Quasi-stationary states in temporal correlations for traffic systems: Cologne orbital motorway as an example

Shanshan Wang, Sebastian Gartzke, Michael Schreckenberg and Thomas Guhr

shanshan.wang@uni-due.de

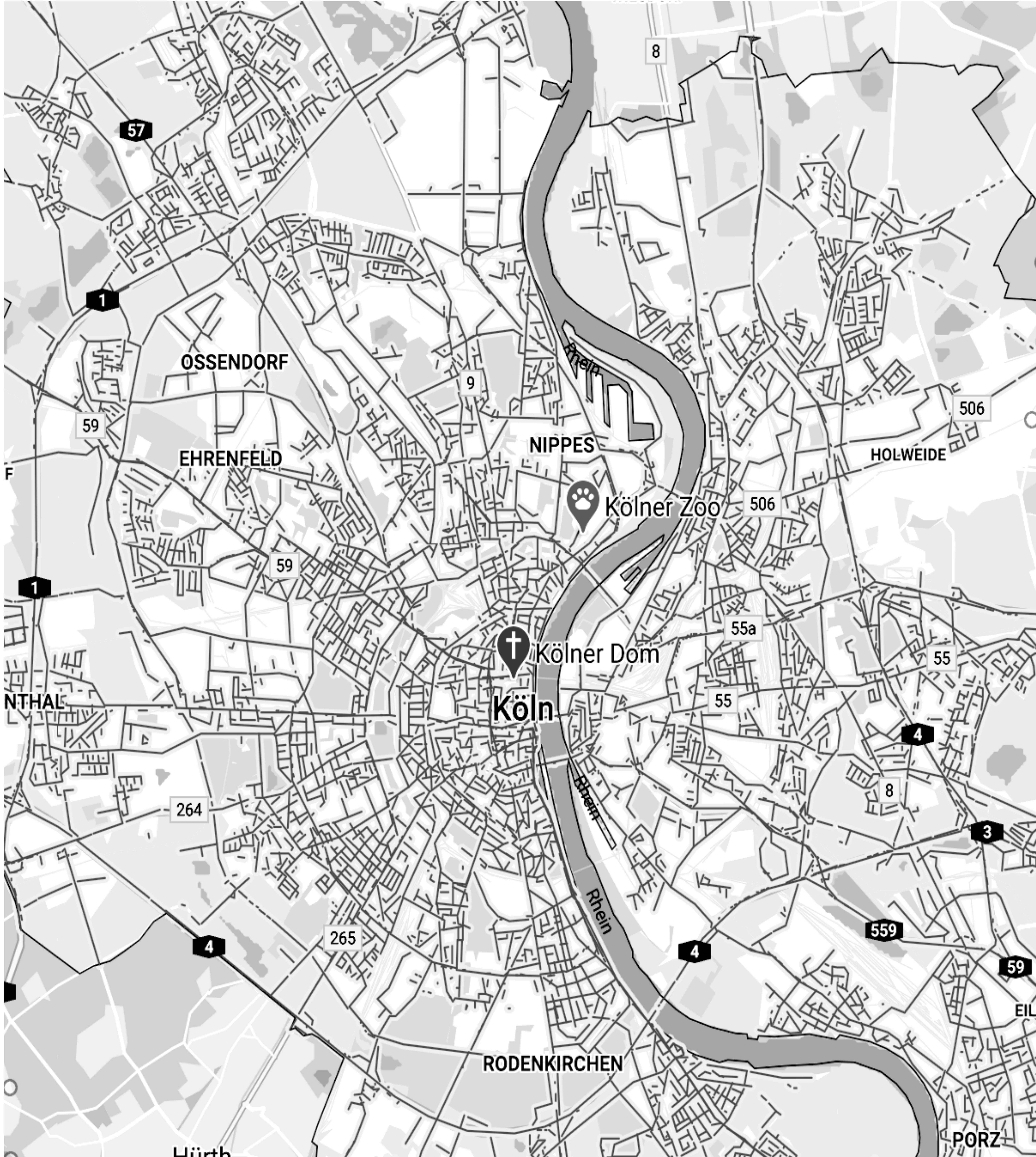
Faculty of Physics, University of Duisburg-Essen

DPG Spring Meetings, 23. 03. 2021

DPG Straßen.NRW. Funded by DFG Deutsche Forschungsgemeinschaft German Research Foundation

UNIVERSITÄT DUISBURG ESSEN Open-Minded

Contents



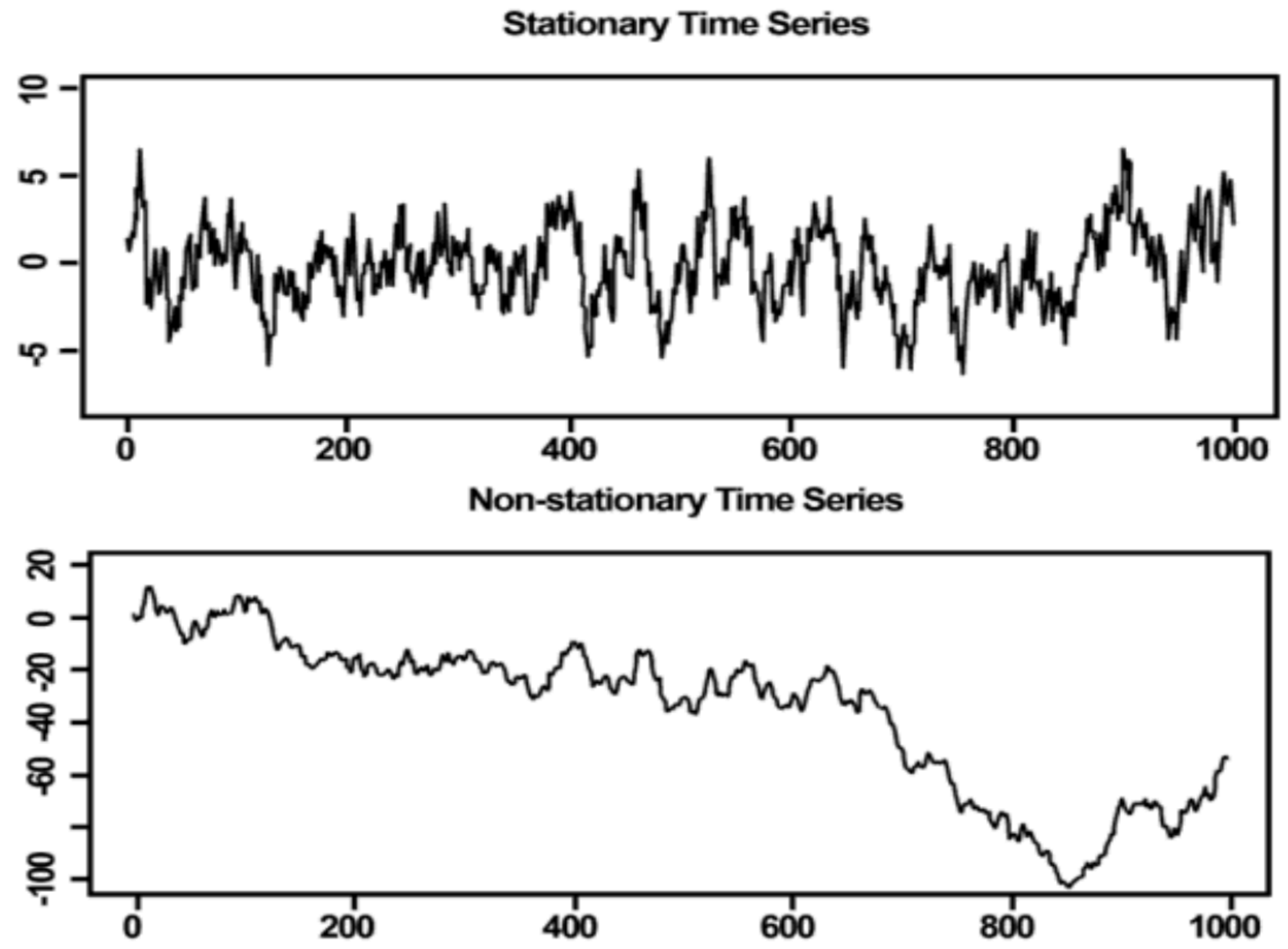
57
8
1
OSSENDORF
9
Rhein
59
EHRENFELD
NIPPES
506
HOLWEIDE
Kölner Zoo
506
59
1
55a
Kölner Dom
55
Köln
55
NTHAL
4
8
Rhein
264
3
Rhein
265
4
4
559
59
RODENKIRCHEN
PORZ

Background—non-stationary time series

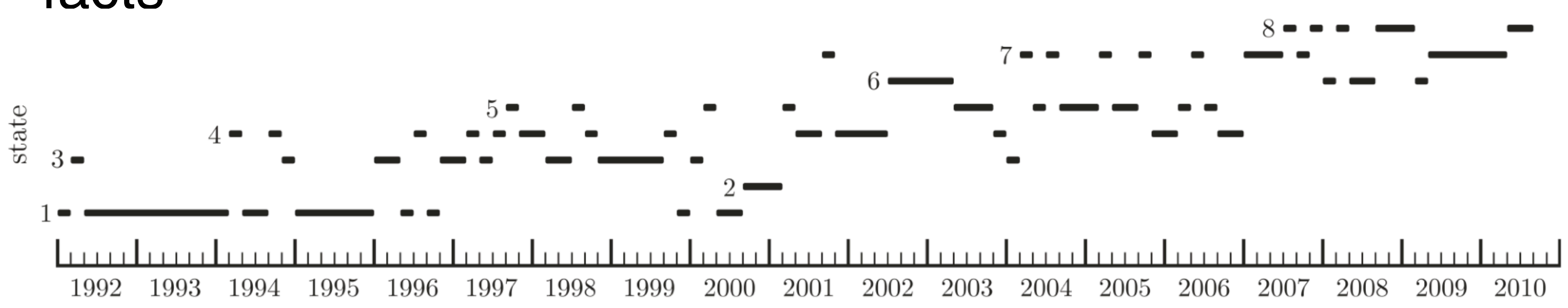
- Non-stationary time series have attracted much attention
- In non-stationary systems, parameters, e.g., mean and variance, may change over time
- Over a very short time period, non-stationary systems may be quasi-stationary

stationary → quasi-stationary → non-stationary → t

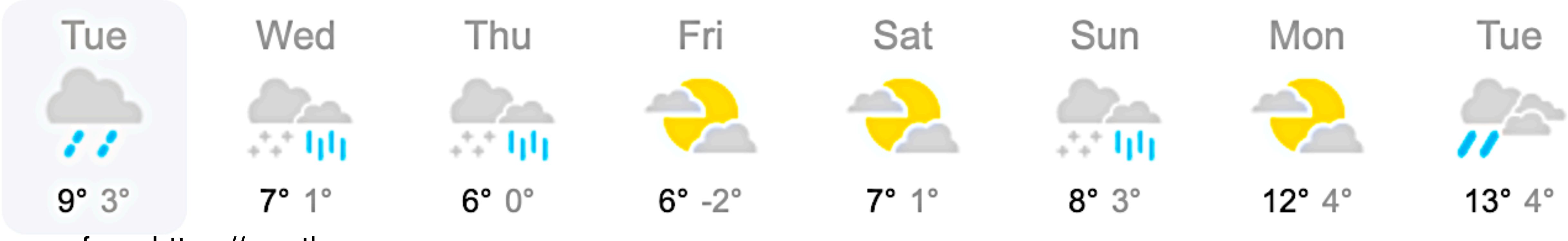
- Identifying the abrupt non-stationary transitions help us to obtain warnings before some events happen or to obtain post-event learning from a correlation analysis of known facts



Ref. Halder and Ozdemir, Packt Publishing Ltd., 2018



Ref. Münnix, Shimada, Schäfer, Leyvraz, Seligman, Guhr, and Stanley. Scientific Reports, 2:644, 2012



from https://weather.com

SCIENTIFIC REPORTS

OPEN

Identifying States of a Financial Market

Michael C. Münnix[1,2], Takashi Shimada[1,3], Rudi Schäfer[2], Francois Leyvraz[4], Thomas H. Seligman[4], Thomas Guhr[2] & H. Eugene Stanley[1]

SUBJECT AREAS:

New Journal of Physics

The open access journal at the forefront of physics

IOP Institute of Physics

PAPER

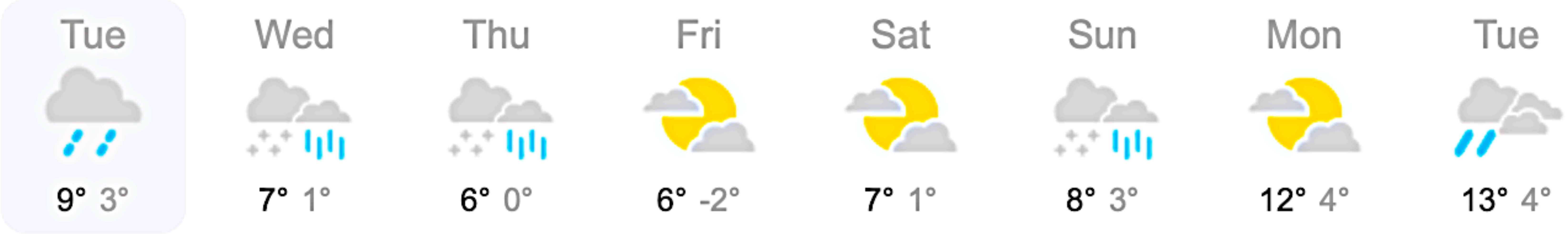
Identifying long-term precursors of financial market crashes using correlation patterns

Hirdesh K Pharasi[1,5], Kiran Sharma[2], Rakesh Chatterjee[1,3], Anirban Chakraborti[2,5], Francois Leyvraz[1,4] and Thomas H Seligman[1,4]

Financial markets

Background—motivation

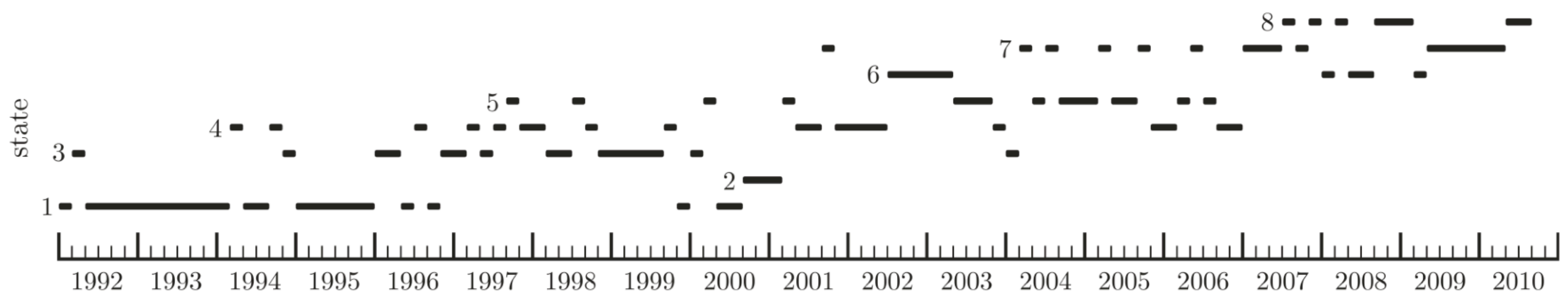
Weather



from https://weather.com

Traffic systems

state 1	state 3	state 2	state 2	state 3	state 1	state 1	state 2	→ quasi-stationary states
free	heavy congested	light congested	light congested	heavy congested	free	free	light congested	→ traffic states for one section



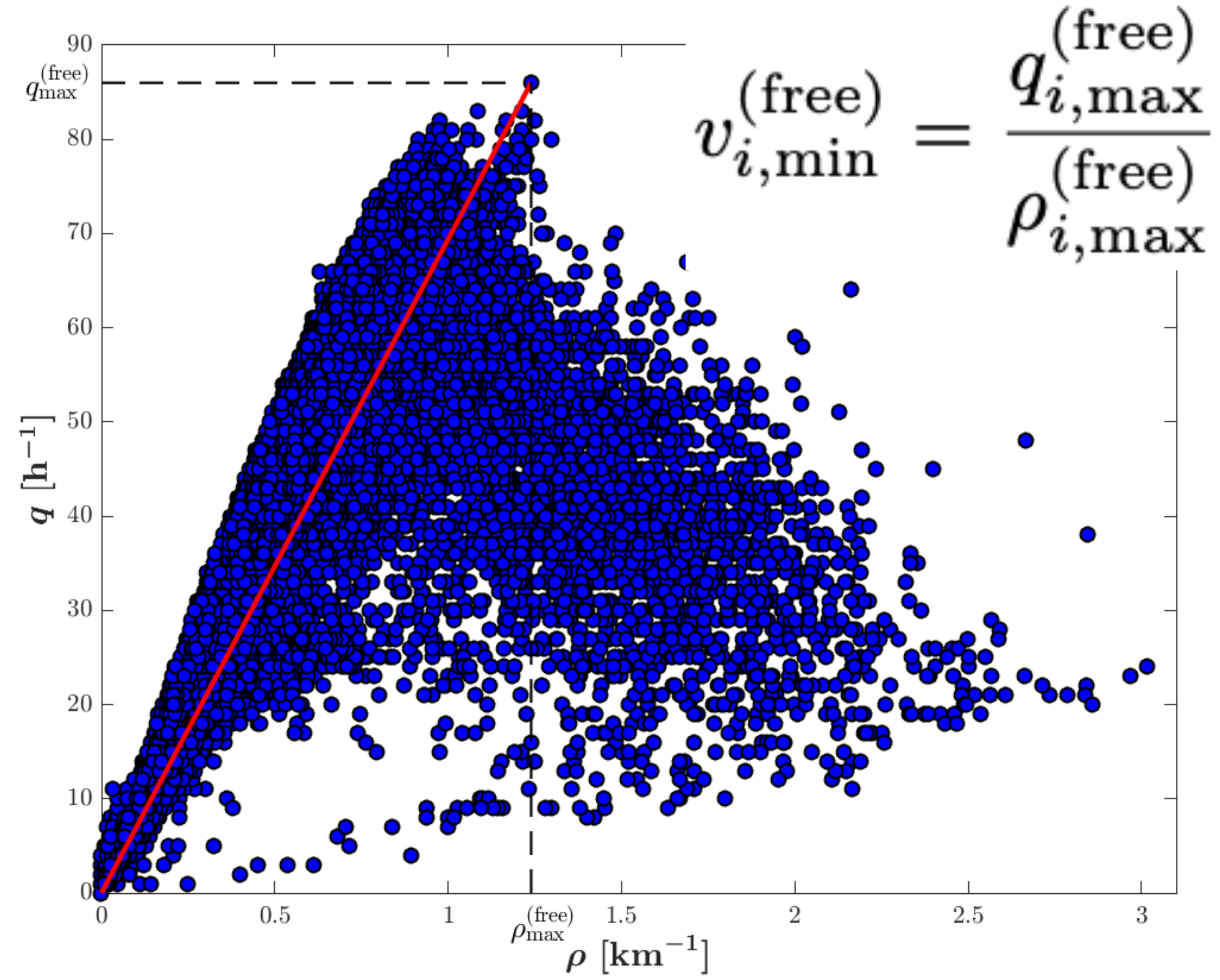
find quasi-stationary states

Ref. Münnix, Shimada, Schäfer, Leyvraz, Seligman, Guhr, and Stanley. Scientific Reports, 2:644, 2012

- as a precursor in traffic states
- for predication through models

- guide traffic planning, traffic control, traveling strategy
- improve traffic efficiency

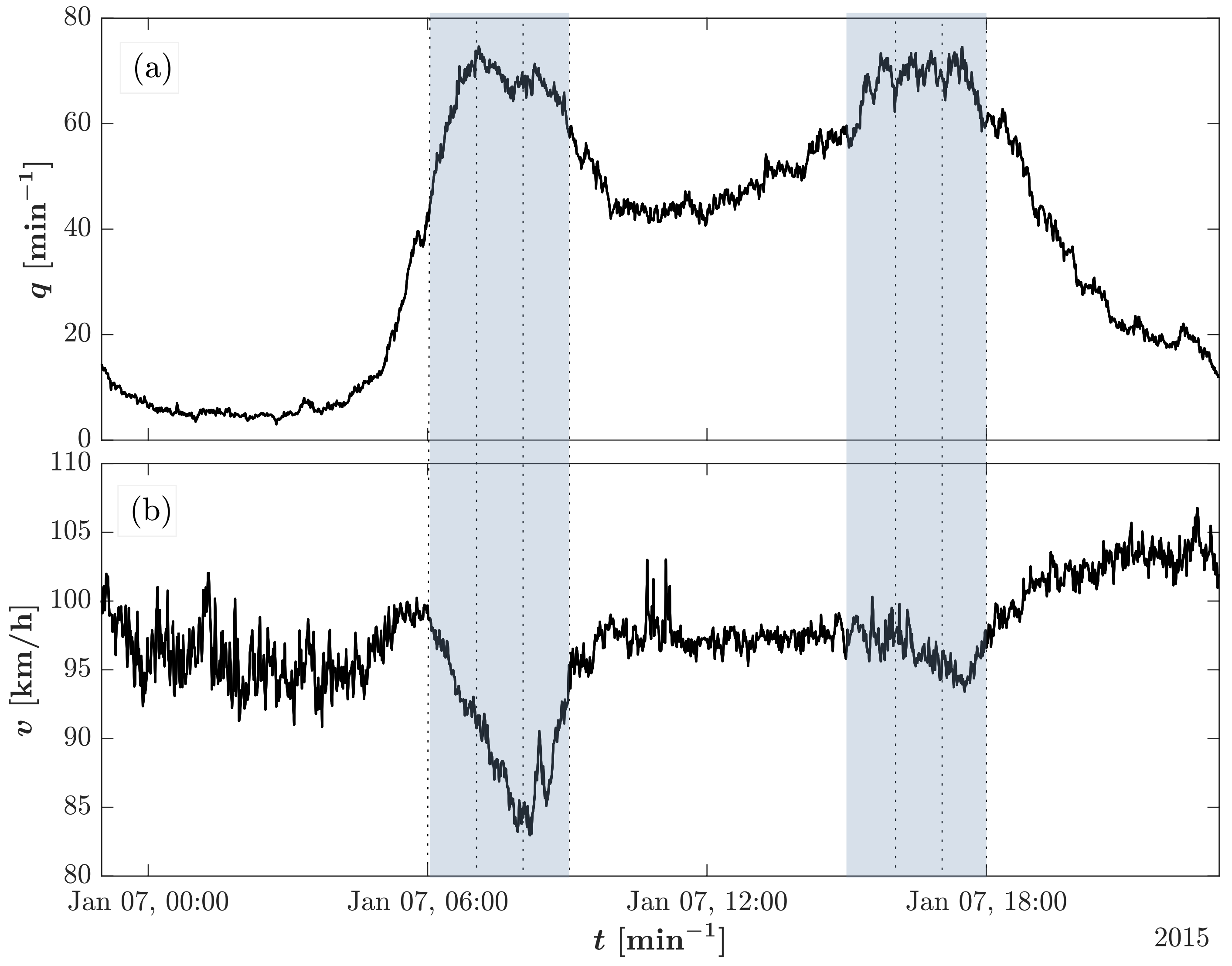
Background—traffic systems



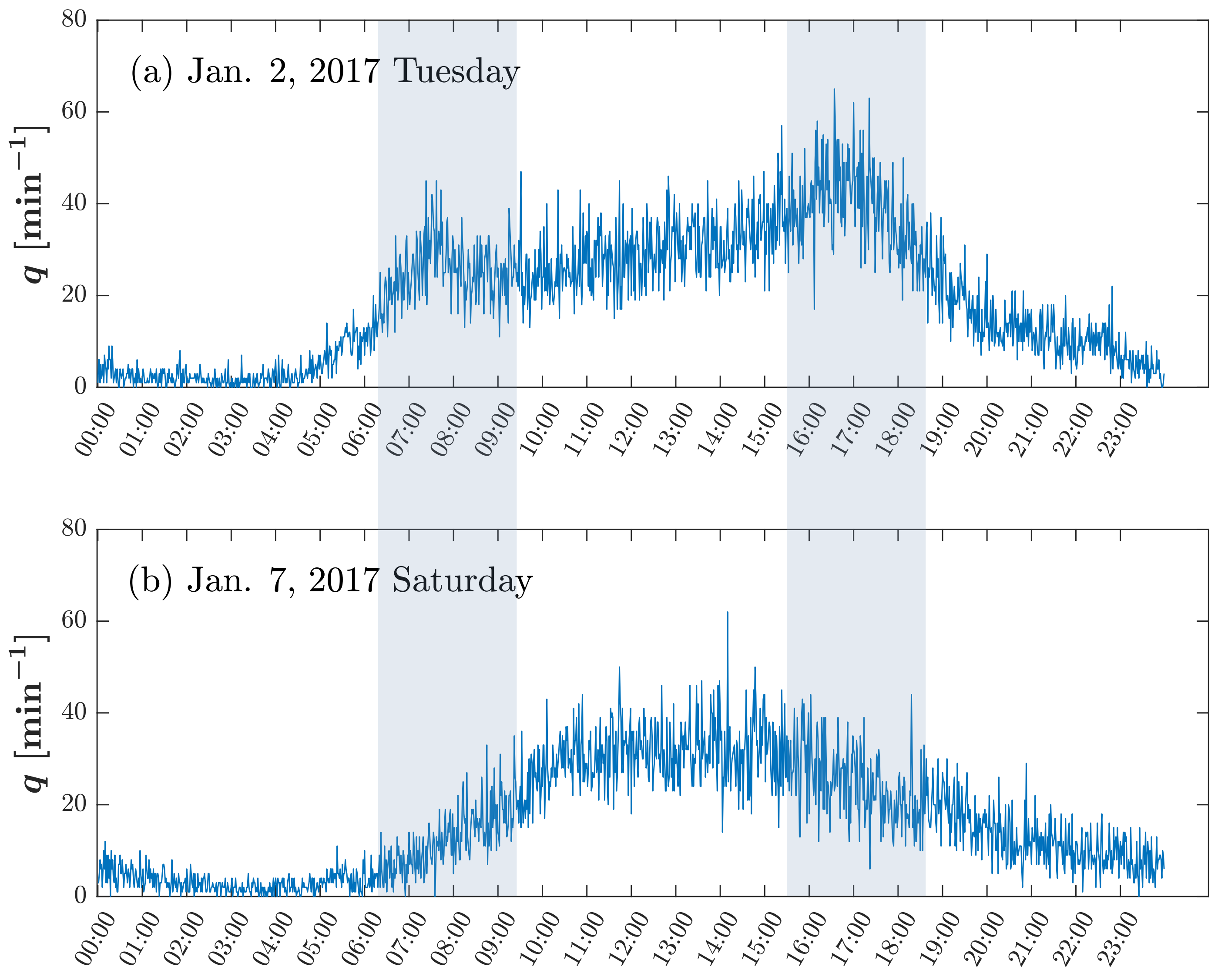
- flow: number of vehicles at unit time
- density: number of vehicles at unit length
- velocity = flow / density
- traffic states: free or congested states

- critical velocity: $v_{i,\min}^{(\text{free})}$
- $v_i \geq v_{i,\min}^{(\text{free})}$ free traffic state
- $v_i < v_{i,\min}^{(\text{free})}$ congested traffic state

Ref. Kerner, The Physics of Traffic, Springer, 2012

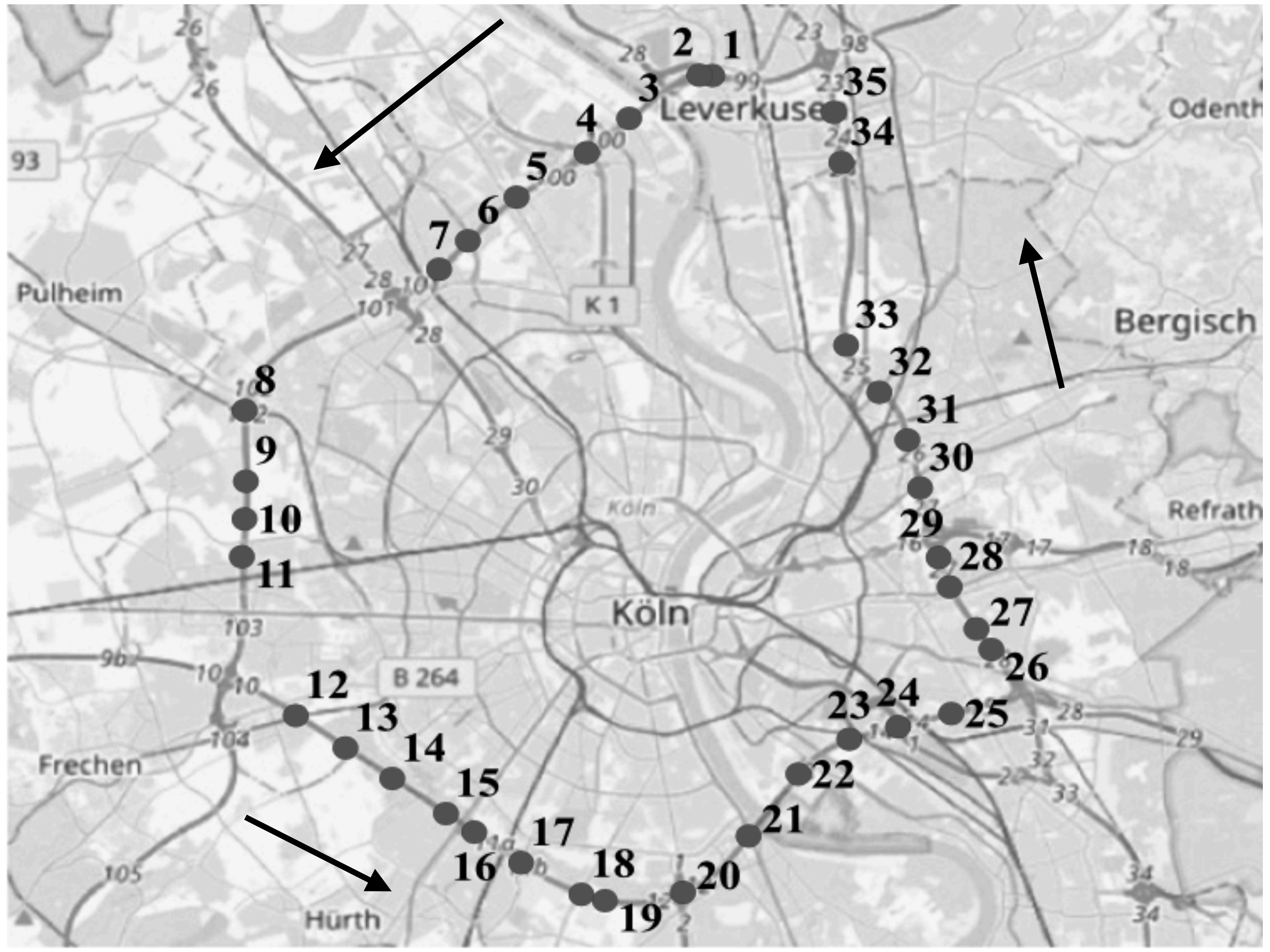


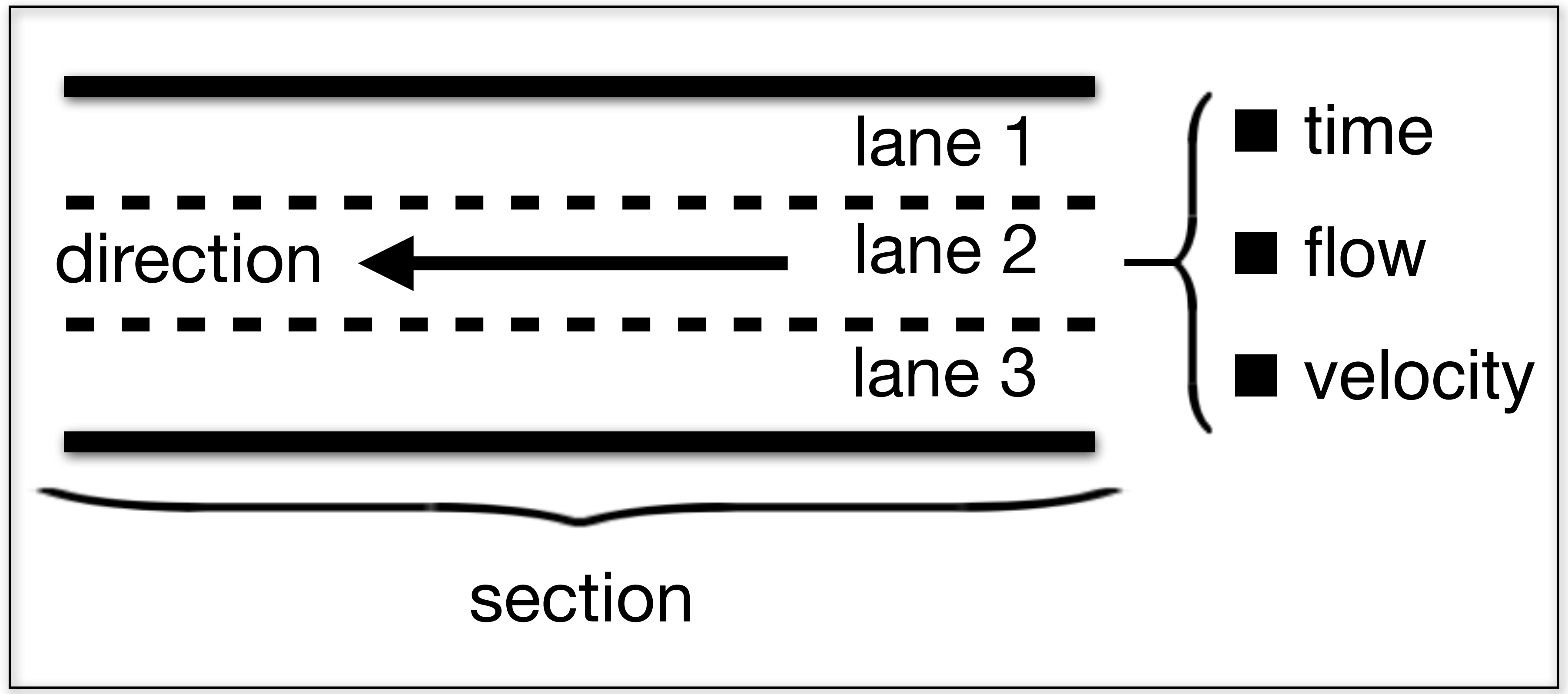
- quasi-stationary time series of traffic flow and velocity
- flow and velocity in rush hours



- different traffic behaviors on workdays and weekends

Datasets





- loop data detected at 35 sections of Cologne orbital motorway
- resolution of time: one minute
- combine data of multiple lanes to one effective lane at one minute
- combined flow (number of vehicles at unit time) in one minute

$$q_k(t) = \sum_l q_{kl}(t)$$

- density: number of vehicles at unit length

$$\rho_{kl}(t) = \frac{q_{kl}(t)}{v_{kl}(t)}$$

- combined velocity in one minute

$$v_k(t) = \frac{q_k(t)}{\sum_l \rho_{kl}(t)}$$

- combine data of one minute to 15 minutes
- Each data point is for one section in a time interval of 15 minutes

$K \times T$ data matrix, rows: sections, columns: time

$$G = \begin{bmatrix} G_1(1) & G_1(2) & \dots & G_1(T) \\ G_2(1) & G_2(2) & \dots & G_2(T) \\ \vdots & \vdots & \ddots & \vdots \\ G_K(1) & G_K(2) & \dots & G_K(T) \end{bmatrix}$$

$T = 96$

$K = 35$

normalize $G_k(t)$ to zero mean and unit variance

$$M_k(t) = \frac{G_k(t) - \langle G_k(t) \rangle_K}{\sqrt{\langle G_k(t)^2 \rangle_K - \langle G_k(t) \rangle_K^2}}$$

$K \times T$ normalized data matrix

$$M = \begin{bmatrix} M_1(1) & M_1(2) & \dots & M_1(T) \\ M_2(1) & M_2(2) & \dots & M_2(T) \\ \vdots & \vdots & \ddots & \vdots \\ M_K(1) & M_K(2) & \dots & M_K(T) \end{bmatrix}$$

$T \times T$ temporal correlation matrix

$$D = \frac{1}{K} M^\dagger M$$

normalize $G_k(t)$ to zero mean

$$A_k(t) = G_k(t) - \langle G_k(t) \rangle_K$$

$T \times T$ temporal covariance matrix

$$\Sigma = \frac{1}{K} A^\dagger A$$

spectrum decomposition

$$\Sigma = \sum_{t=1}^{T} \Theta_t V(t) V^\dagger(t)$$

reduced-rank covariance matrix

$$\tilde{\Sigma} = \sum_{t=a}^{b} \Theta_t V(t) V^\dagger(t)$$

diagonal matrix of the square roots of the diagonal elements in $\tilde{\Sigma}$ $\quad \tilde{\sigma} = \mathrm{diag}\,(\tilde{\sigma}_1, \dots, \tilde{\sigma}_T)$

reduced-rank correlation matrix $\quad \tilde{D} = \tilde{\sigma}^{-1} \tilde{\Sigma} \tilde{\sigma}^{-1}$

Correlation Matrices—decomposition of matrices

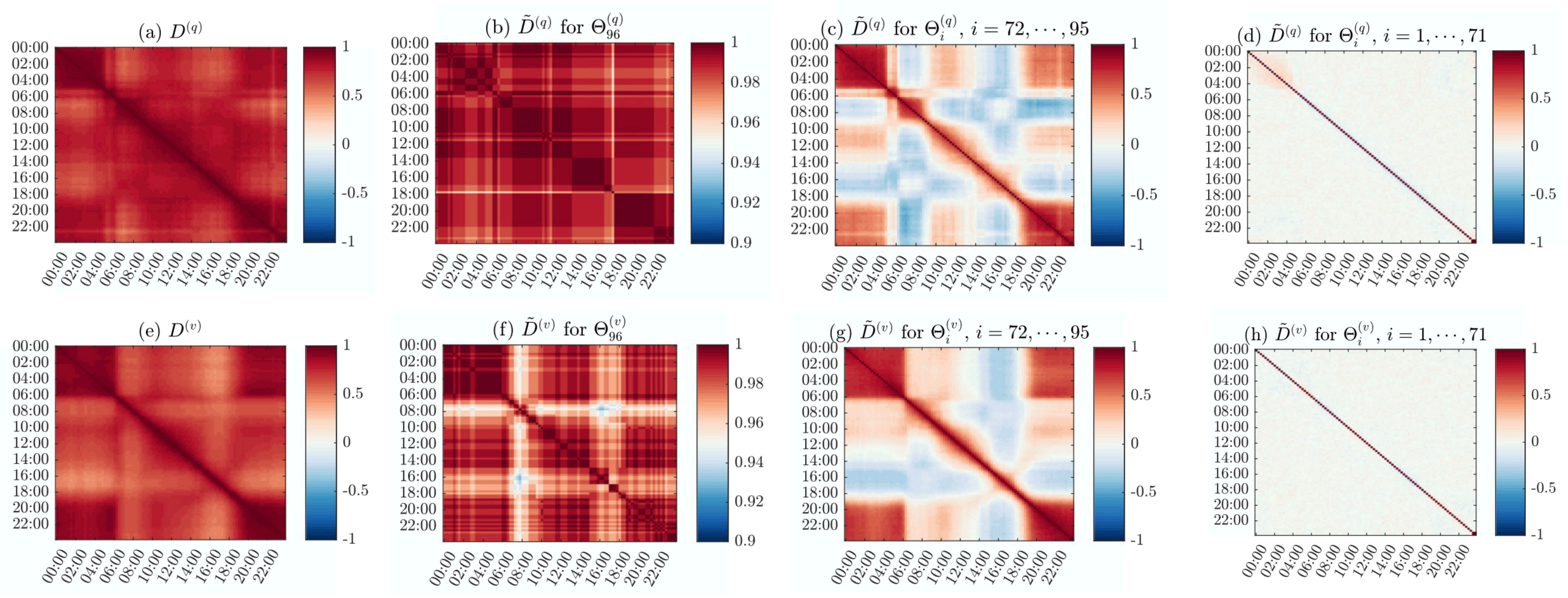
$$D = \frac{1}{K} M^\dagger M$$

$$\tilde{D} = \tilde{\sigma}^{-1} \tilde{\Sigma} \tilde{\sigma}^{-1}$$

all eigenvalues | largest eigenvalue | middle eigenvalues | small eigenvalues

flow

velocity



What kind of information each reduced-correlation matrix contains?

Correlation Matrices—roles of eigenvalues

reduced-rank data matrix

$$\tilde{A} = \sum_{t=a}^{b} S_t U(t) V^{\dagger}(t)$$

reduced-rank covariance matrix can be derived from reduced-rank data matrix

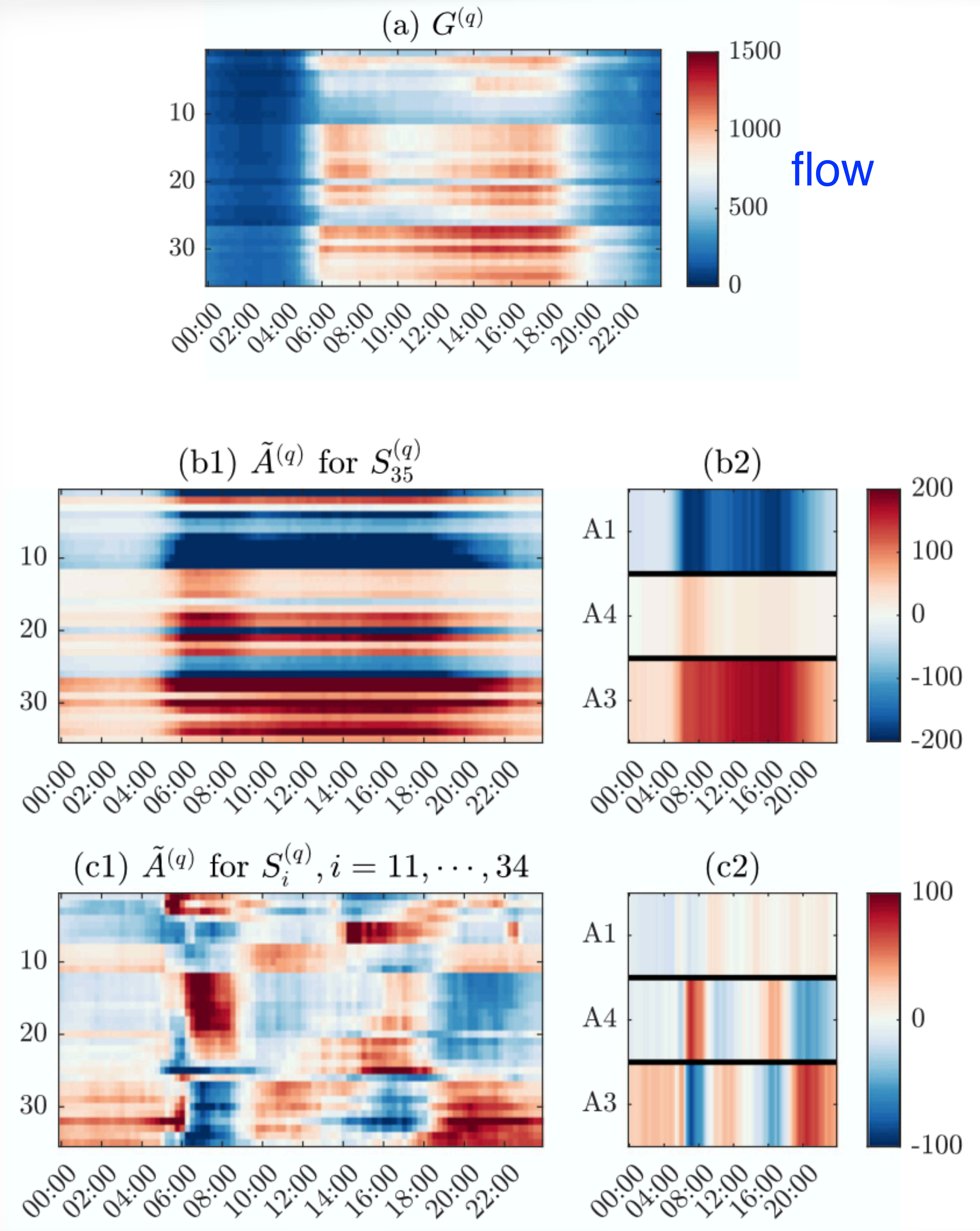
$$\tilde{\Sigma} = \sum_{t=a}^{b} \Theta_t V(t) V^{\dagger}(t) = \frac{1}{K} \tilde{A}^{\dagger} \tilde{A}$$

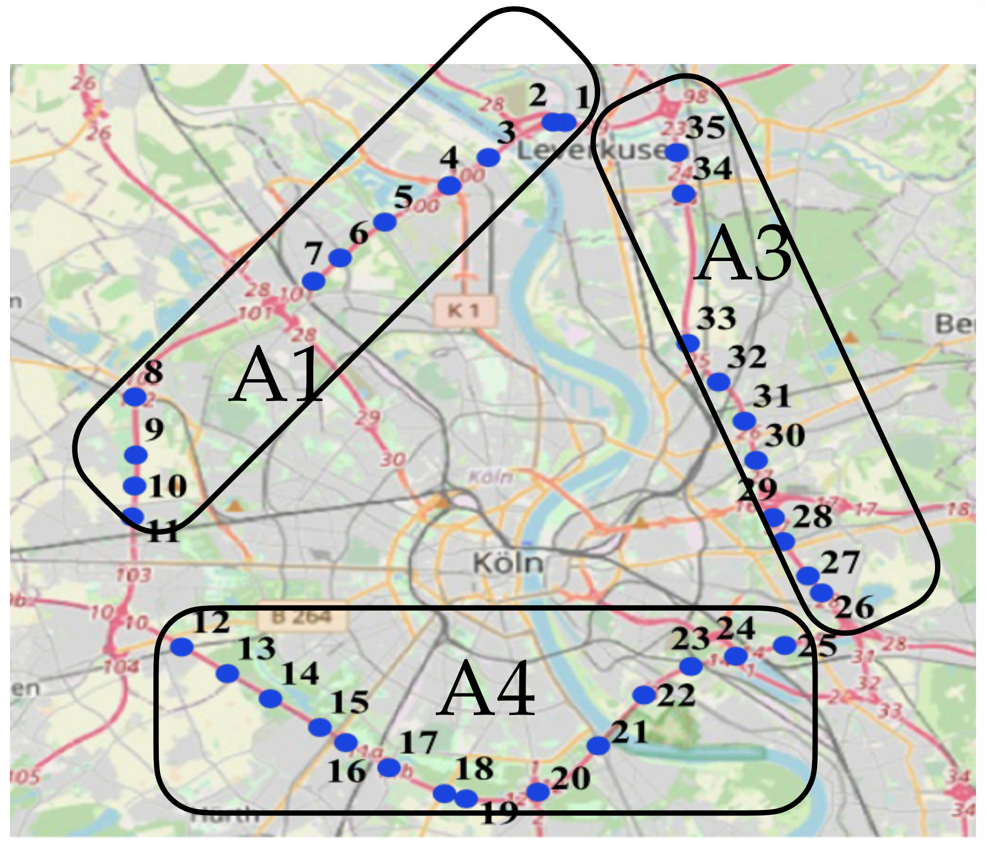
where each eigenvalue corresponds to a singular value

$$S_t = \sqrt{K \Theta_t}$$

information of reduced-correlation matrix can be trace back to the information of reduced-rank data matrix

$$\tilde{D} \longrightarrow \tilde{\Sigma} \longrightarrow \tilde{A}$$

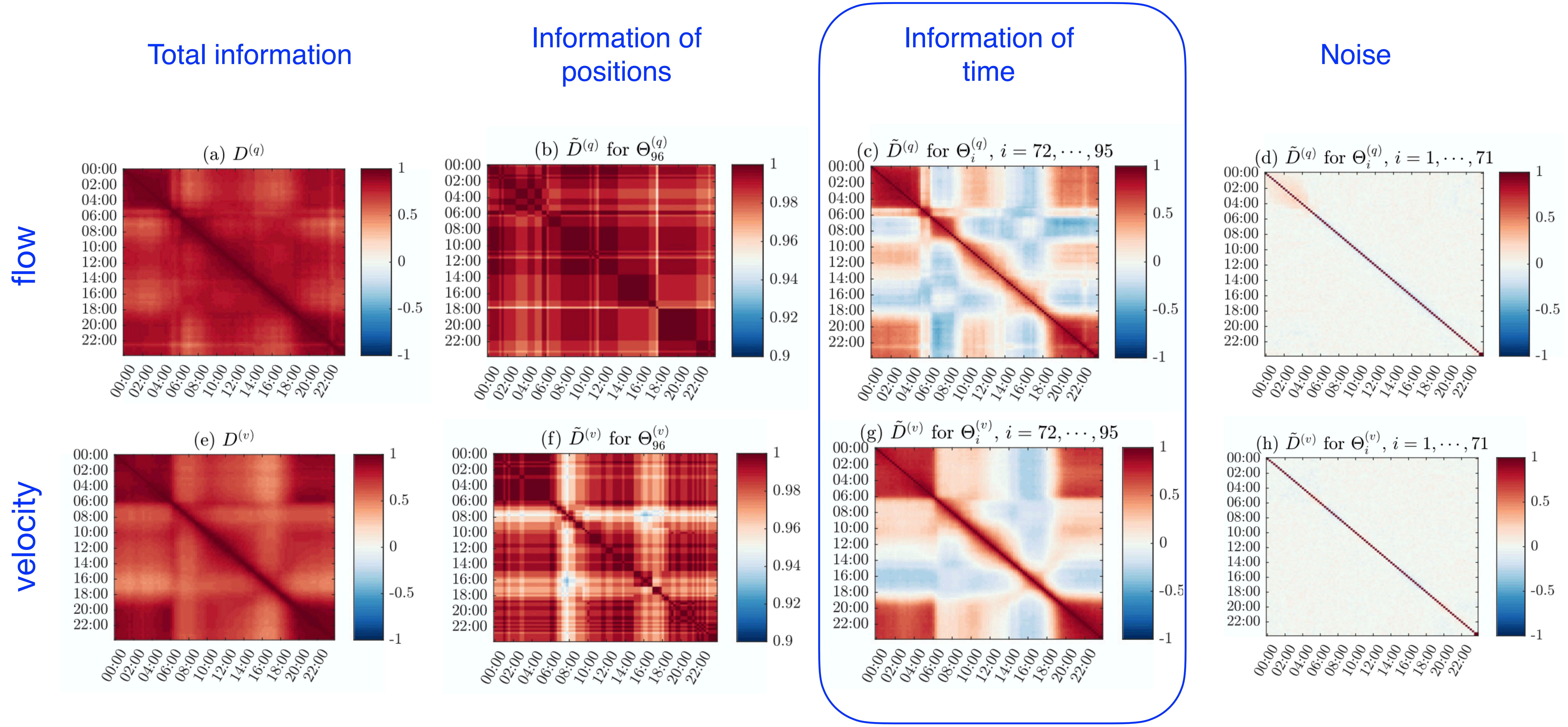




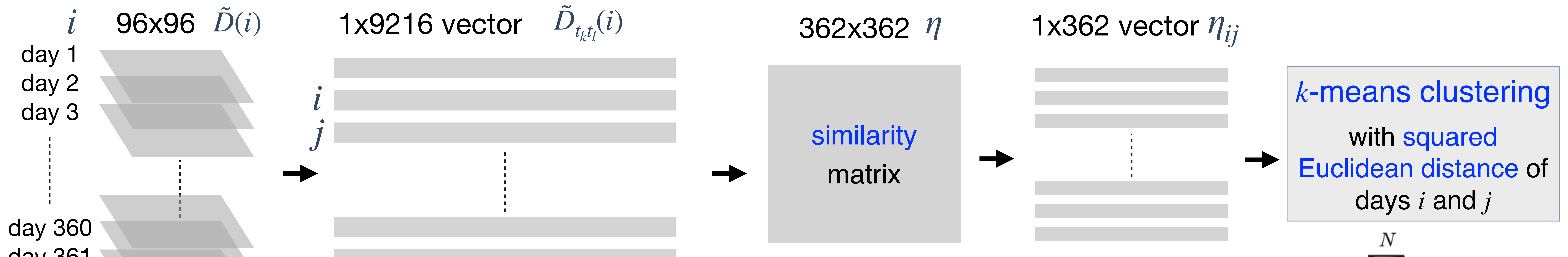
for largest singular value
Information of positions dominates

for middle singular values
Information of time is revealed

Correlation Matrices—decomposition of matrices



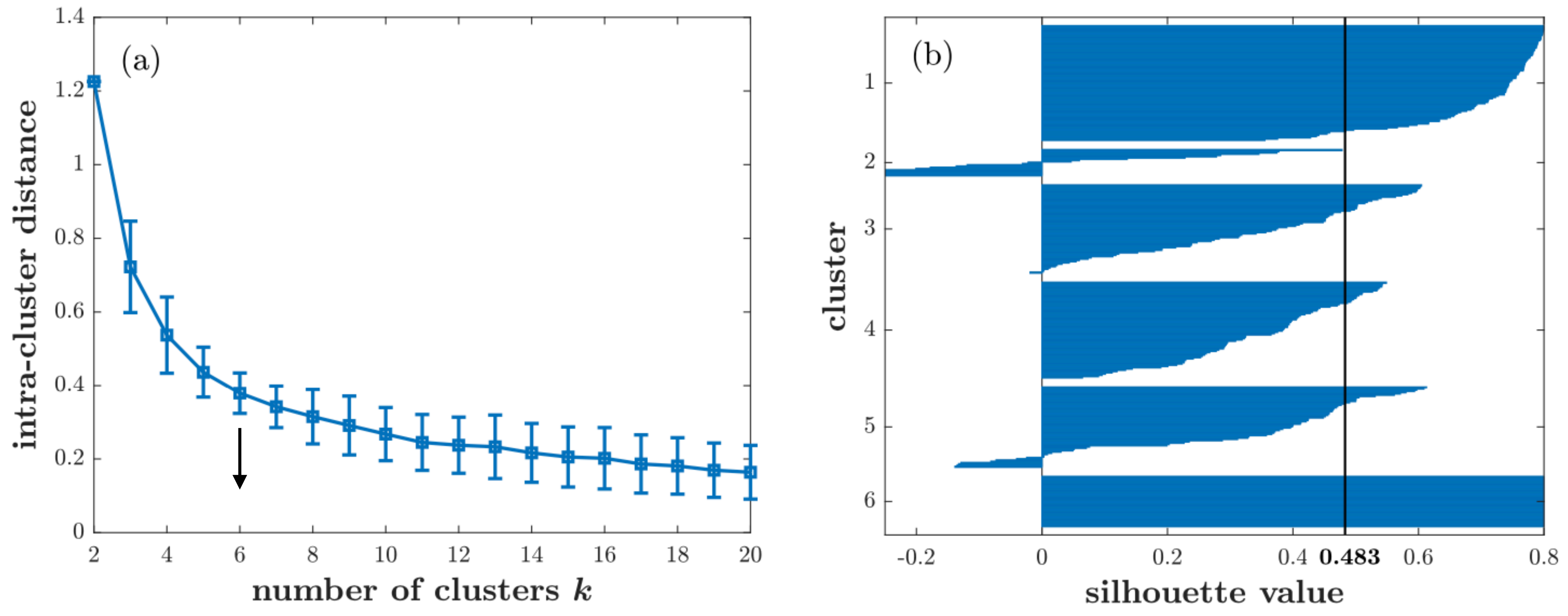
Clustering—method and validation



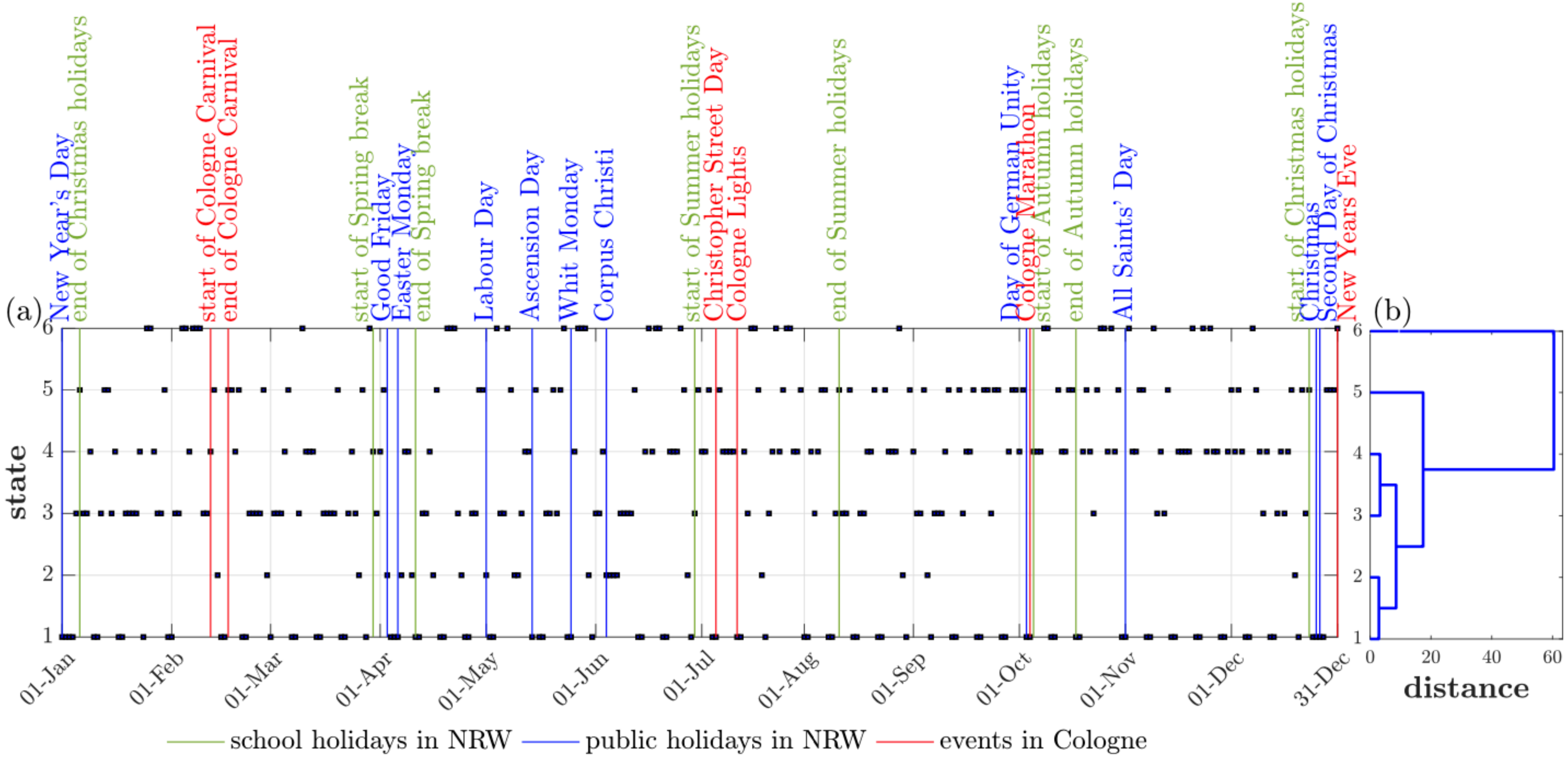
$$\eta_{ij} = \left\langle \left| \tilde{D}_{t_k t_l}(i) - \tilde{D}_{t_k t_l}(j) \right| \right\rangle_{t_k t_l}$$

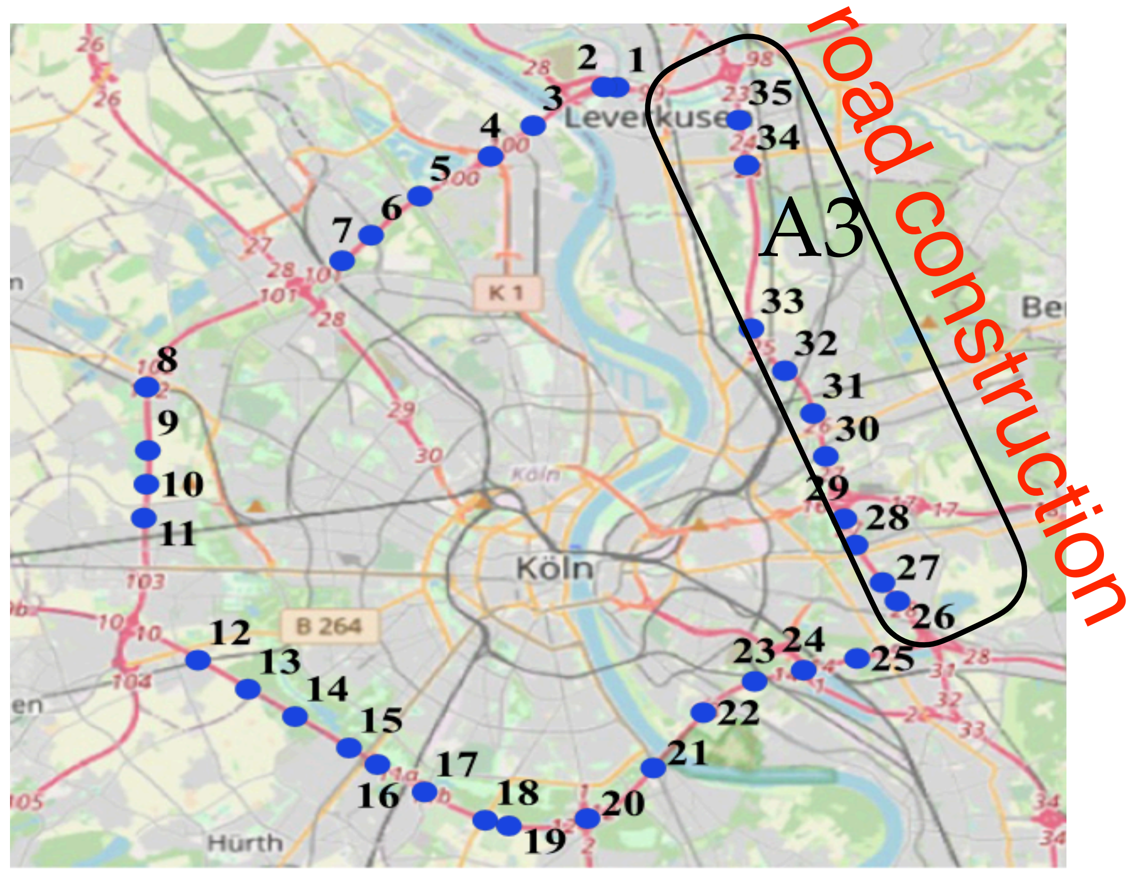
$$d_{ij} = \sum_{n=1}^{N} (\eta_{in} - \eta_{jn})^2$$

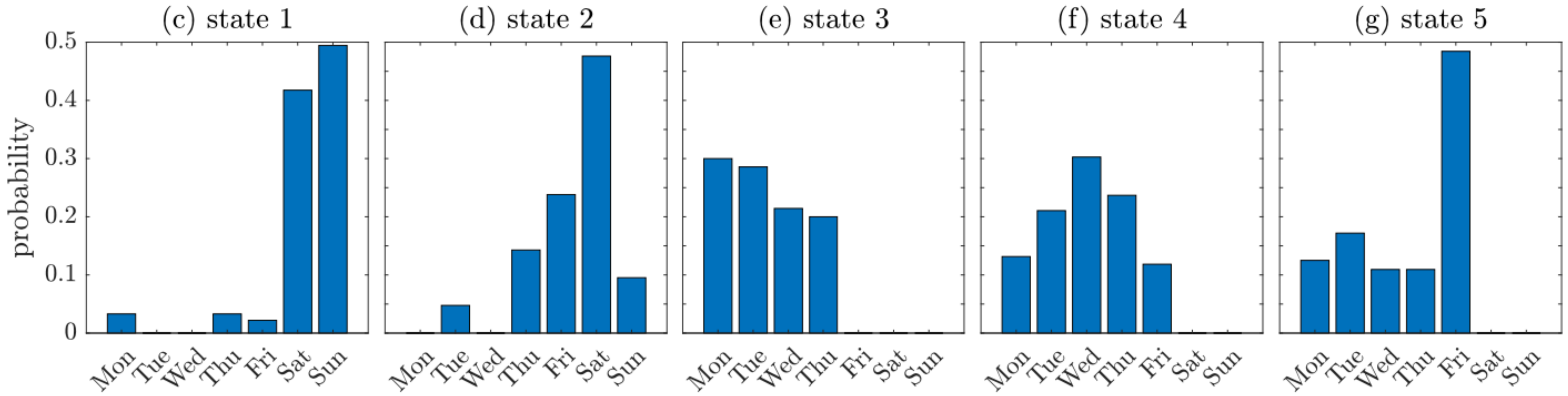
- determine number of clusters:
  - given a k, run 500 times of k-means clustering
  - obtain 500 averaged intra-cluster distances
  - obtain a mean value and a standard deviation
  - find the k corresponding to the minimal standard deviation
- with the best k , perform k-means clustering
- validate the consistency within clusters by silhouette values



Quasi-stationary states—evolution of states with time





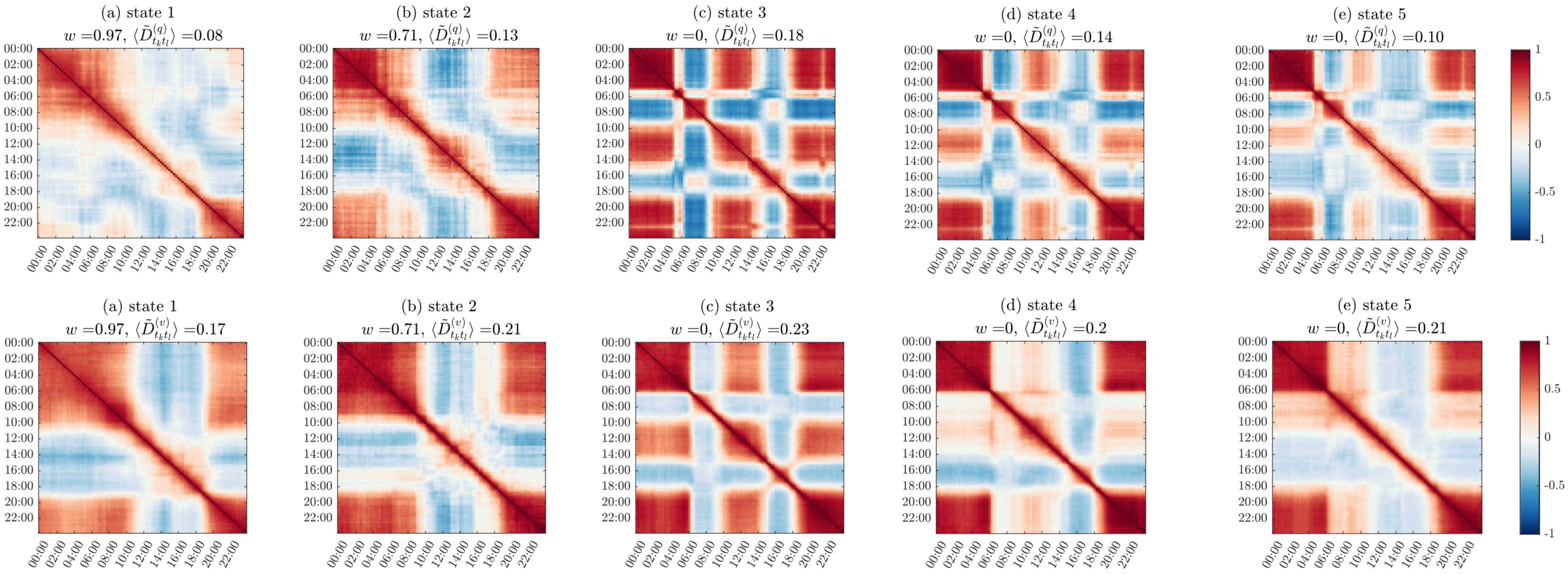


- state 1: almost for holidays
- state 2: both workdays and holidays
- states 3,4,5: only for workdays

Quasi-stationary states—correlation structures

holiday state | mixed state | workday state | workday state | workday state

flow / velocity



- strongly positive correlation between two time points:
  a free (congested) flow at a time is more likely to be followed by a free (congested) flow at another time
- strongly negative correlation between two time points:
  a free (congested) flow at a time is more likely to be followed by a congested (free) flow at another time

Quasi-stationary states—Mapping five states onto traffic states

critical velocity

$$v_i^{(0)}(t) = \frac{q_{i,\max}^{(\text{free})}(t)}{\rho_{i,\max}^{(\text{free})}(t)}$$

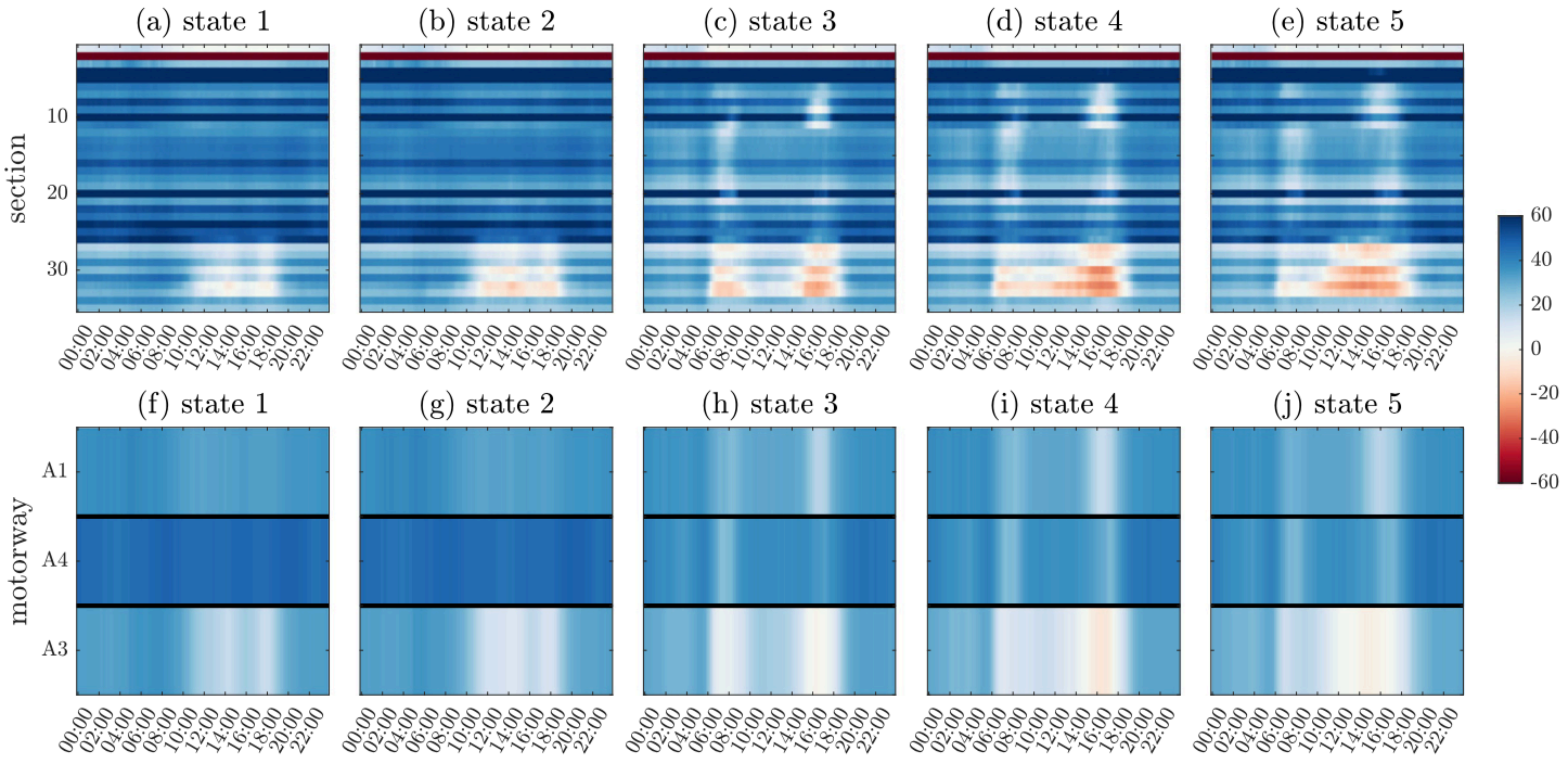
traffic state quantified by

$$\Delta v_i(t) = v_i(t) - v_i^{(0)}(t)$$

$\Delta v_i(t) \geq 0$ free state

$\Delta v_i(t) < 0$ congested state

holiday state | mixed state | workday state | workday state | workday state



traffic states in time and space

Conclusions

- Found distinct structural features depending on time, i.e., the rich non-Markovian features of traffic, in the reduced-rank correlation matrix of traffic flows

- Identified five quasi-stationary states by k-means clustering

- The five states present three types: the holiday state (state 1), the workday states (states 3–5) and a mixed state of holidays and workdays (state 2)

- Revealed free or congested states in both space and time by mapping the five quasi-stationary states onto traffic states

- Our study provides a proof of concept and a basis for further study in traffic systems.

Journal of Statistical Mechanics: Theory and Experiment
An IOP and SISSA journal

PAPER: Interdisciplinary statistical mechanics

Quasi-stationary states in temporal correlations for traffic systems: Cologne orbital motorway as an example

Shanshan Wang*, Sebastian Gartzke, Michael Schreckenberg and Thomas Guhr

Fakultät für Physik, Universität Duisburg-Essen, Lotharstraße 1, 47048 Duisburg, Germany

Wang, Gartzke, Schreckenberg and Guhr, J. Stat. Mech. **2020** 103404 (2020)

DPG Straßen.NRW. Funded by DFG Deutsche Forschungsgemeinschaft German Research Foundation

UNIVERSITÄT DUISBURG ESSEN Open-Minded