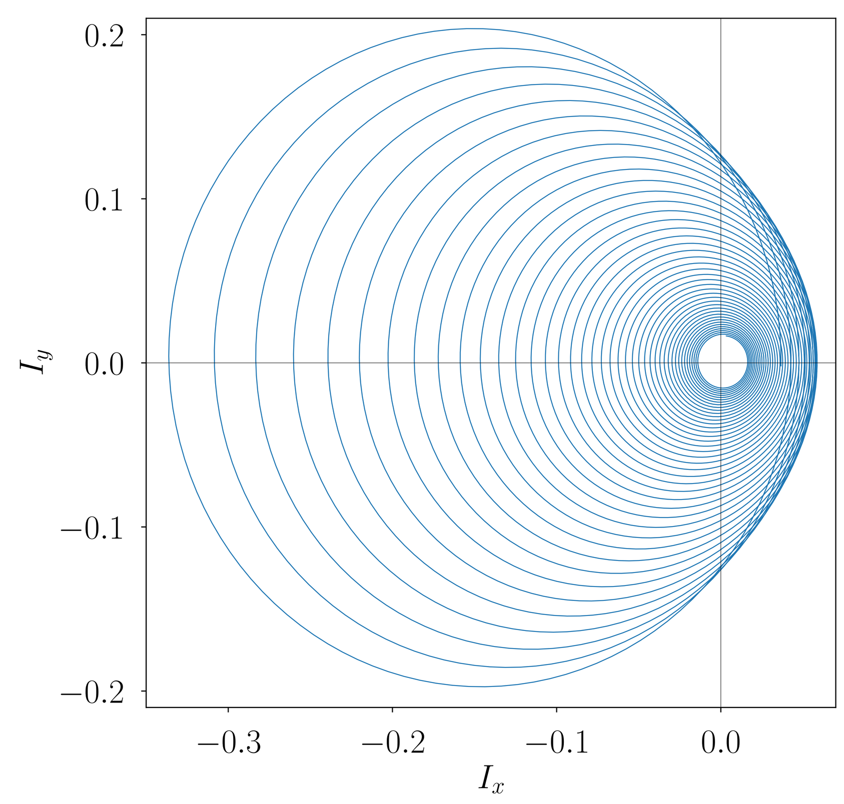
- Introduce pseudospin representation:
- $P_{\text{e}} = P_0 + P_{\text{d}}$, $P_{\text{o}} = P_\uparrow + P_\downarrow$,
  $I_{\text{x}} = \frac{1}{2}(P^0_{\text{d}} + P^{\text{d}}_0)$, $I_{\text{y}} = \frac{i}{2}(P^0_{\text{d}} - P^{\text{d}}_0)$, $I_{\text{z}} = \frac{1}{2}(P_0 - P_{\text{d}})$
- Decoupling of $P_{\text{e,o}}$ and $\mathbf{I}$.

$$\frac{d\mathbf{I}}{dt} = \mathbf{A} - \mathbf{R}\cdot\mathbf{I} + \mathbf{B}\times\mathbf{I}$$

- Pseudospin accumulation $\mathbf{A}$
- Relaxation tensor $\mathbf{R}$
- Exchange field $\mathbf{B}$

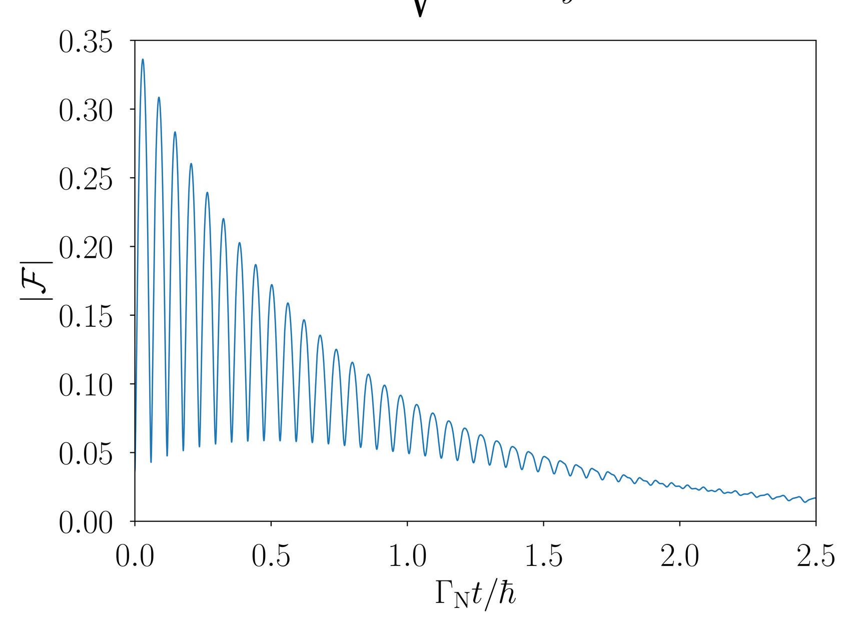
Quench

- Prepare QD in a specific state, e.g. doubly occupied



- Pseudospin dynamics in x-y-plane
- Interplay of precession and decay
- Pair amplitude

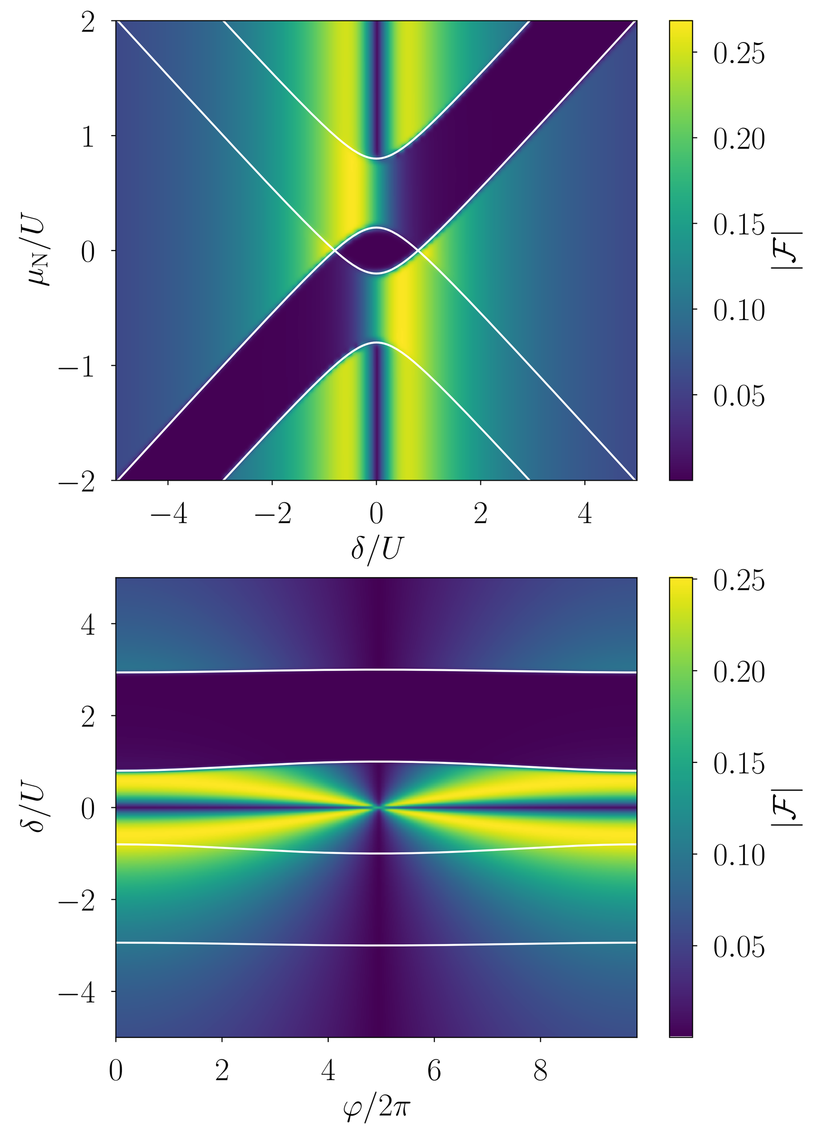
$$|\mathcal{F}| = \sqrt{I_x^2 + I_y^2}$$



- Oscillation frequency determined by Γ_{S}
- Exponential decay proportional to $1/\Gamma_{\text{N}}$
- Long time value depends on μ_{N}, δ, φ
- Visible Higgs dynamics compared to the weakly coupled case [3]

Adiabatic Driving

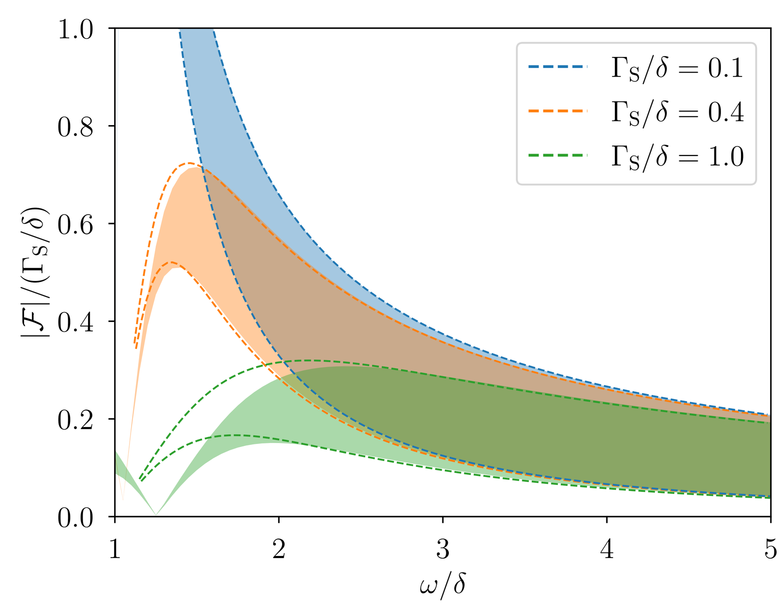
- Slow external, periodic driving of phase $\varphi(t) = \omega t$ or detuning $\delta(t) = \delta_0 + \delta_1\cos(\omega t)$
- Adiabatic limit $\omega \ll \frac{(k_{\text{B}}T)^2}{\Gamma_S}$ for phase driving and $\omega \ll \frac{(k_{\text{B}}T)^2}{\delta_1}$ for detuning



- Identify Andreev bound states in infinite-gap limit
- Higgs mode is linked to pair-amplitude oscillations

Fast Driving

- Fast external periodic driving of phase $\varphi(t) = \omega t$ in the infinite bias limit
- Fast driving limit given when $\omega \gg \Gamma_S$



- Visible oscillation amplitude to high frequencies
- QD can follow the external driving different from the weak coupling case [3]

Conclusion and Outlook

- Coherent pair-amplitude oscillations on a time scale $\hbar/\Gamma_{\text{S}}$ accompanied by an exponential decay on a time scale $\hbar/\Gamma\text{N}$ after a quench
- Overcome damping by external driving
- Adiabatic driving allows to link the dynamics to the statical properties of the system
- Pair-amplitude dynamics can follow fast external driving up to frequencies signficantly larger than the coupling strength

References

[1] Matsunaga et al., Sience **345**, 1145 (2014).
[2] Futterer et al., PRB **87**, 014509 (2013).
[3] Kamp, Sothmann, PRB **103**, 045414 (2021).
[4] Heckschen, Sothmann, PRB **105**, 045420 (2022).