Price Cross-responses in Correlated Financial Markets

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Nov. 18th, 2016

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How to buy and sell stocks in the financial market?

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impatient traders ➔ market orders
patient traders ➔ limit orders

- best ask and best bid
- spread
- midpoint price

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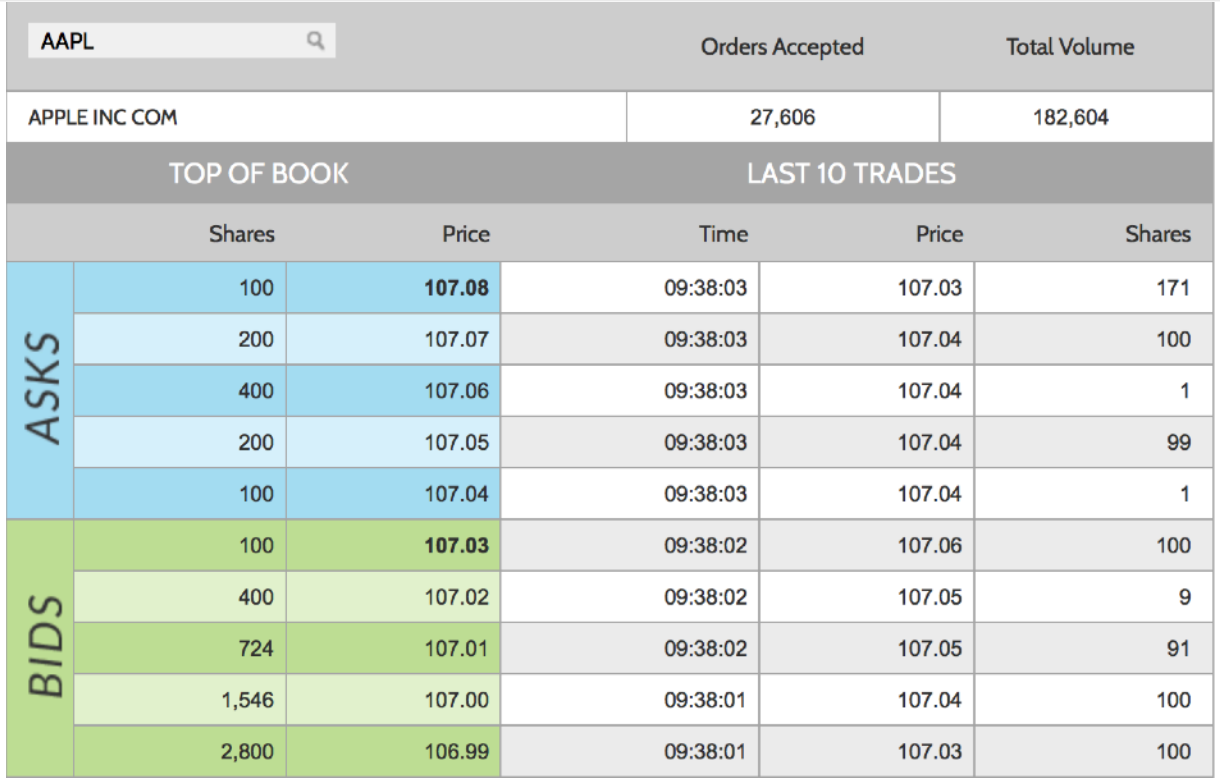

AAPL	Orders Accepted	Total Volume
APPLE INC COM	27,606	182,604

TOP OF BOOK			LAST 10 TRADES		
	Shares	Price	Time	Price	Shares
ASKS	100	**107.08**	09:38:03	107.03	171
	200	107.07	09:38:03	107.04	100
	400	107.06	09:38:03	107.04	1
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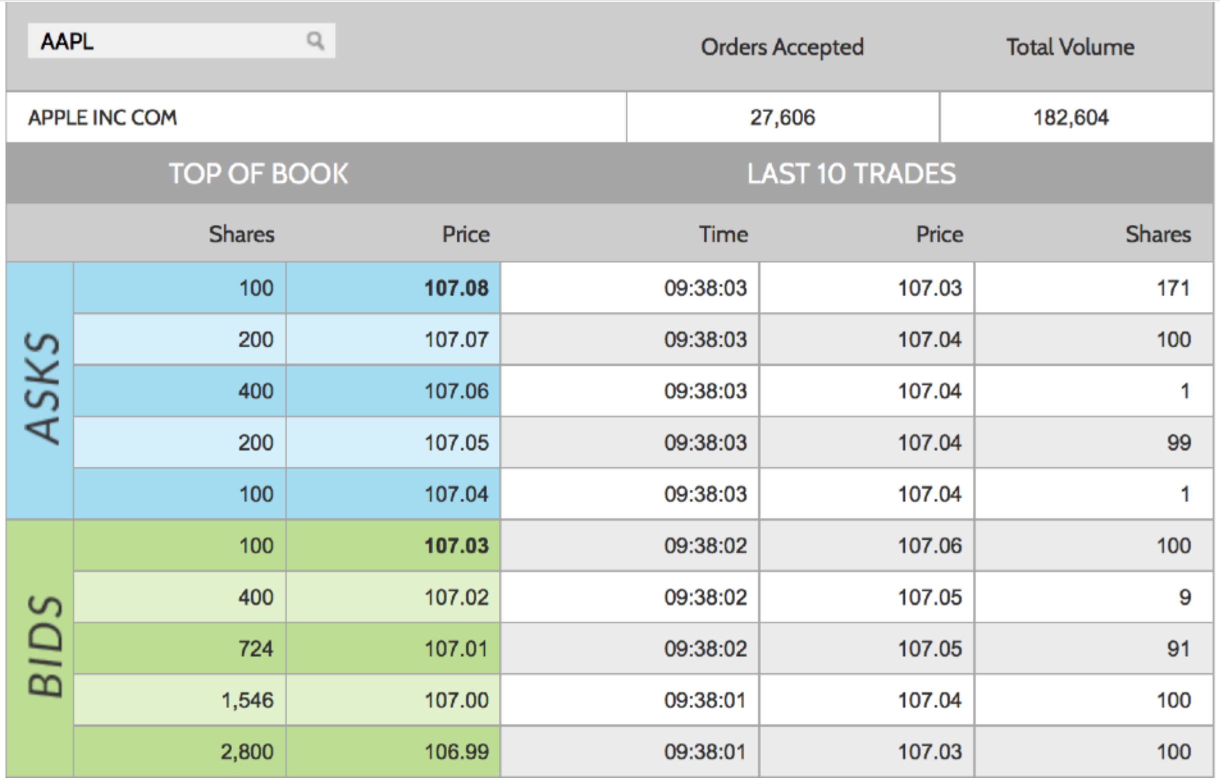

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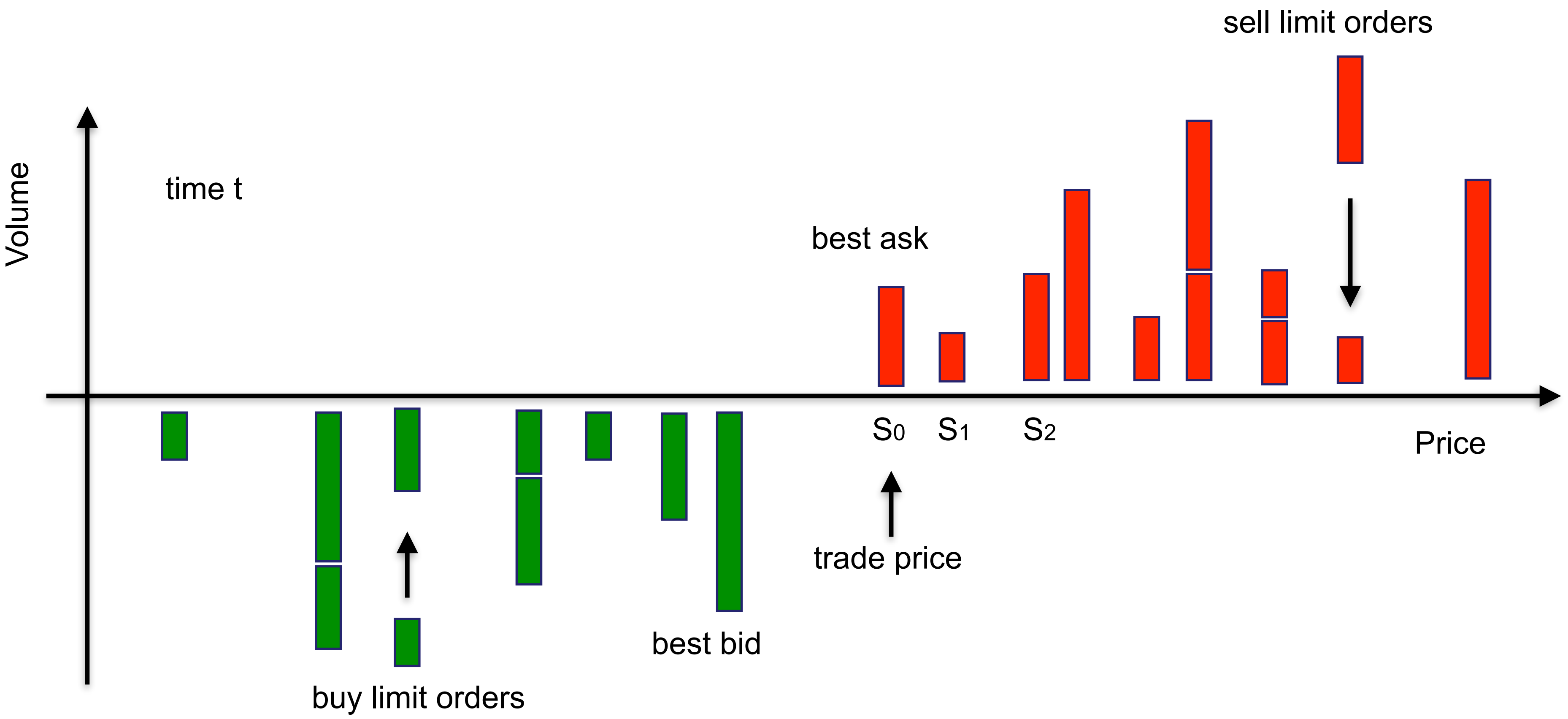

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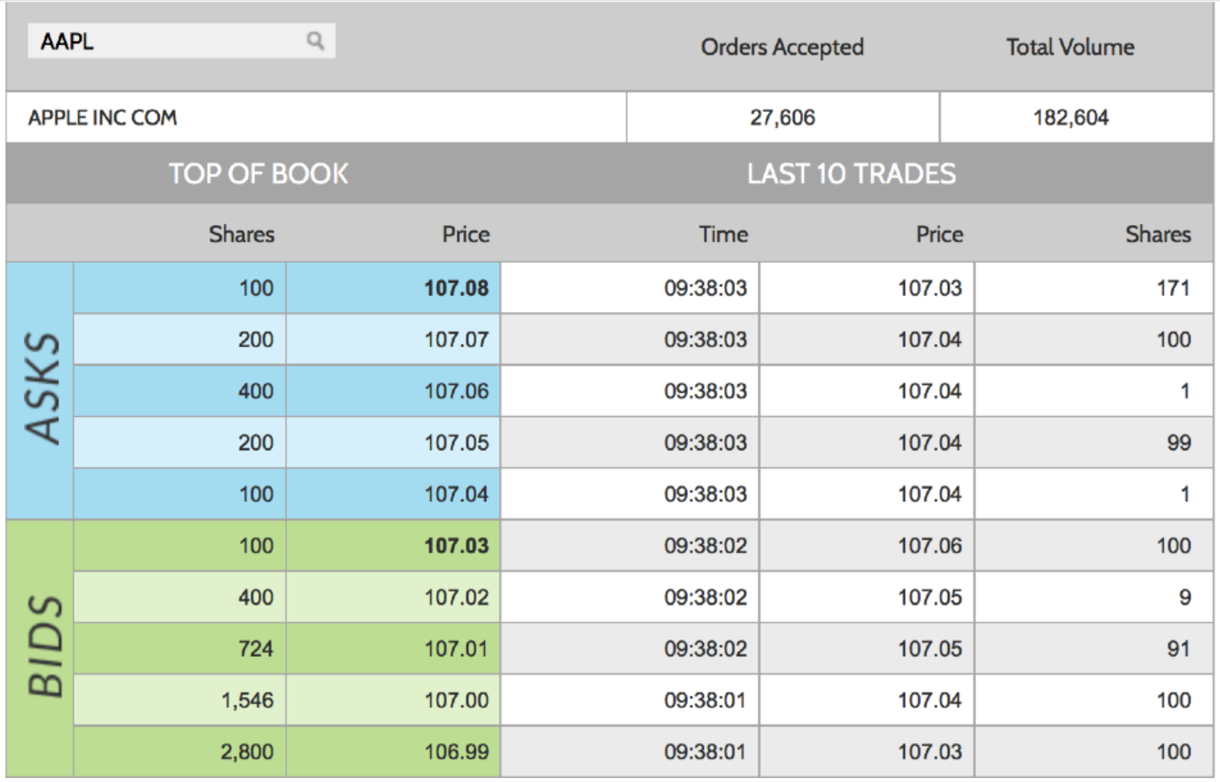

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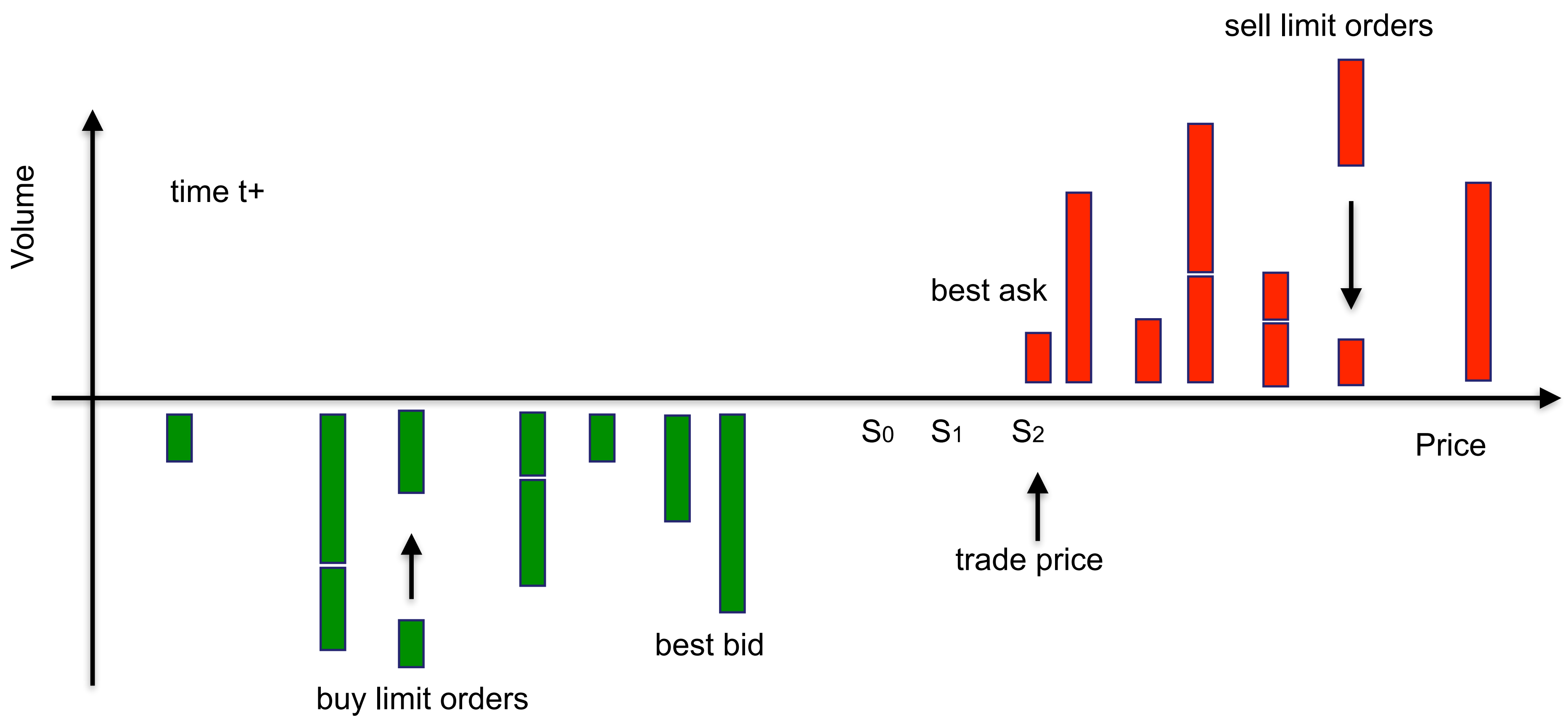

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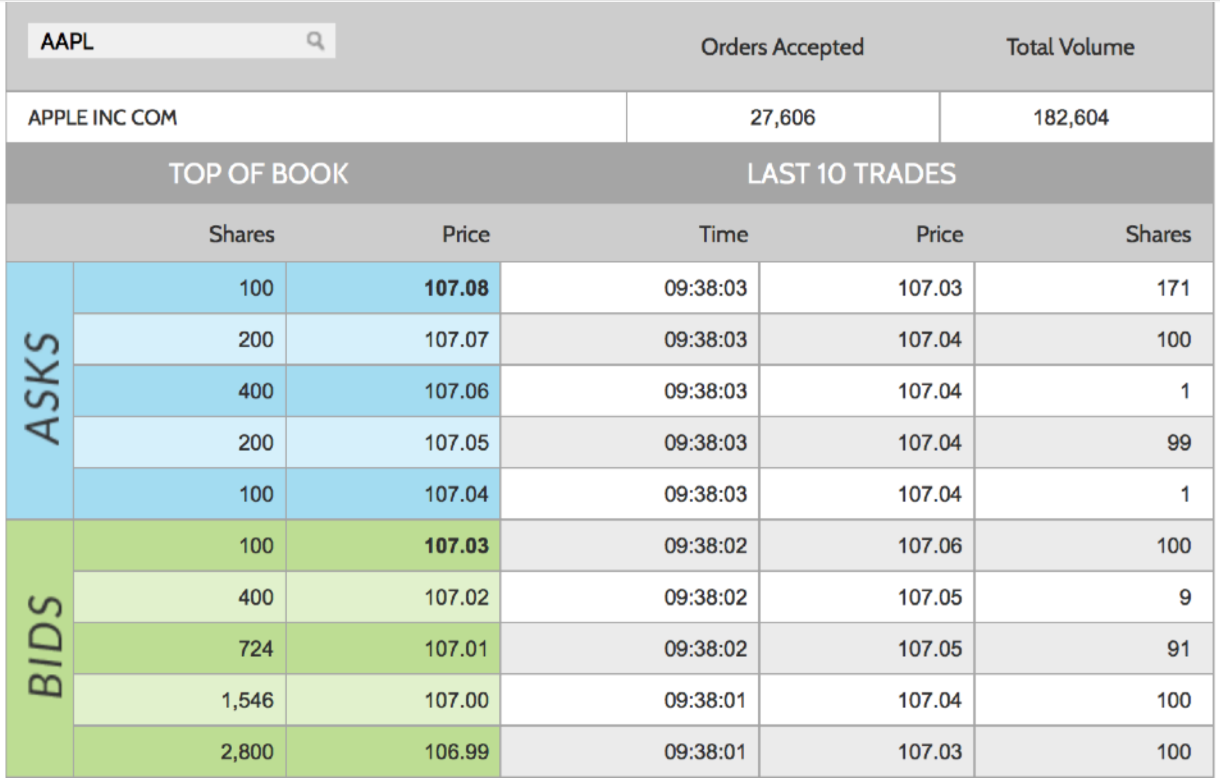

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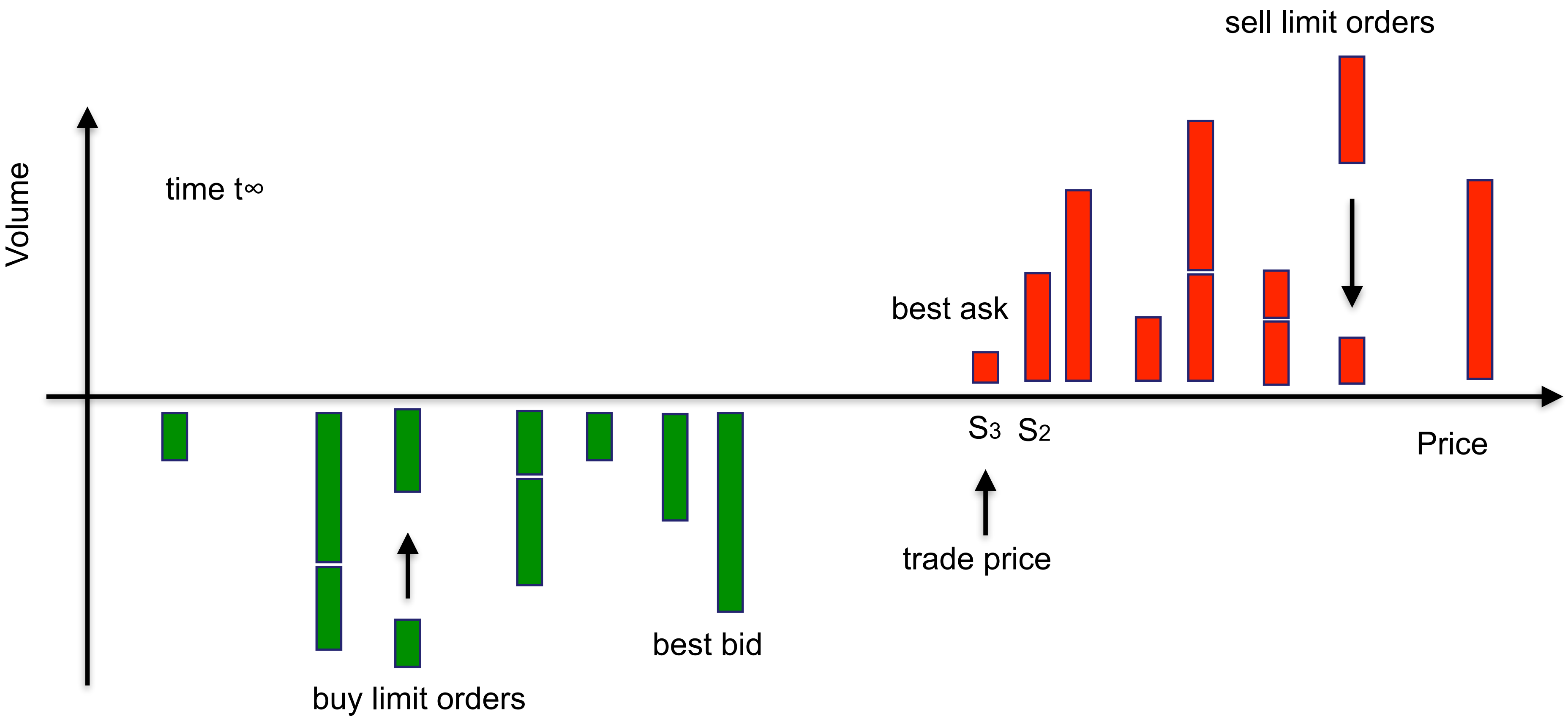

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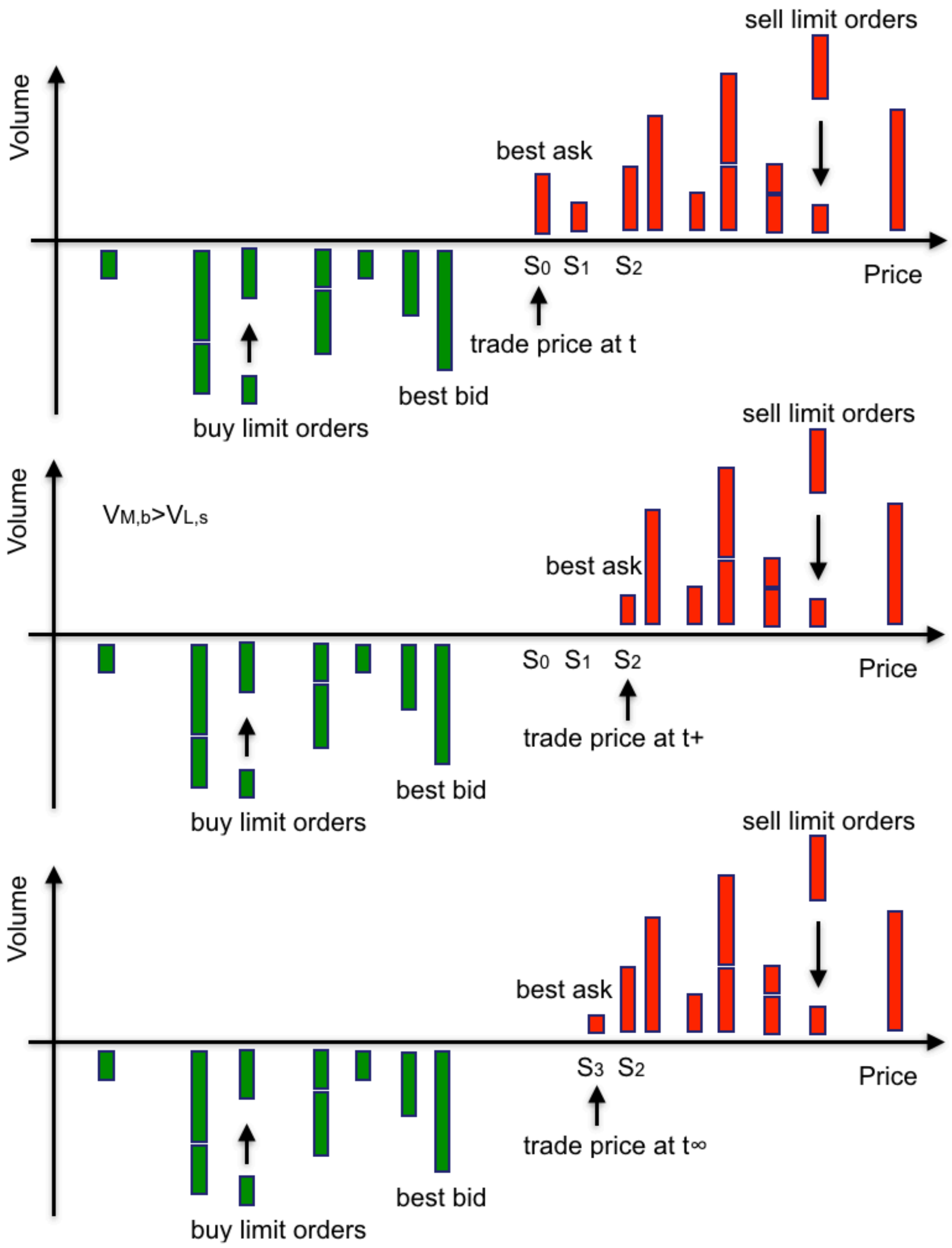

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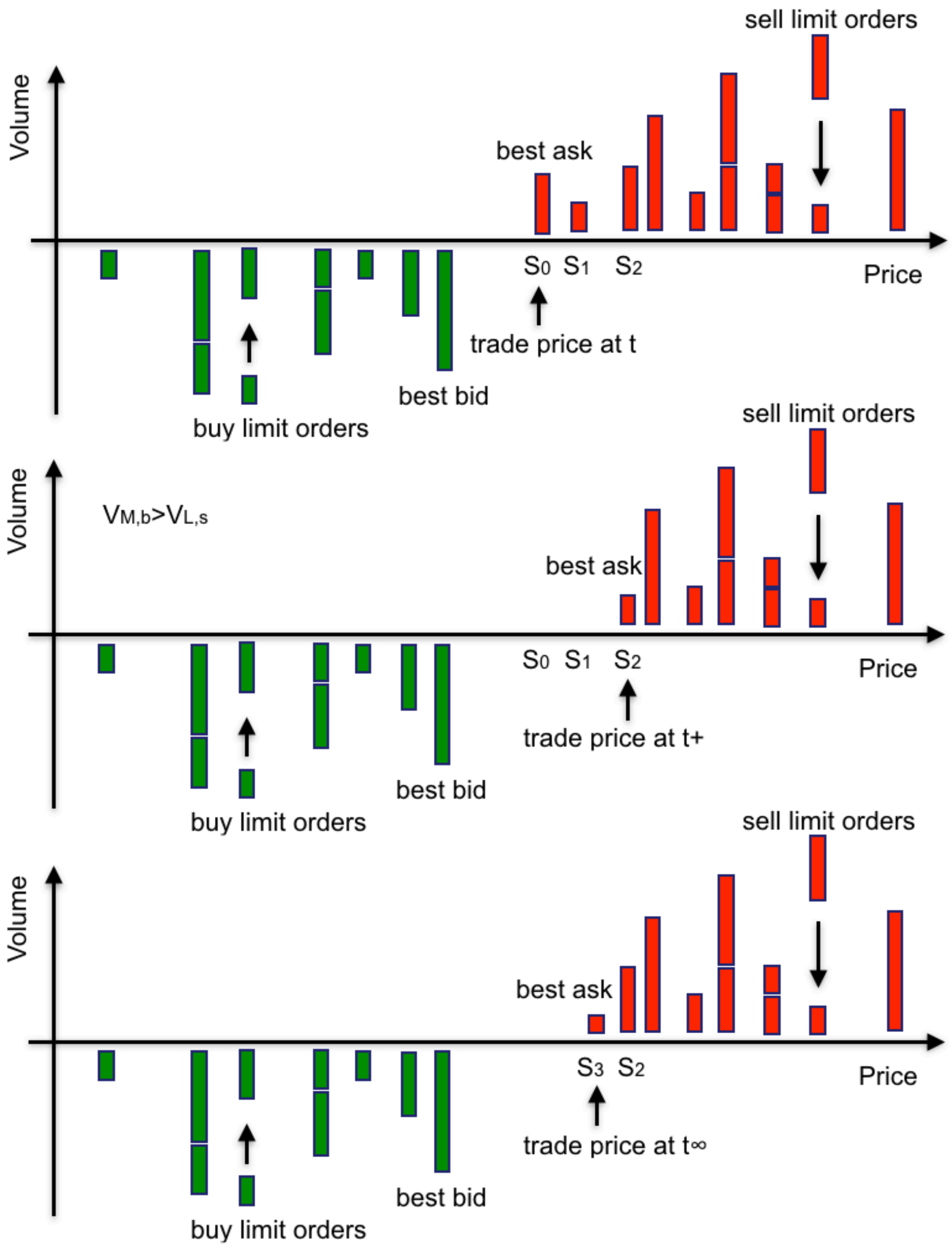

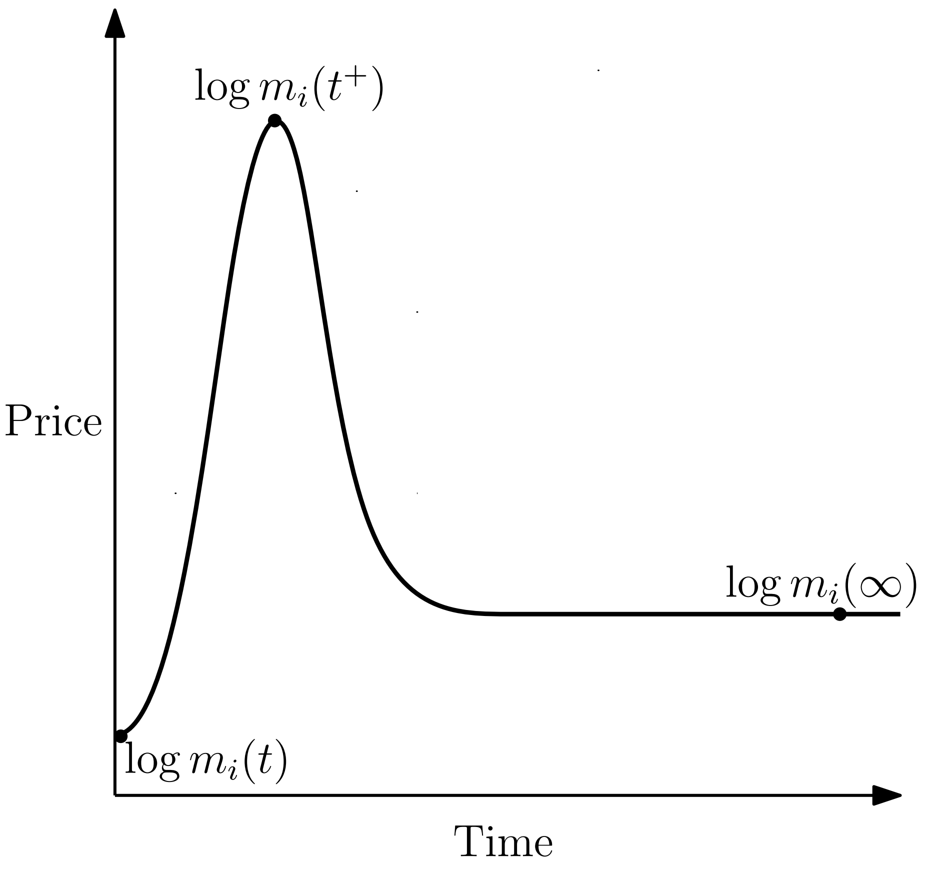

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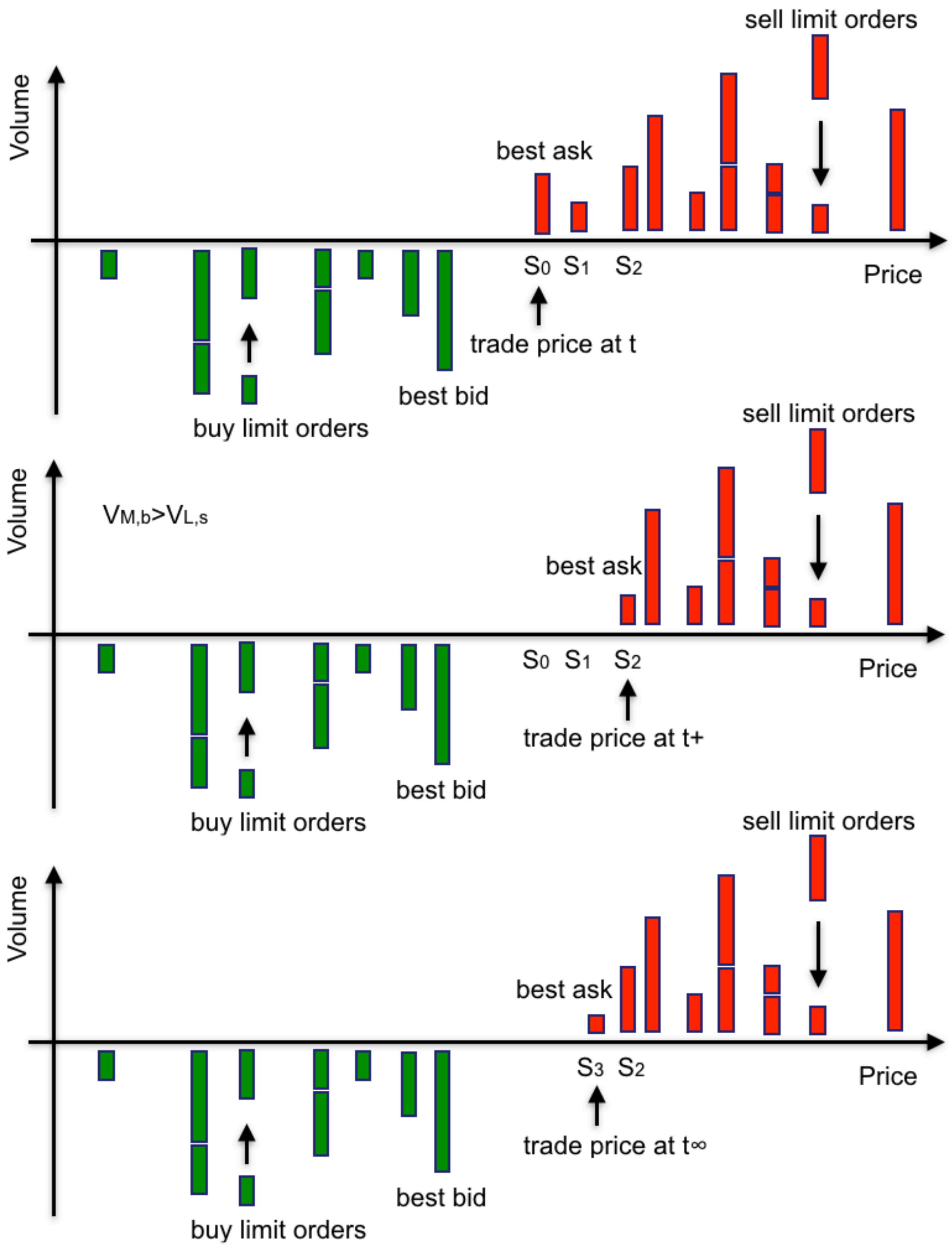

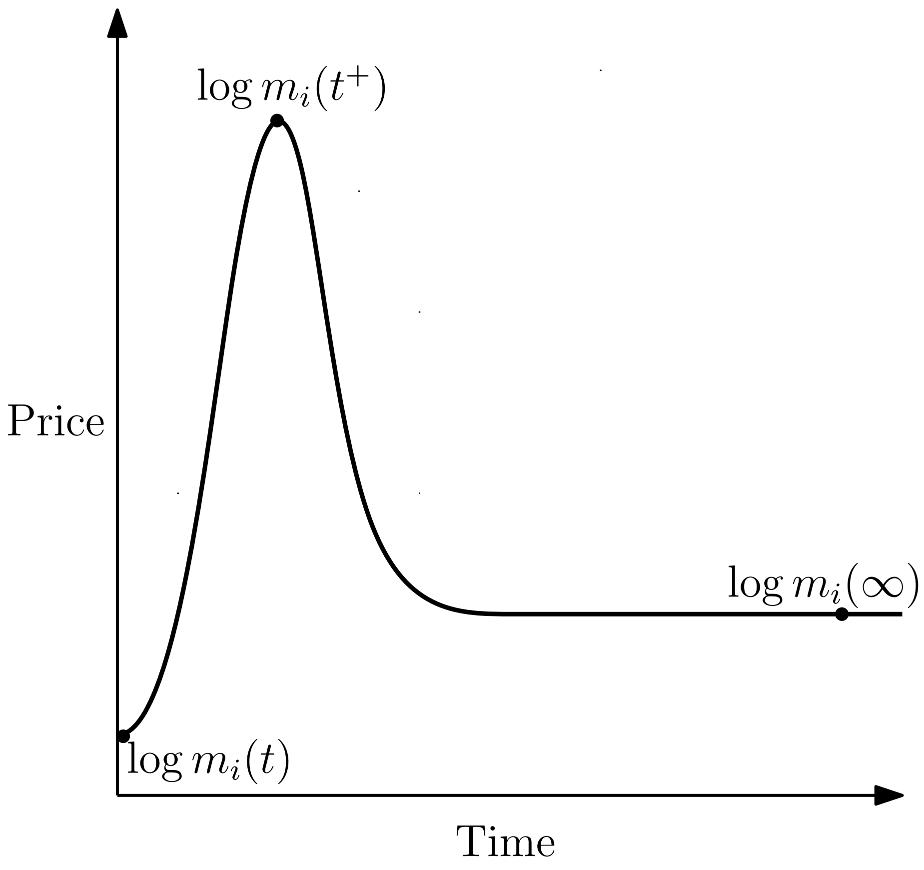

Market liquidity:

- For a liquid market, the shares of a stock can be rapidly bought or sold with little impact on the stock price.
- The market liquidity can be measured by the spread between the best ask and the best bid

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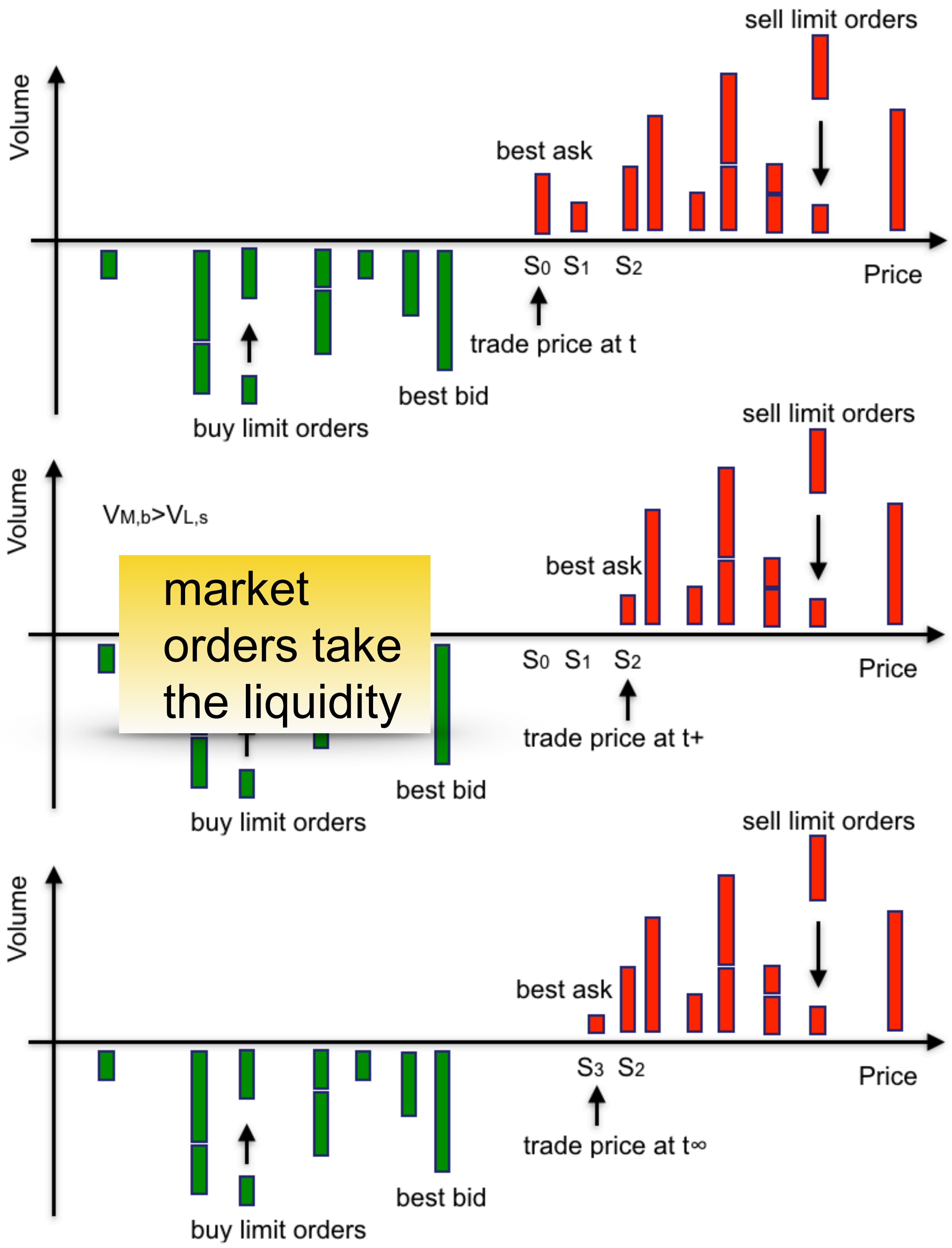
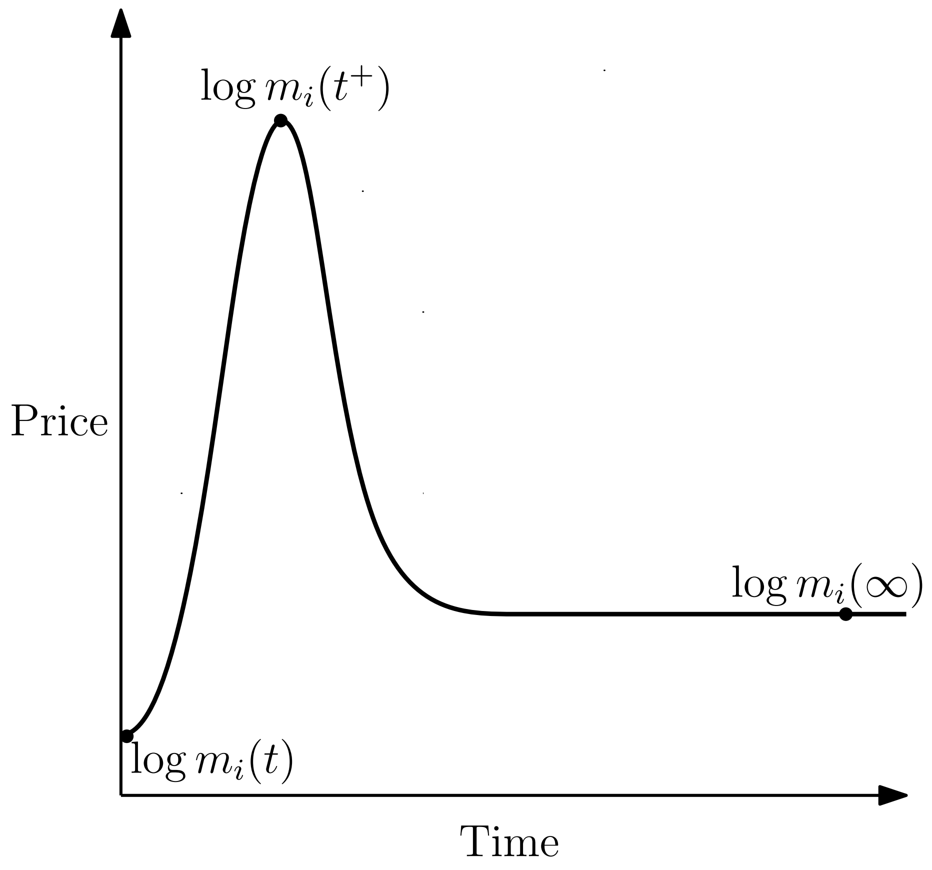


market orders take the liquidity



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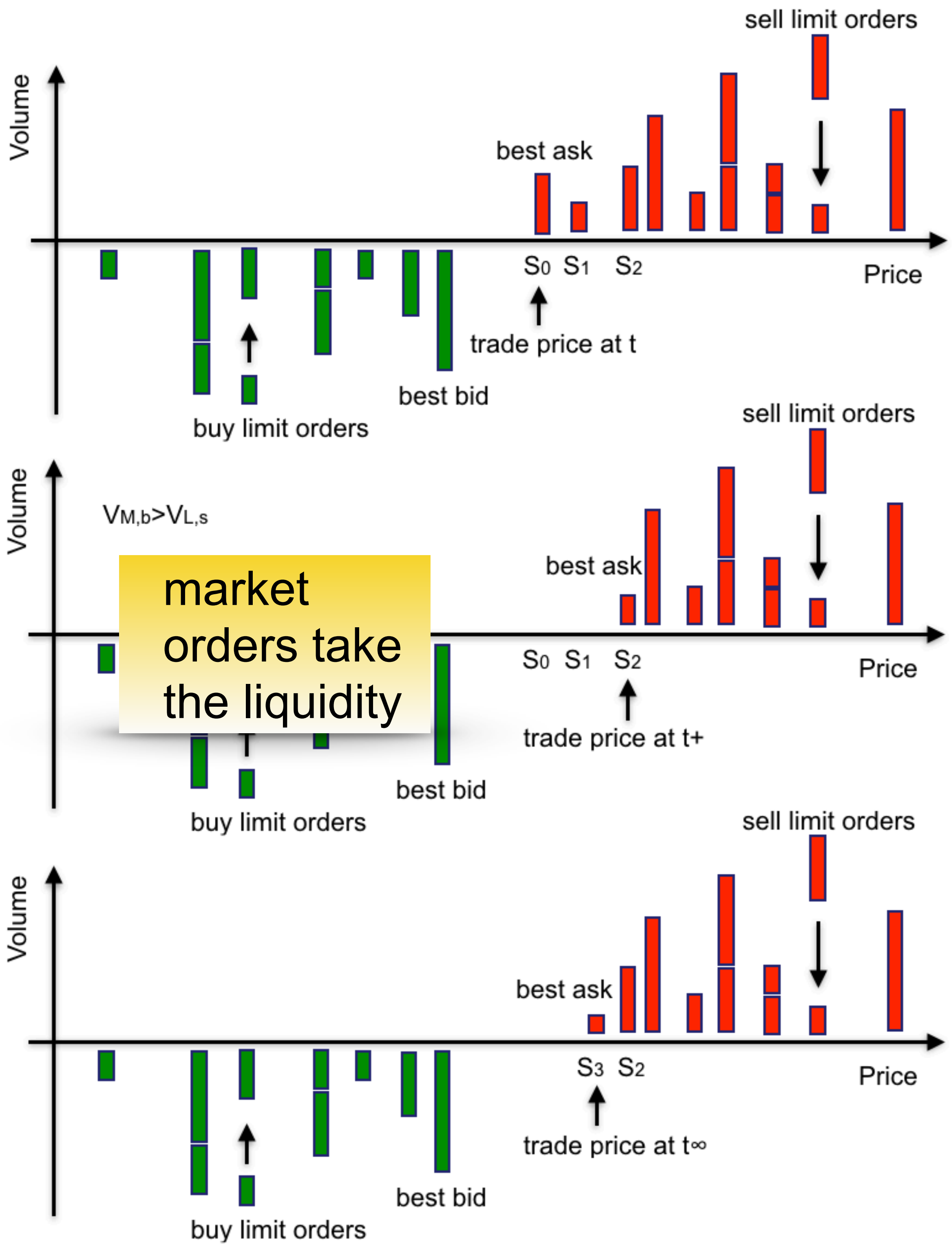

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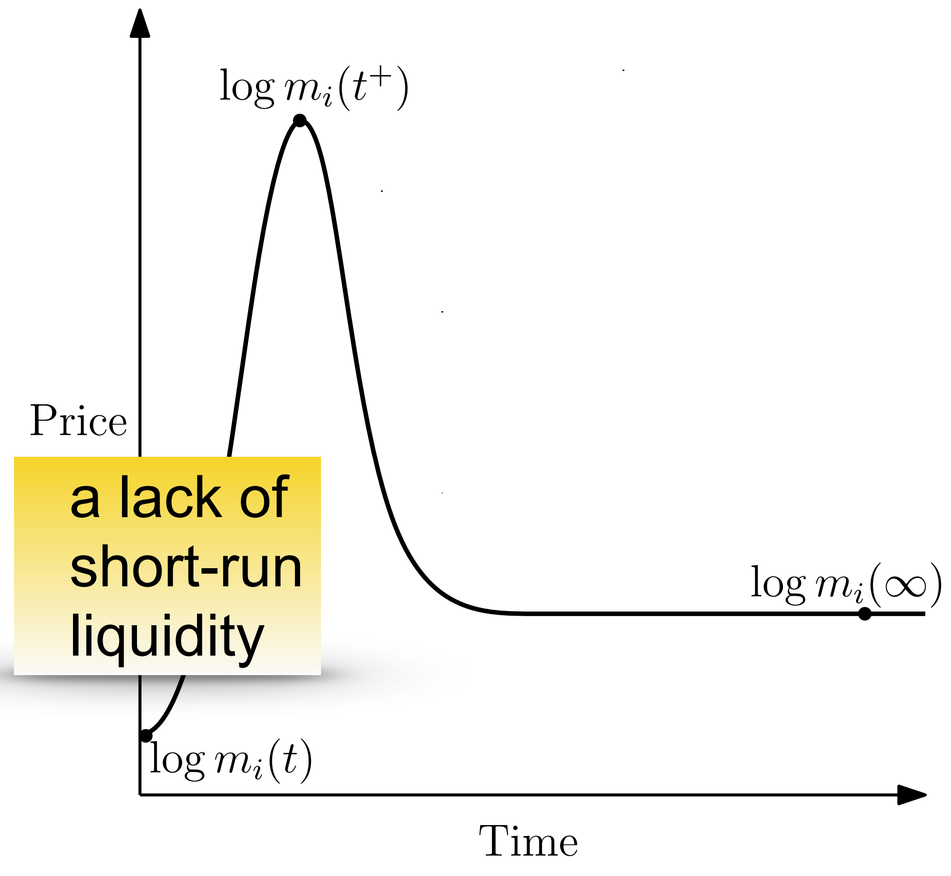

a lack of short-run liquidity

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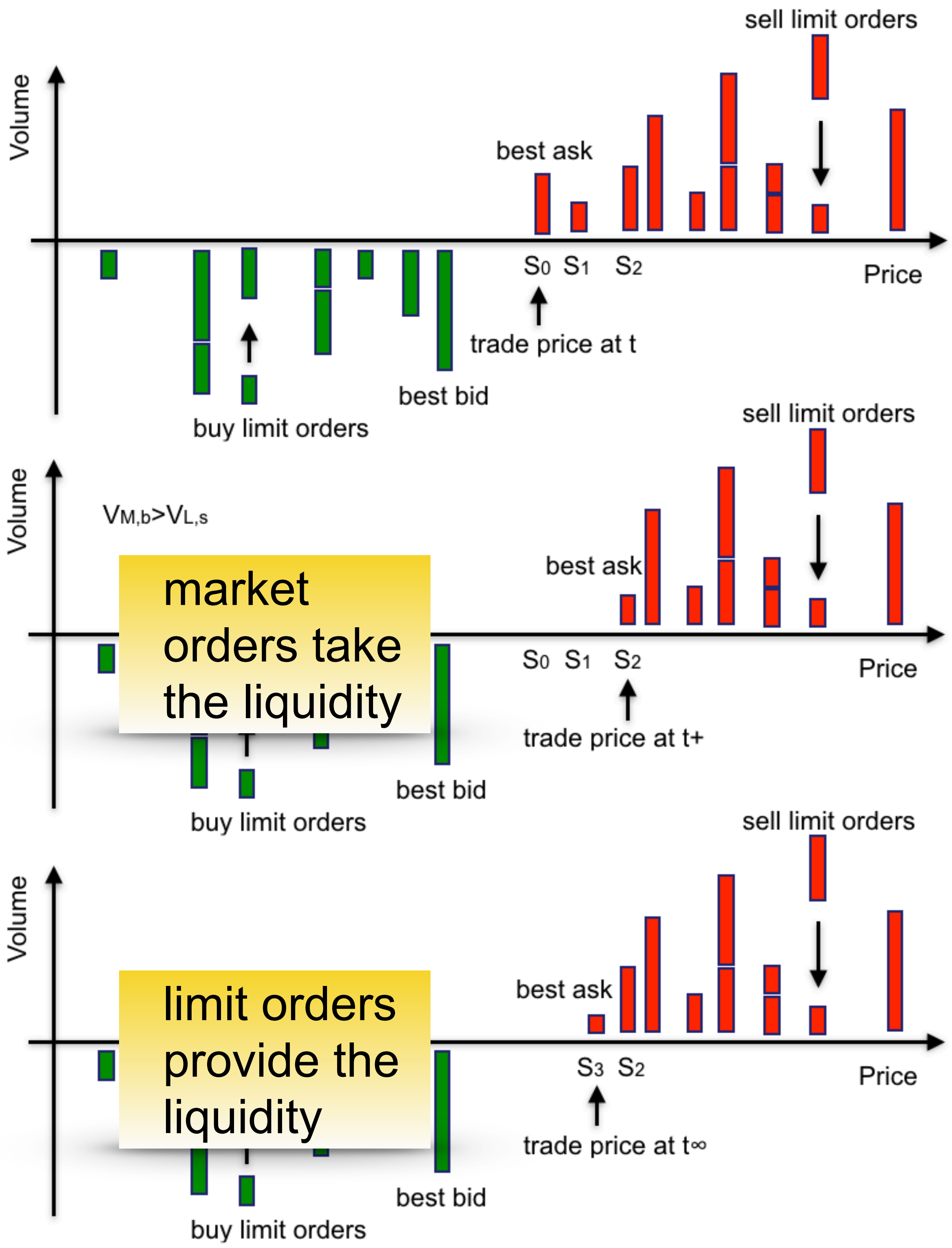
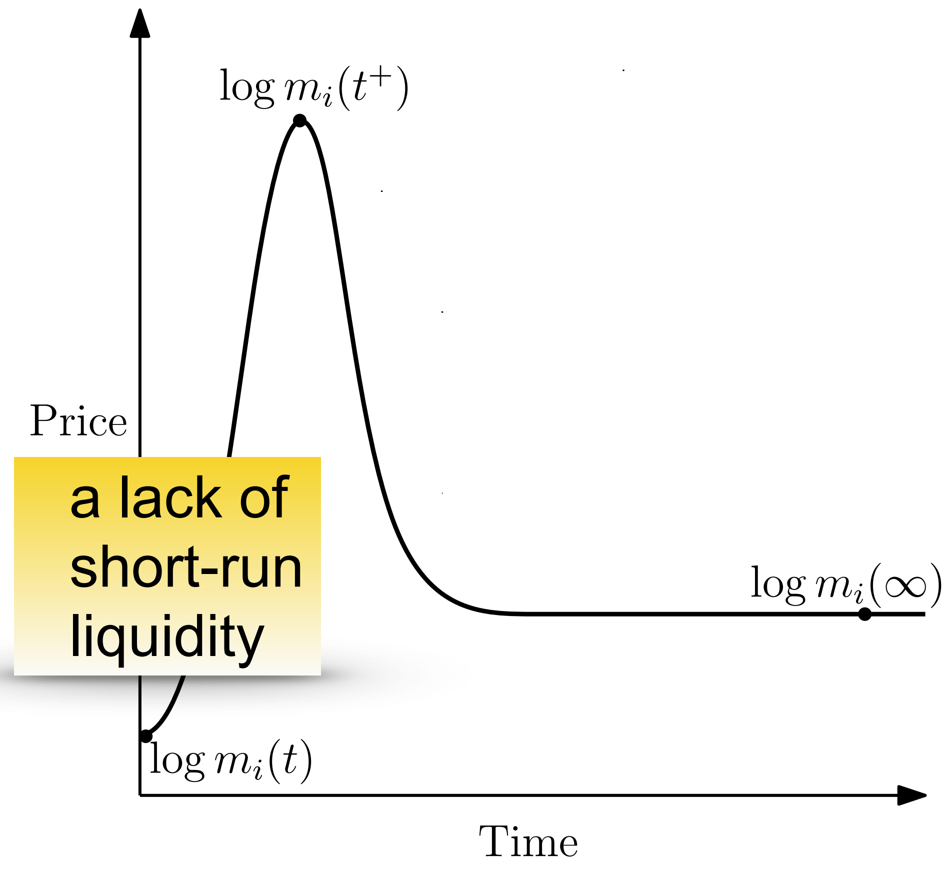


market orders take the liquidity

limit orders provide the liquidity



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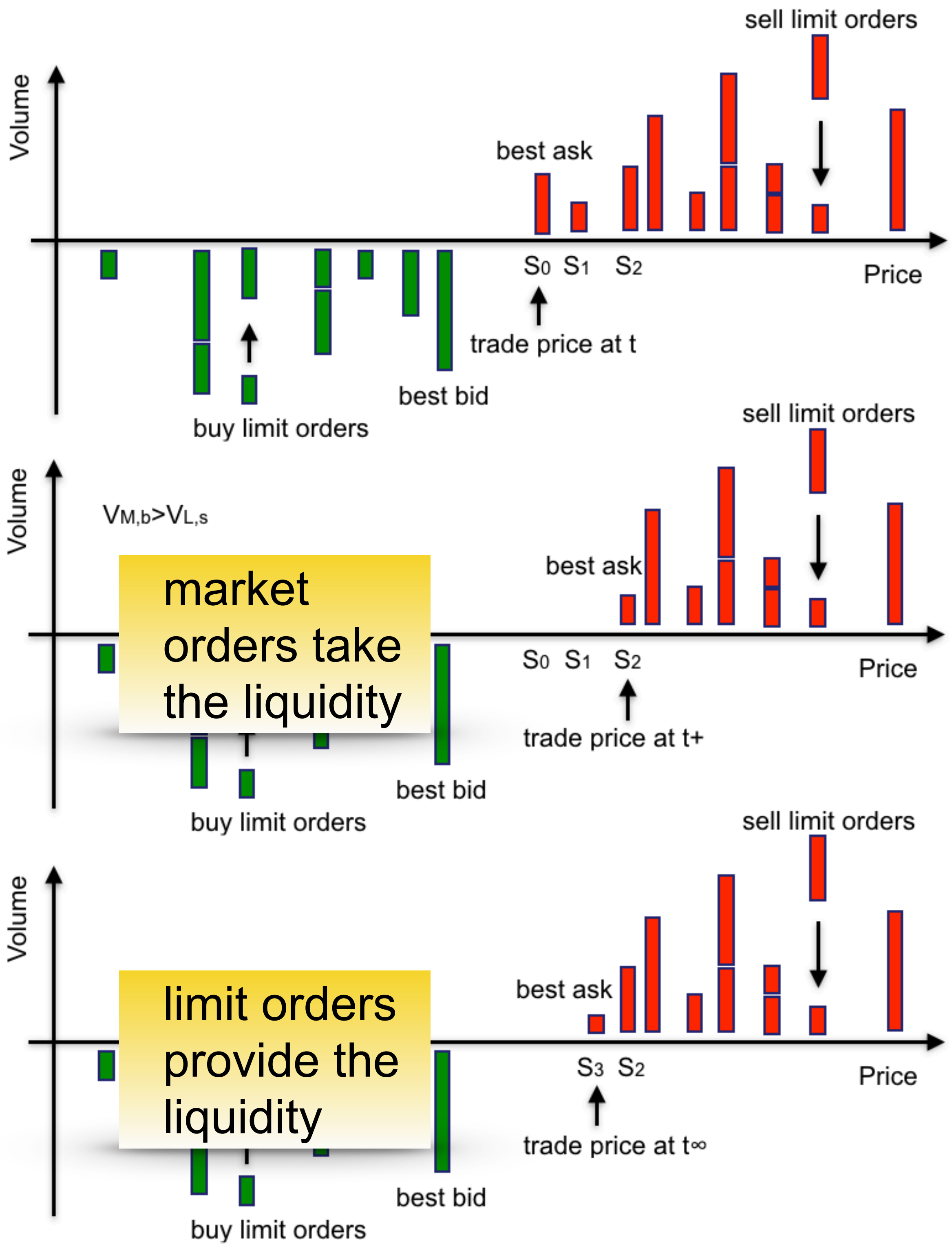

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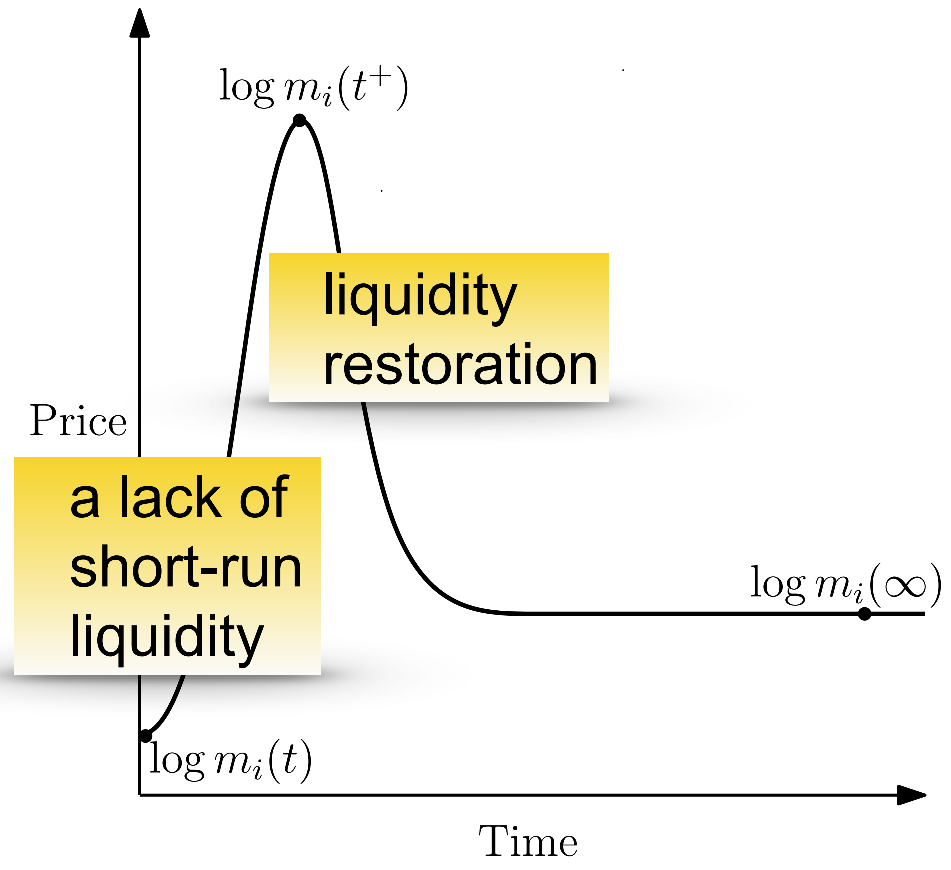

liquidity restoration

a lack of short-run liquidity

Market liquidity:

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Background—correlation of trade signs

How does the liquidity influence trades?

liquidity	volume	price ($)	cost ($)	total cost ($)	liquidity cost ($)
high	10000	2	20000	20000	0
low	5000	2	10000	21500	1500
	2000	2.2	4400		
	3000	2.5	7500		

Correlation of trade signs in single stocks

$$\mathcal{C}_0(\ell) = \langle \varepsilon_{n+\ell}\varepsilon_n \rangle - \langle \varepsilon_n \rangle^2$$

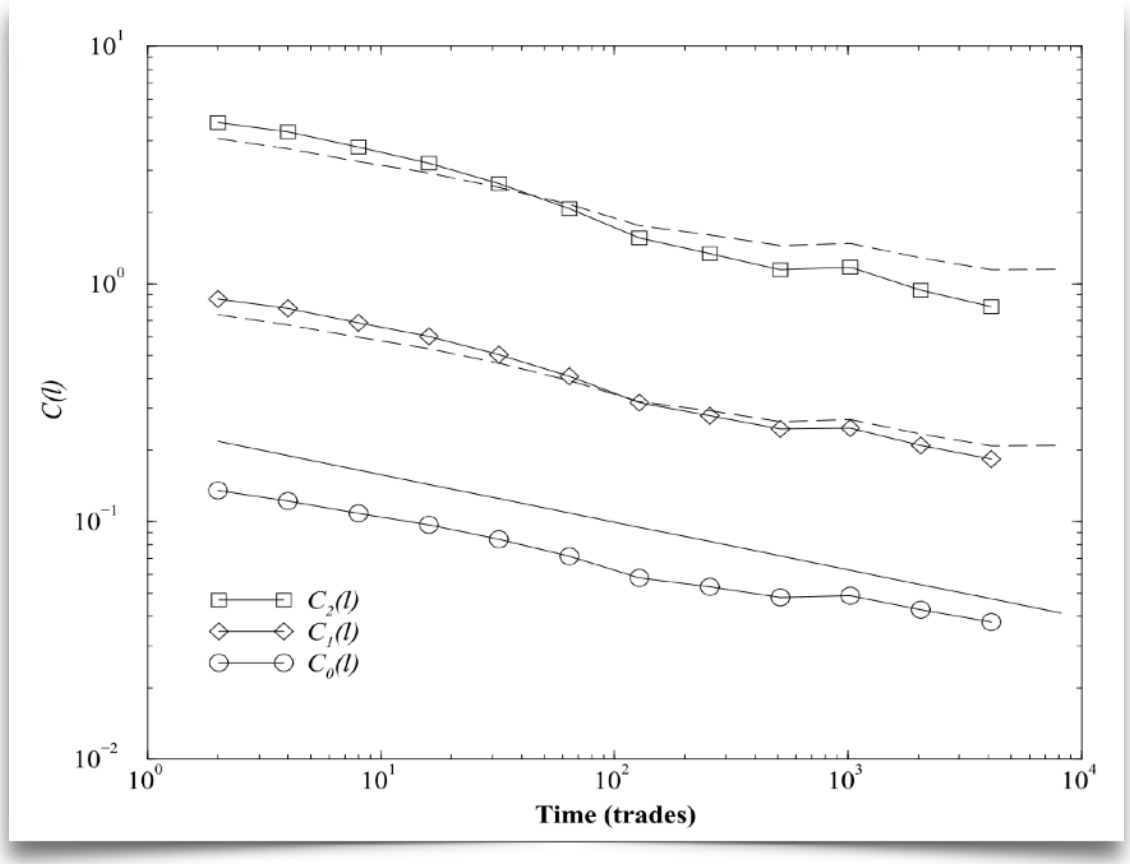
$$\mathcal{C}_1(\ell) = \langle \varepsilon_{n+\ell}\ \varepsilon_n \ln V_n \rangle$$

$$\mathcal{C}_2(\ell) = \langle \varepsilon_{n+\ell} \ln V_{n+\ell}\ \varepsilon_n \ln V_n \rangle$$

fitted by

$$\mathcal{C}_0(\ell) \simeq \frac{C_0}{\ell^\gamma}, \qquad (\ell \geq 1)$$

$$\gamma = 1/5$$



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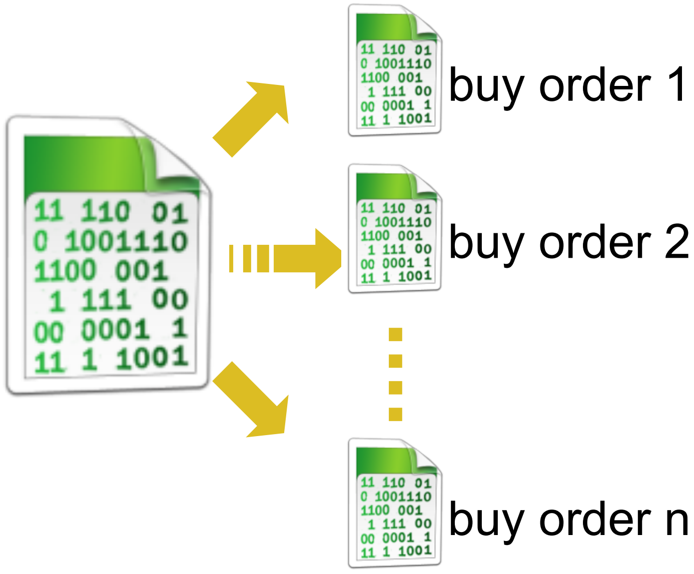
Order splitting



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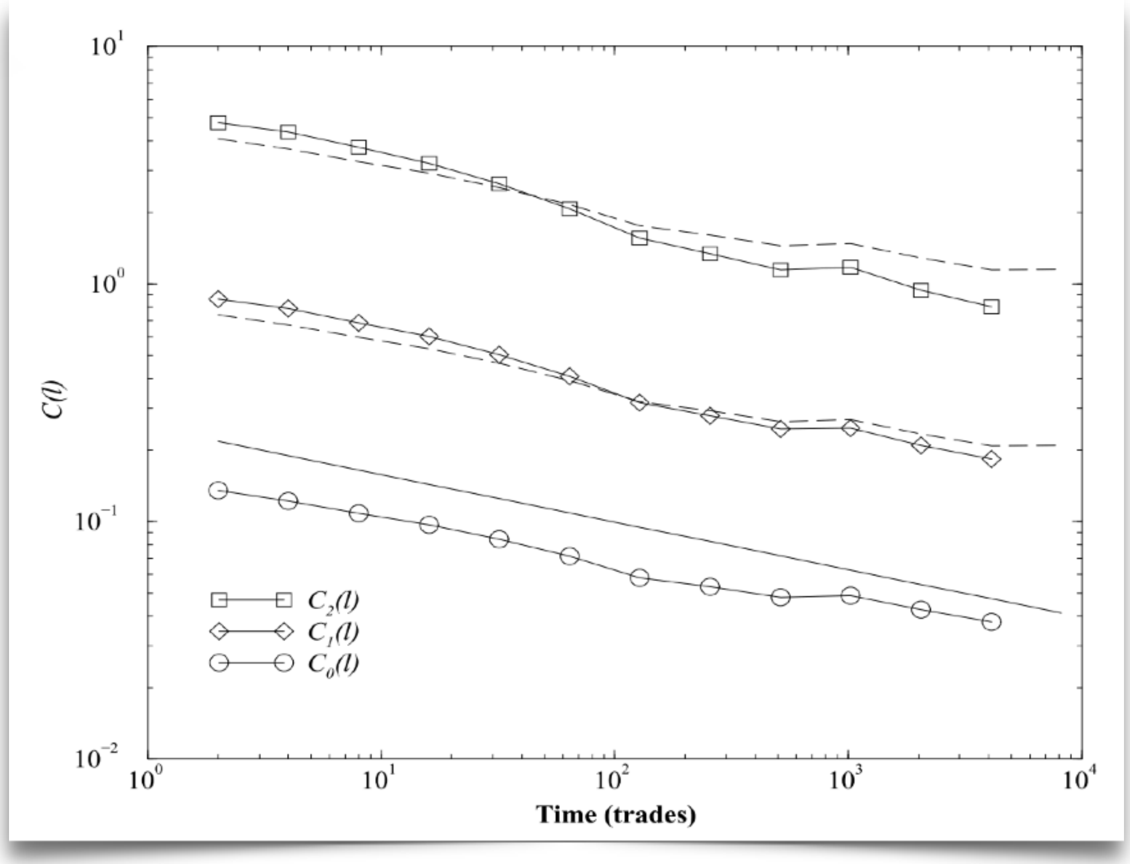

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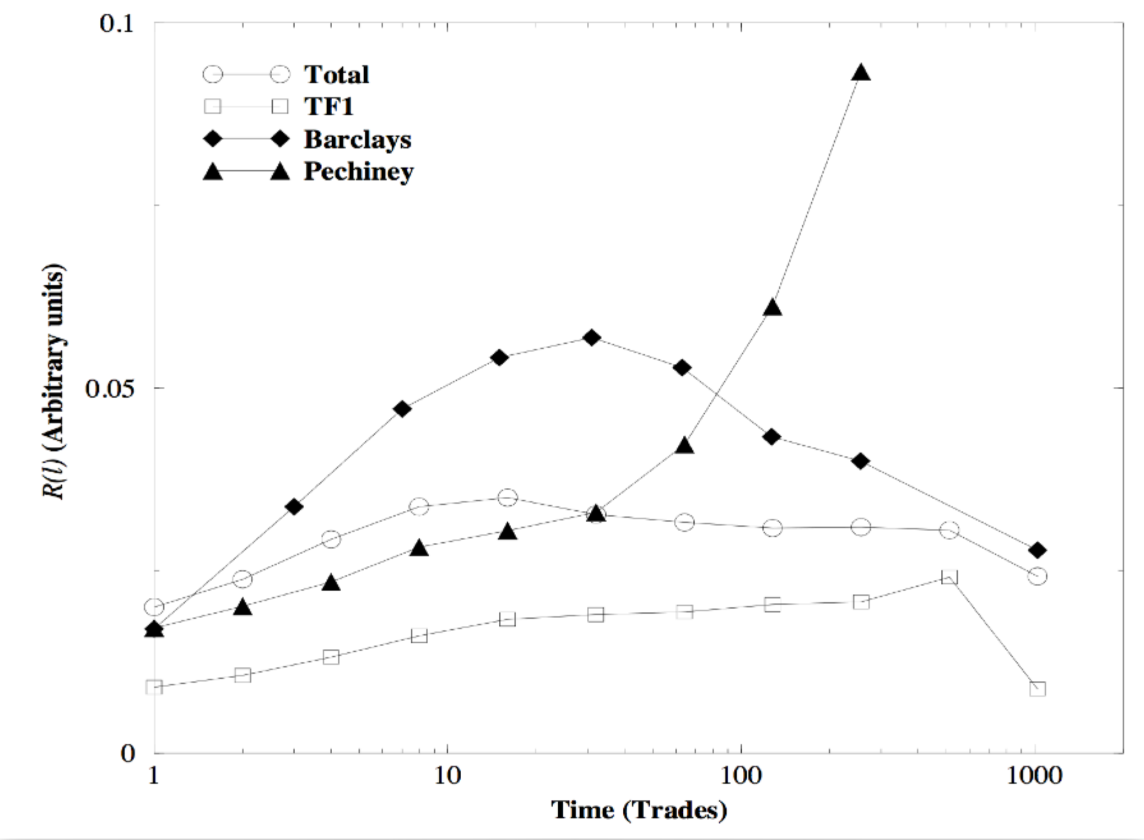
Background—price responses



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Price response

measures how much the price change after time τ, on average, conditioned on an initial buy or sell trade.

$$R_{ii}(\tau) = \Big\langle \big(S_i(t+\tau) - S_i(t)\big)\varepsilon_i(t) \Big\rangle_t$$

- The price reversion is contradictory to the long-memory sign correlation.
- To solve the paradox, a decaying quantity, i.e. an impact function, is required to reverse the price.

Some questions

- Is it possible that the price of one stock is impacted by the trades of other stocks?
- Is it the long or short memory for the sign cross-correlation between stocks?

Empirical results—data sets

- Our study is based on the Trades and Quotes (TAQ) data set from NASDAQ stock market

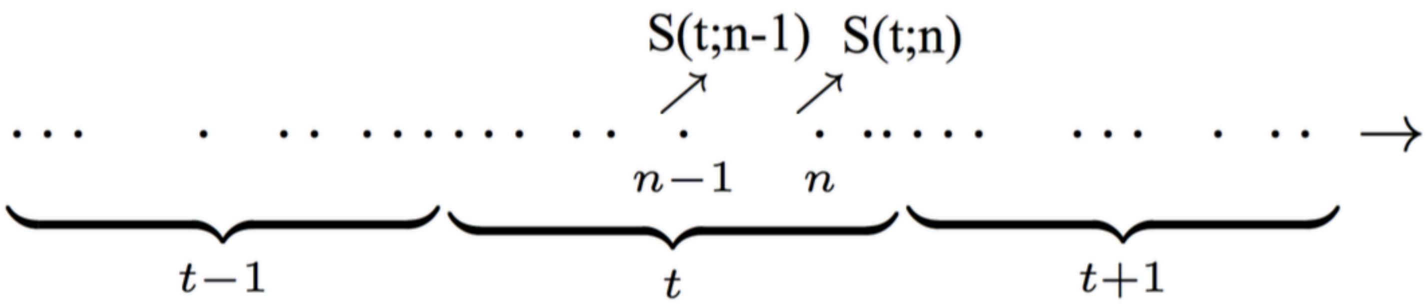
- The stocks we used are from S&P 500 index in the year 2008

- We use the intraday data with the trading time from 9:40 to 15:50 of New York time

- For a stock pair, we consider the common trading days that the two stocks have trades

Empirical results—trade signs

A time series
of trades



The trade sign of n-th trade in time interval t is defined as

$$\varepsilon(t;n) = \begin{cases} \operatorname{sgn}\big(S(t;n) - S(t;n-1)\big) & , \quad \text{if} \quad S(t;n) \neq S(t;n-1)\,, \\ \varepsilon(t;n-1) & , \quad \text{otherwise}\,. \end{cases}$$

The trade sign at time interval t is

$$\varepsilon(t) = \begin{cases} \operatorname{sgn}\left(\sum\limits_{n=1}^{N(t)} \varepsilon(t;n)\right) & , \quad \text{if} \quad N(t) > 0\,, \\ 0 & , \quad \text{if} \quad N(t) = 0\,. \end{cases}$$

$$\varepsilon(t) = \begin{cases} +1, & \text{for a majority of buy market orders,} \\ 0, & \text{for a lack of trading or a balance} \\ & \text{of buy and sell market orders} \\ -1\,, & \text{for a majority of sell market orders.} \end{cases}$$

Empirical results—response function and sign correlator

The midpoint price at time t is

$$m_i(t) = \frac{1}{2}\left(a_i(t) + b_i(t)\right) .$$

The price change from t to $t + \tau$ is

$$r_i(t,\tau) = \log m_i(t+\tau) - \log m_i(t) = \log \frac{m_i(t+\tau)}{m_i(t)} .$$

The price cross–response function is defined as

$$\boxed{R_{ij}(\tau) = \left\langle r_i(t,\tau)\varepsilon_j(t)\right\rangle_t} .$$

The cross–correlator of trade signs between stocks i and j is

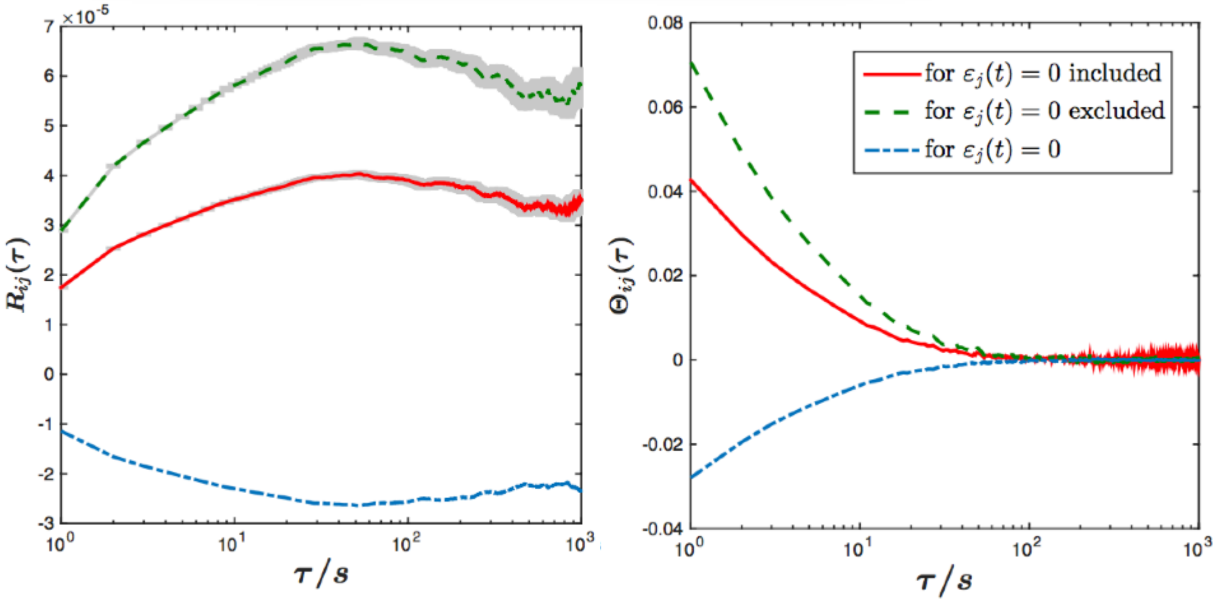
$$\boxed{\Theta_{ij}(\tau) = \left\langle \varepsilon_i(t+\tau)\varepsilon_j(t)\right\rangle_t} ,$$

where

$$\Theta_{ij}(0) = \Theta_{ji}(0) \qquad \text{and} \qquad \Theta_{ij}(\tau) = \Theta_{ji}(-\tau) .$$

Empirical results—price cross-responses for stock pairs

The influence of zero trade signs



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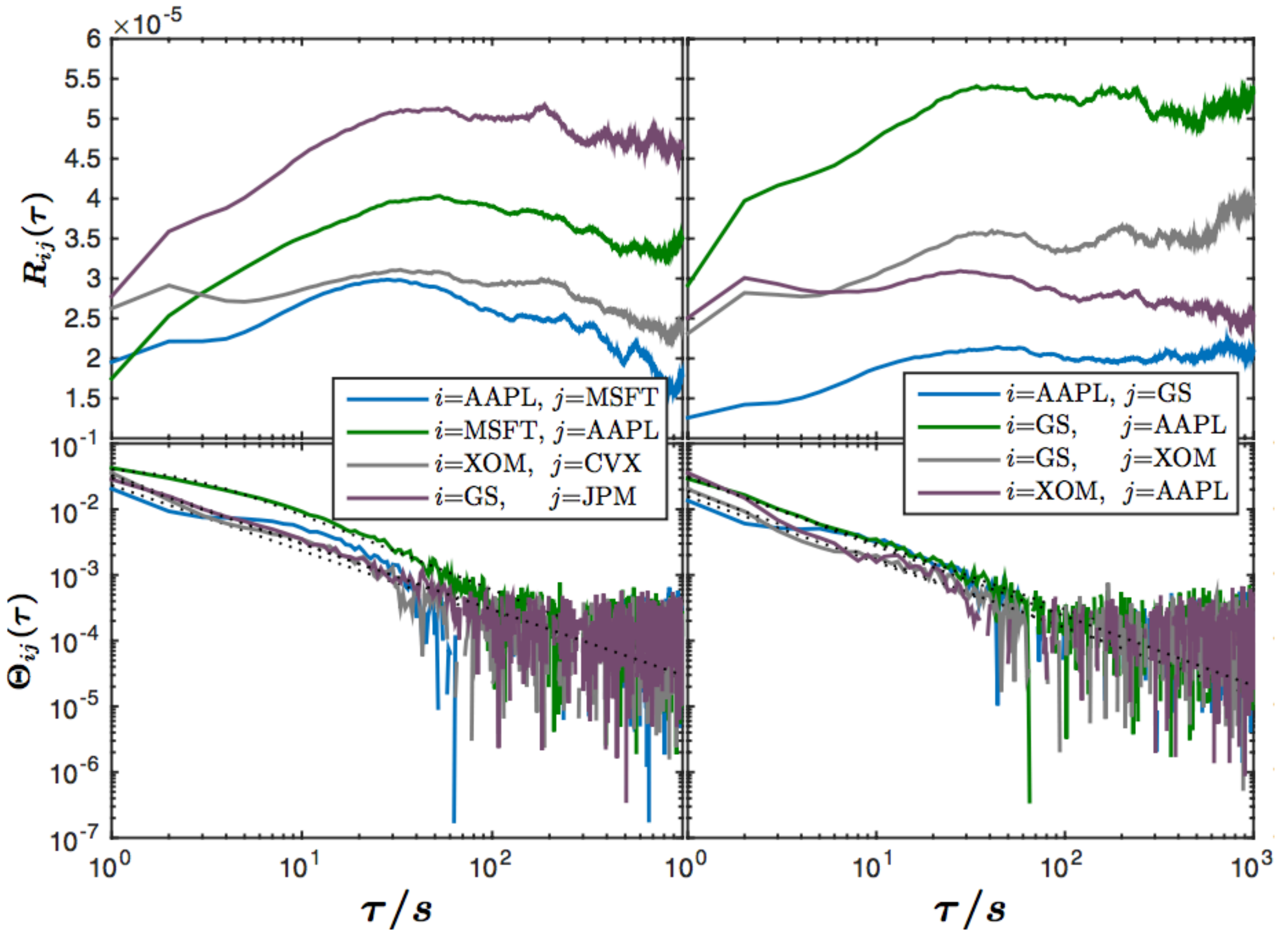

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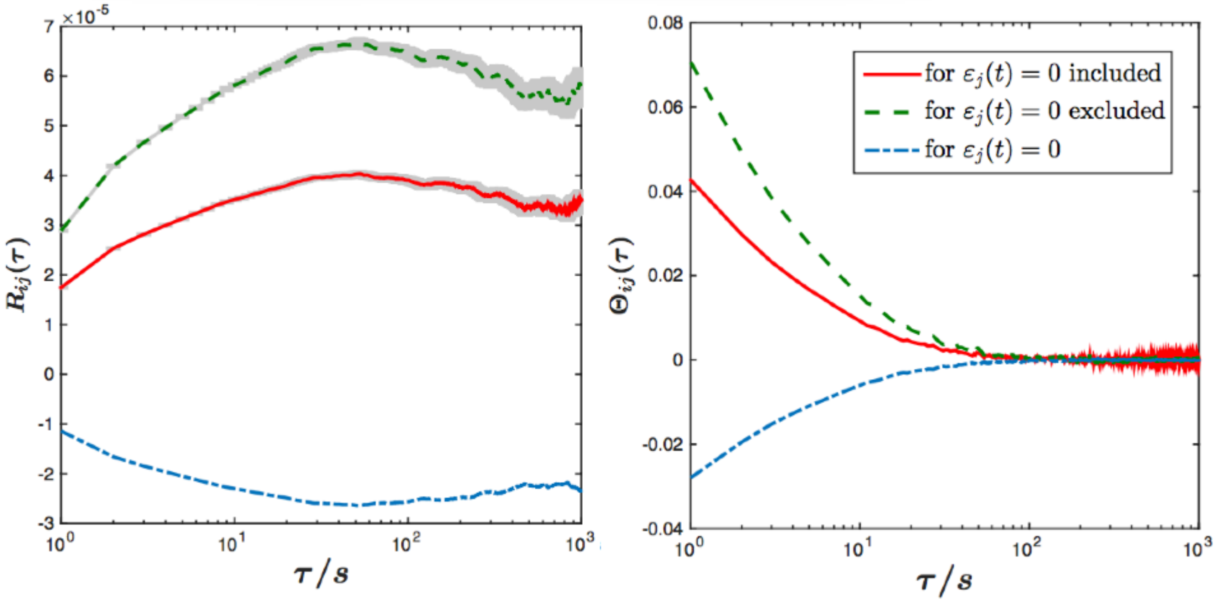

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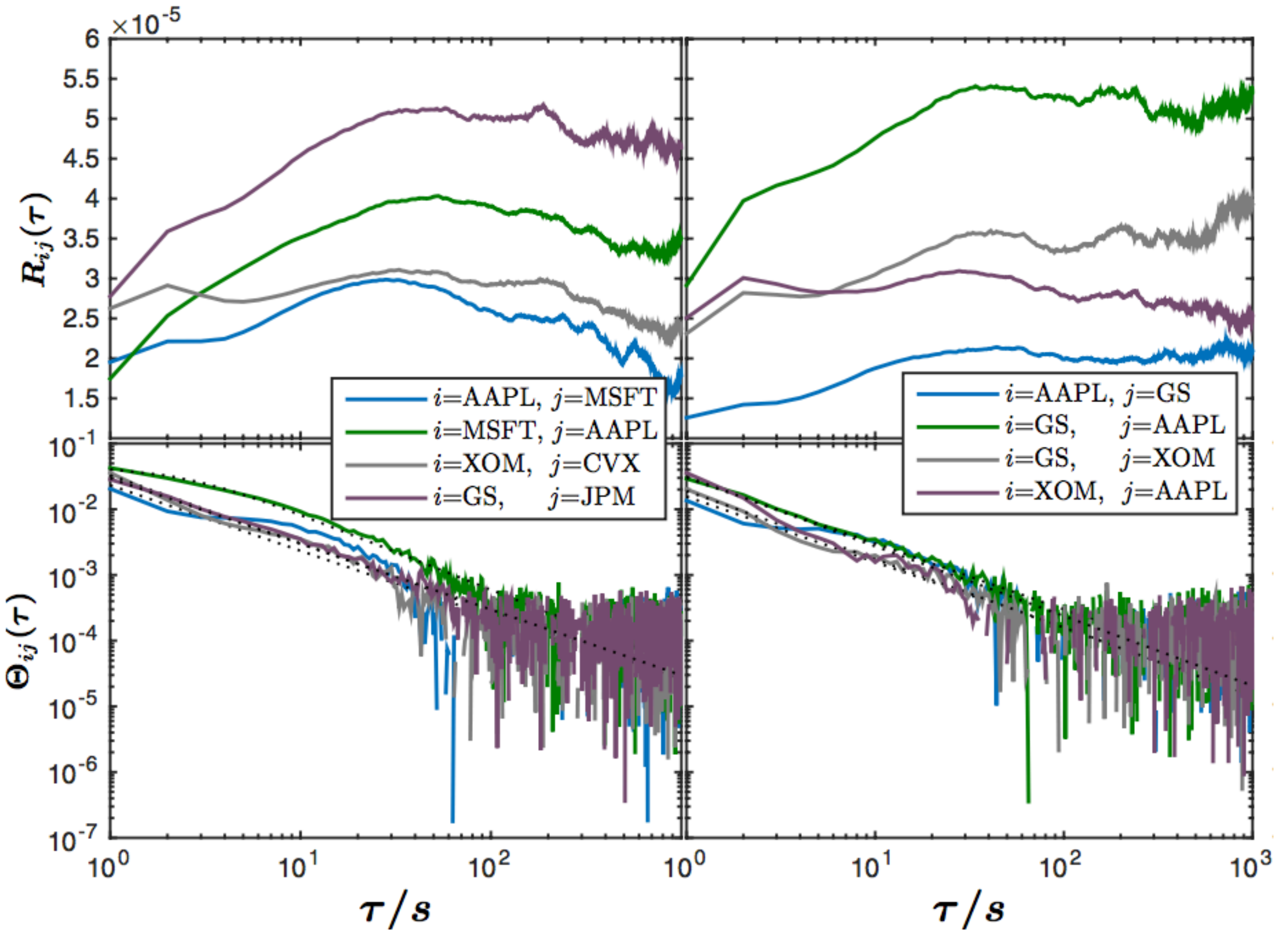

Memory properties of sign cross-correlators

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$$\Theta_{ij}(\tau) = \frac{\vartheta_{ij}}{\left(1+(\tau/\tau_{ij}^{(0)})^2\right)^{\gamma_{ij}/2}}$$

$$\begin{cases} 0<\gamma_{ij}<1 & \text{a long-memory process} \\ \gamma_{ij}\geq 1 & \text{a short-memory process} \end{cases}$$

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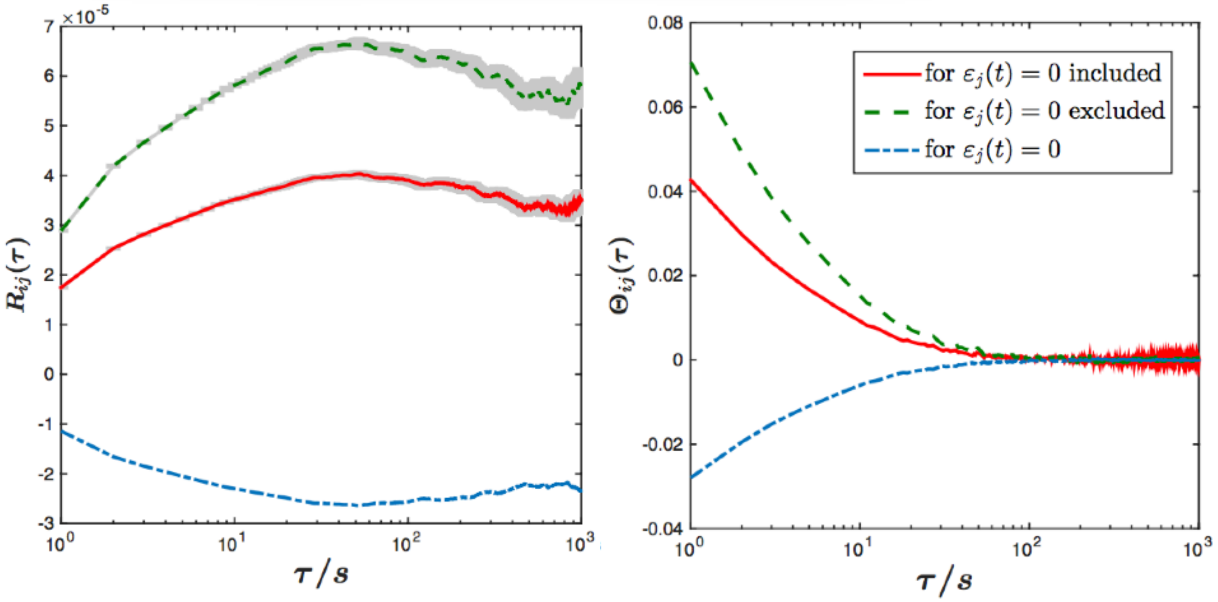

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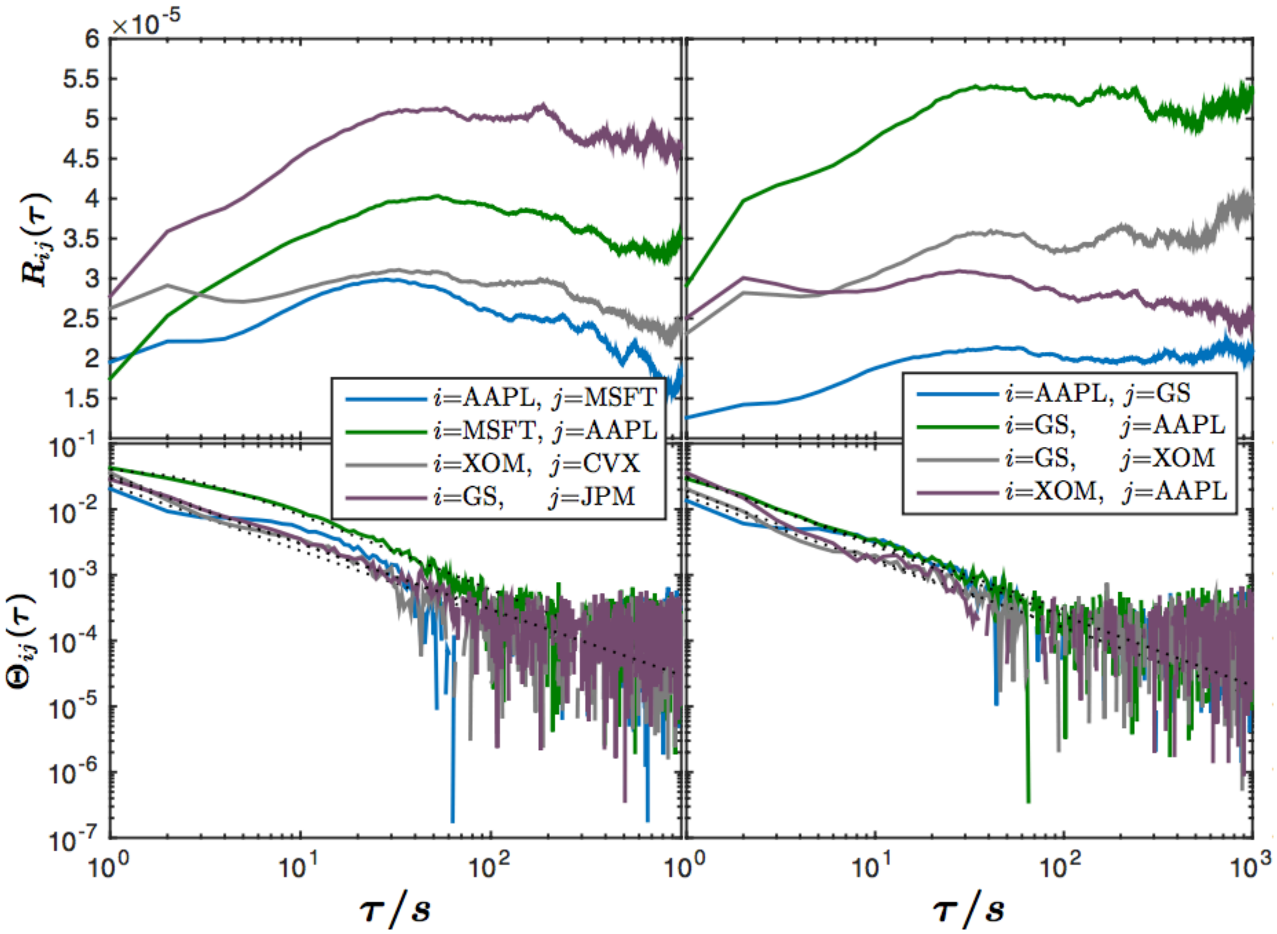

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Sign Correlators	Stock i	Stock j	ϑ_{ij}		$\tau_{ij}^{(0)}$ [s]		γ_{ij}		χ_{ij}^2 ($\times10^{-6}$)	
			inc. 0	exc. 0	inc. 0	exc. 0	inc. 0	exc. 0	inc. 0	exc. 0
Cross	AAPL	MSFT	0.46	0.05	0.05	3.46	1.00	1.35	0.23	1.52
	MSFT	AAPL	0.04	0.07	2.34	2.34	1.15	1.15	0.10	0.27
	XOM	CVX	0.61	0.67	0.06	0.21	1.04	1.16	0.07	0.52
	GS	JPM	0.45	0.48	0.07	0.13	1.00	1.00	0.04	0.18
	AAPL	GS	0.46	0.28	0.03	0.14	1.00	0.91	0.11	0.99
	GS	AAPL	0.49	0.49	0.06	0.10	1.00	1.00	0.05	0.13
	GS	XOM	0.61	0.73	0.04	0.08	1.04	1.10	0.04	0.20
	XOM	AAPL	0.76	0.29	0.05	0.34	1.09	1.42	0.12	0.18

Empirical results—average cross-responses

Passive and active average cross-responses

$$R_i^{(p)}(\tau) = \langle R_{ij}(\tau) \rangle_j$$

$$R_j^{(a)}(\tau) = \langle R_{ij}(\tau) \rangle_i$$

Passive and active average cross-correlators of trade signs

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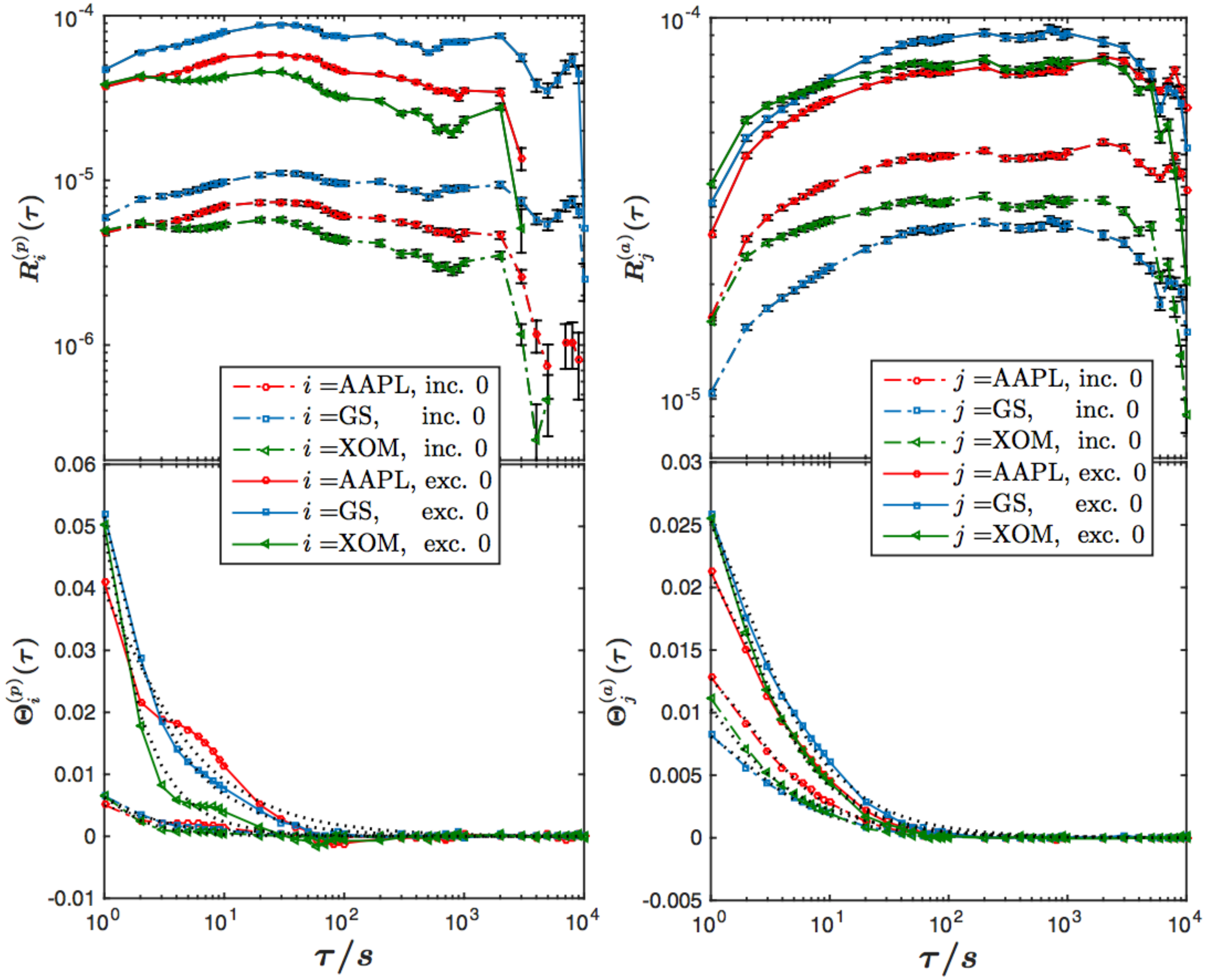

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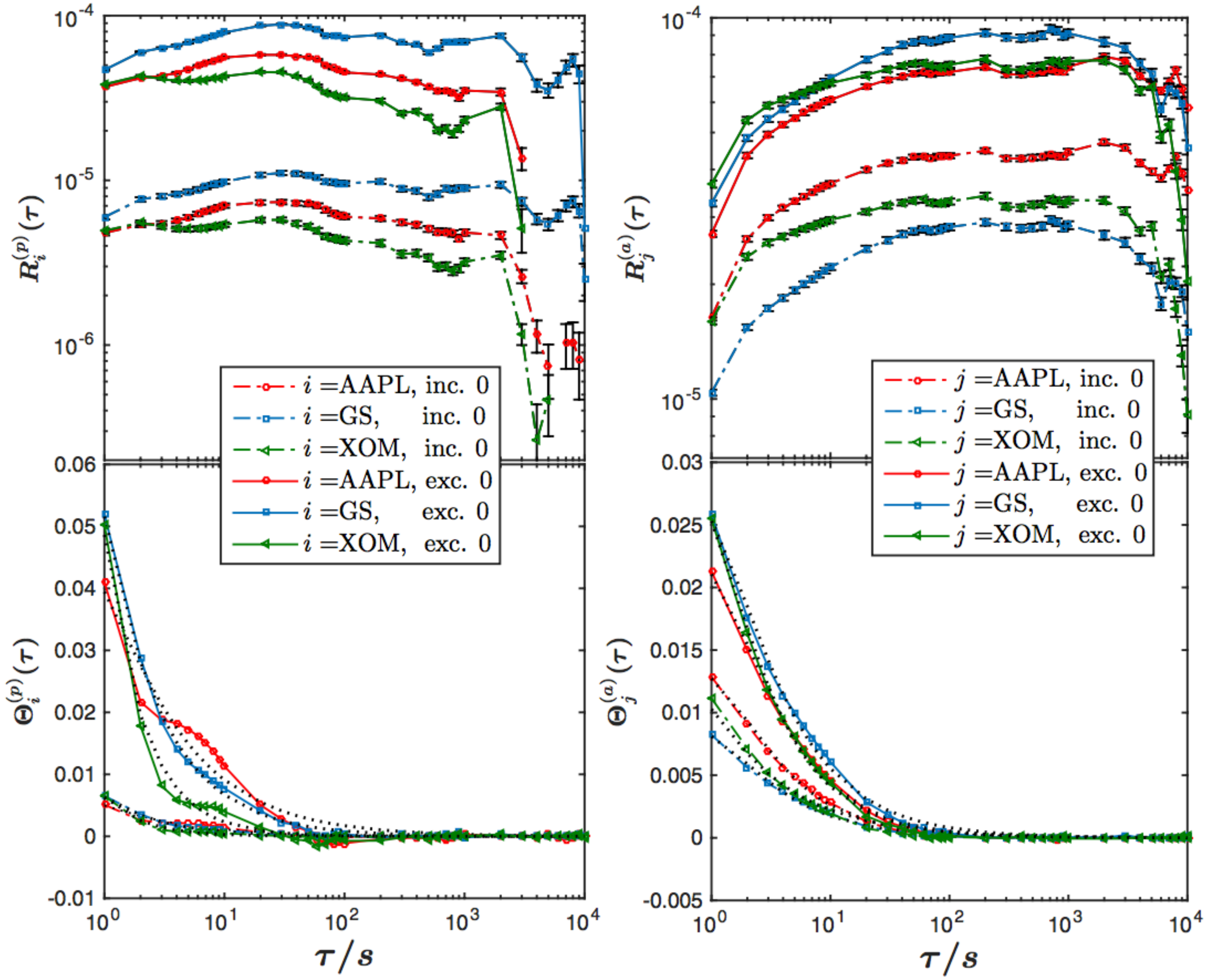

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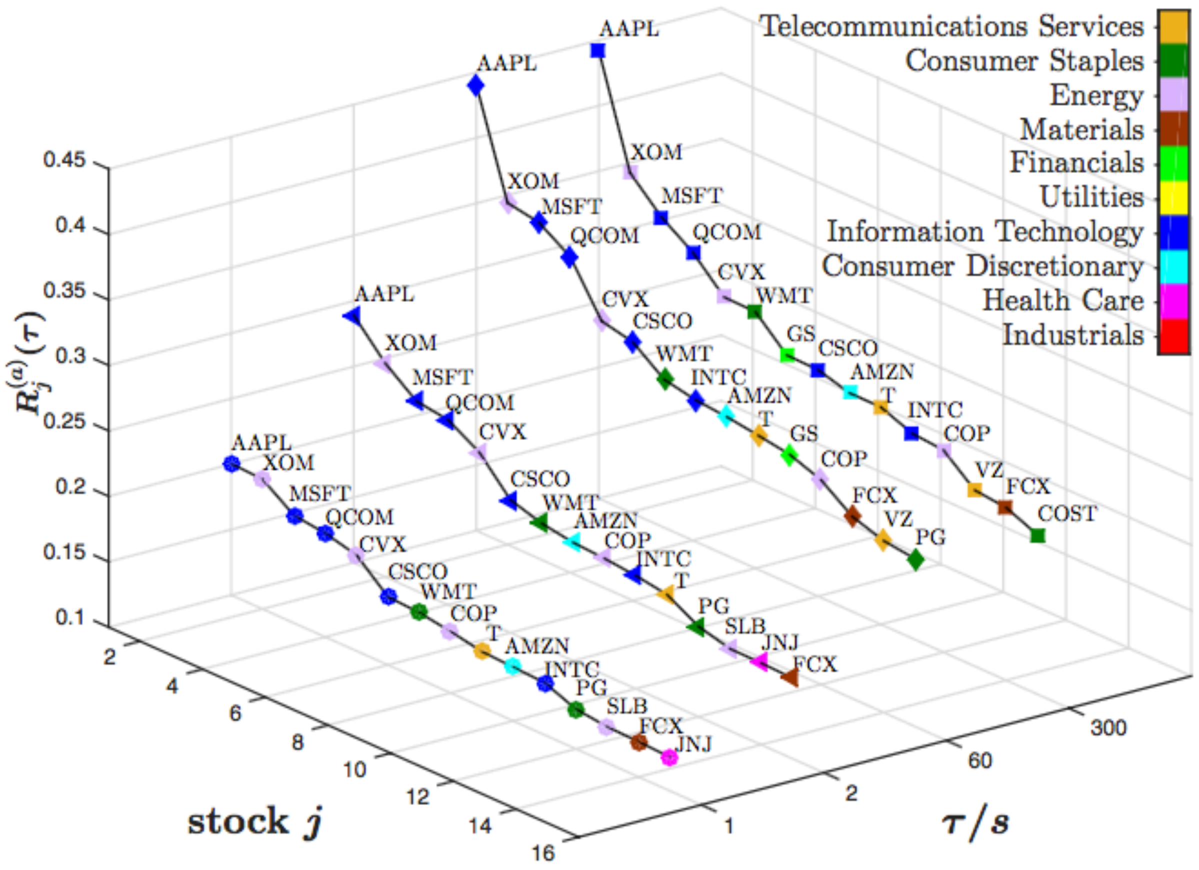
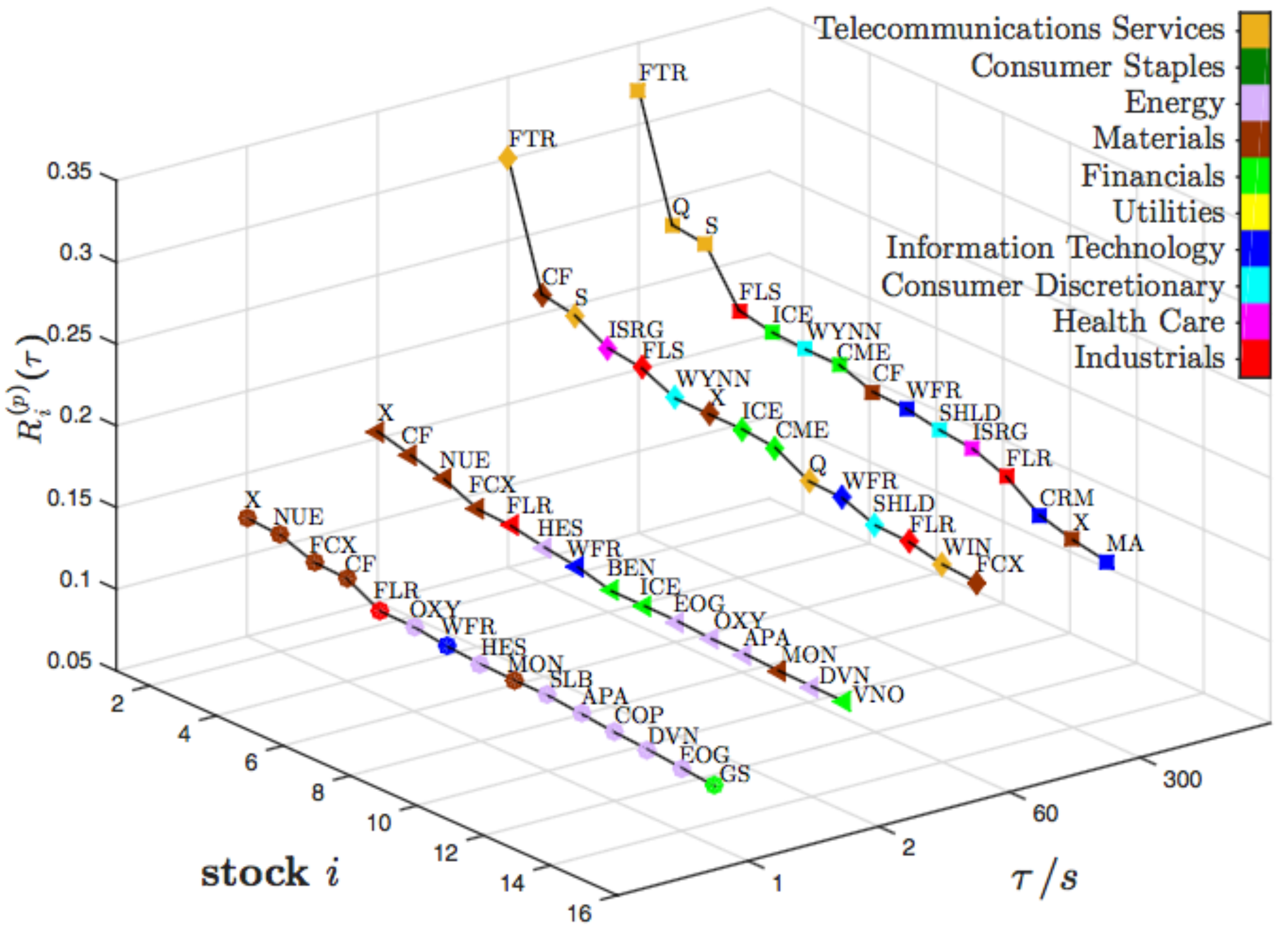
Sign cross-correlators	Stock i, j	ϑ_i or ϑ_j		$\tau_i^{(0)}$ or $\tau_j^{(0)}$ [s]		γ_i or γ_j		χ_i^2 or χ_j^2 ($\times 10^{-6}$)	
		inc. 0	exc. 0	inc. 0	exc. 0	inc. 0	exc. 0	inc. 0	exc. 0
$\Theta_i^{(p)}(\tau)$	AAPL	0.01	0.05	0.47	0.88	0.68	0.73	0.07	4.59
	GS	0.03	0.22	0.23	0.20	0.92	0.90	0.01	0.38
	XOM	0.27	0.83	0.06	0.12	1.32	1.33	0.02	1.20
$\Theta_j^{(a)}(\tau)$	AAPL	0.02	0.03	1.44	1.44	0.90	0.91	0.03	0.08
	GS	0.01	0.03	1.31	1.27	0.85	0.83	0.02	0.18
	XOM	0.02	0.03	0.55	1.08	0.71	0.95	0.11	0.08

Empirical results——identifying the influencing and influenced stocks

Influencing stocks



Influenced stocks



Ref. S. Wang, R. Schäfer, and T. Guhr, Eur. Phys. J. B 89, 207 (2016)

- The calculation is based on 99 stocks from 10 economic sectors in 2008.
- For each sector, we select the first 9 or 10 stocks with the largest average market capitalization.
- The responses are normalized by $\frac{R_{ij}(\tau)}{\max(|R_{ij}(\tau)|)}$

Empirical results—some questions

- How to understand the cross-responses between stocks?

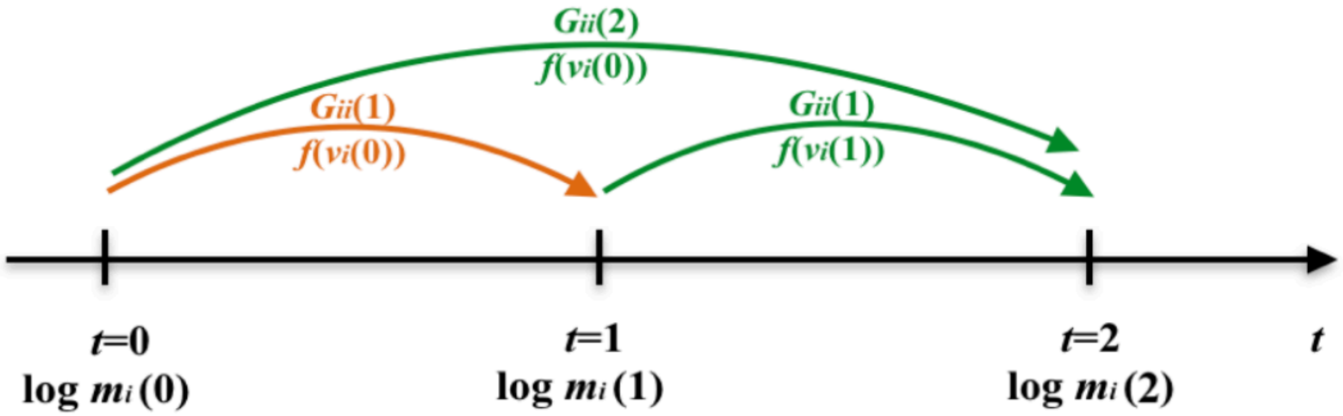
Empirical results—some questions

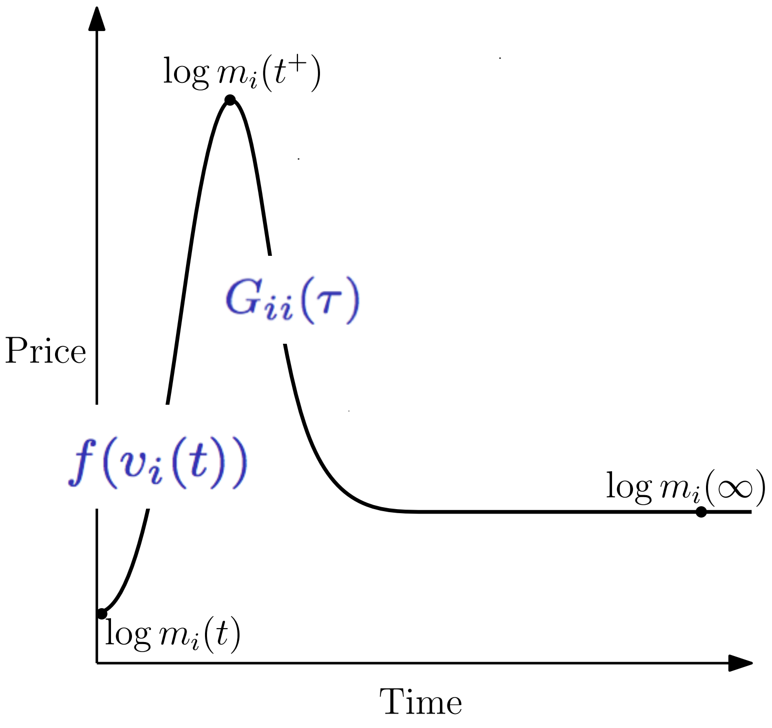
- How to understand the cross-responses between stocks?
- What's the relation between the cross-response and sign correlators?

Empirical results—some questions

- How to understand the cross-responses between stocks?
- What's the relation between the cross-response and sign correlators?
- Why the active and the passive average cross-responses have different behaviours?

A price impact model—setup





$$\log m_i(1) = \log m_i(0) + G_{ii}(1) f(v_i(0)) \varepsilon_i(0) + \eta_{ii}(0)$$

$$\begin{aligned} \log m_i(2) &= G_{ii}(1) f(v_i(1)) \varepsilon_i(1) + \eta_{ii}(1) \\ &+ G_{ii}(2) f(v_i(0)) \varepsilon_i(0) + \eta_{ii}(0) \\ &+ \log m_i(0) \,. \end{aligned}$$

$$\begin{aligned} \log m_i(t) &= \sum_{t'<t} G_{ii}(t-t') f(v_i(t')) \varepsilon_i(t') + \sum_{t'<t} \eta_{ii}(t') \\ &+ \log m_i(-\infty) \,. \end{aligned}$$

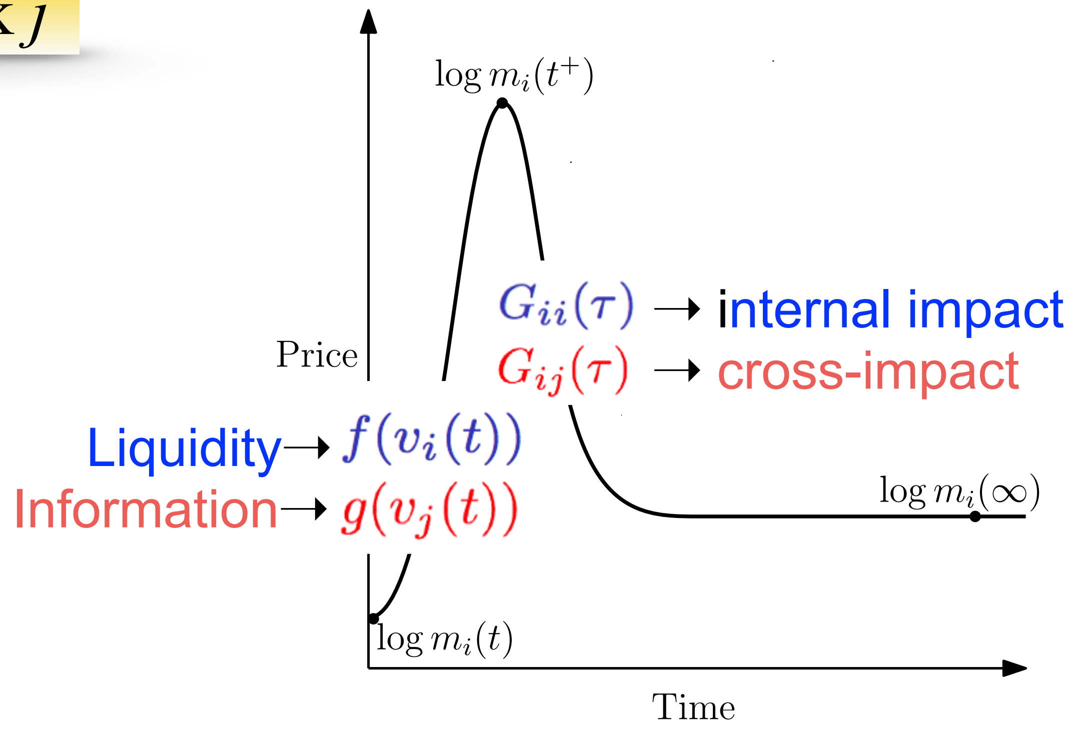
Note that the price is changed only by the stock *i* itself

Equation: ref. J.-P. Bouchaud, Y. Gefen, M. Potters, M. Wyart. Quantitative Finance, 4(2), 176 (2004).

A price impact model—setup

The price is changed by the stock *i* itself and another stock *j*

$$
\begin{aligned}
\log m_i(t) &= \sum_{t'<t}\Big[G_{ii}(t-t')f\big(v_i(t')\big)\varepsilon_i(t') + \eta_{ii}(t')\Big] \\
&+ \sum_{t'<t}\Big[G_{ij}(t-t')g\big(v_j(t')\big)\varepsilon_j(t') + \eta_{ij}(t')\Big] \\
&+ \log m_i(-\infty)
\end{aligned}
$$



$G_{ii}(\tau) \rightarrow$ internal impact

$G_{ij}(\tau) \rightarrow$ cross-impact

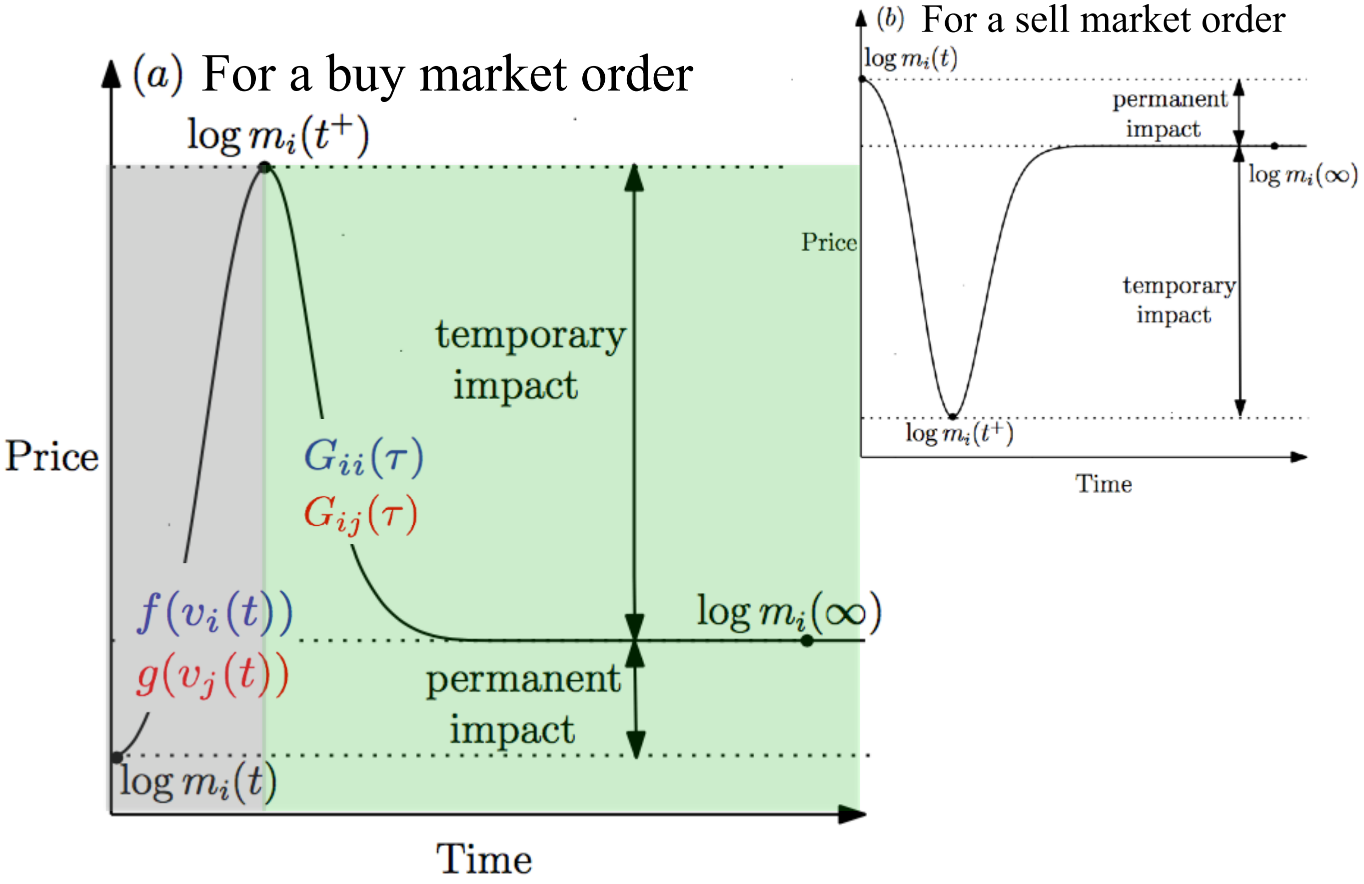
Liquidity→ $f(v_i(t))$

Information→ $g(v_j(t))$

A price impact model—setup

The price is changed by the stock *i* itself and another stock *j*

$$\begin{aligned}\log m_i(t) &= \sum_{t'<t}\Big[G_{ii}(t-t')f\big(v_i(t')\big)\varepsilon_i(t')+\eta_{ii}(t')\Big]\\ &+ \sum_{t'<t}\Big[G_{ij}(t-t')g\big(v_j(t')\big)\varepsilon_j(t')+\eta_{ij}(t')\Big]\\ &+ \log m_i(-\infty)\end{aligned}$$



Assume the impact function

$$G(\tau)=\frac{\Gamma_0}{\left[1+\left(\frac{\tau}{\tau_0}\right)^2\right]^{\beta/2}}+\Gamma$$

The properties of impact function, e.g.
- positive or negative impact
- temporary or permanent impact

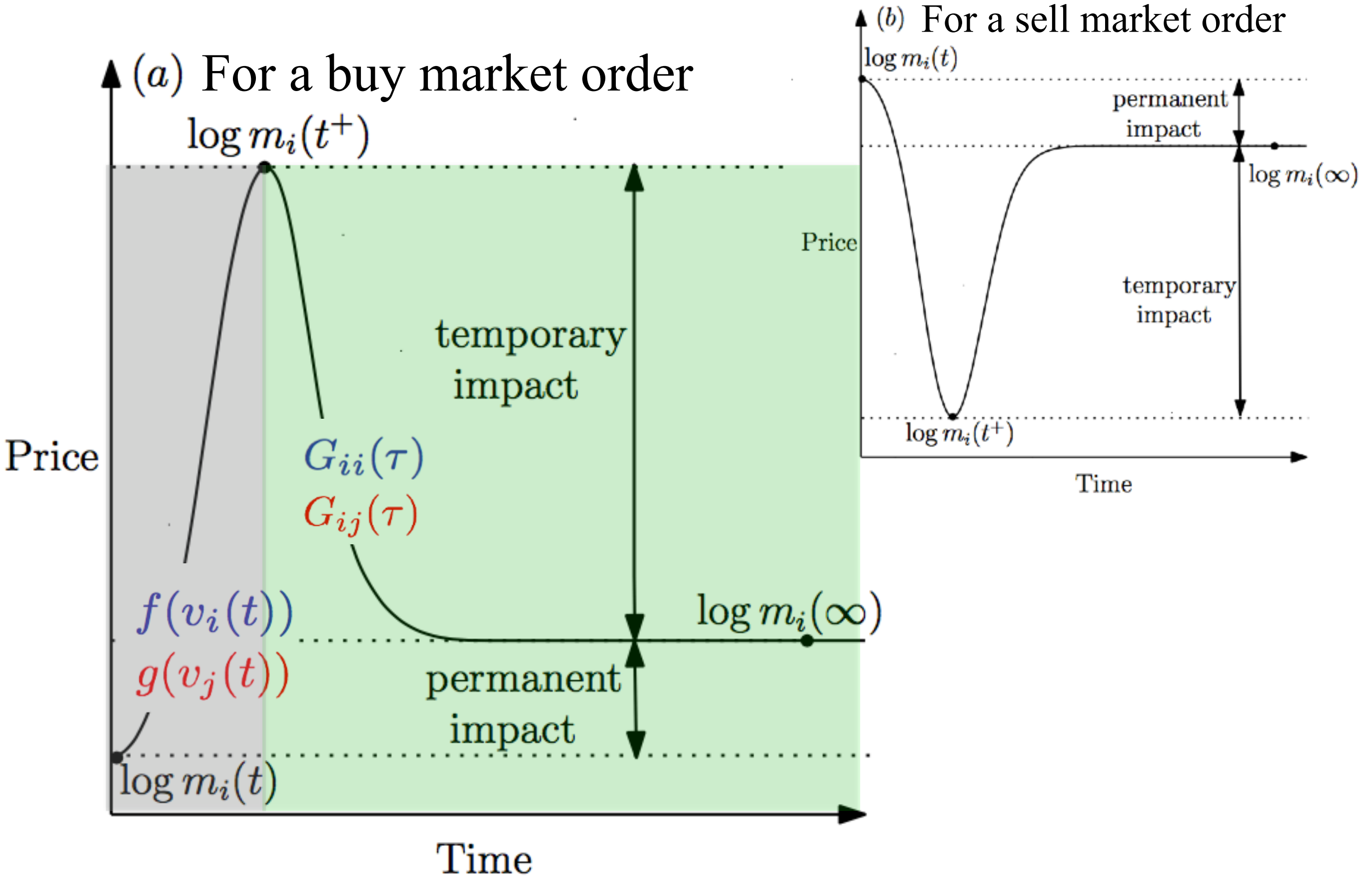
are decided by the fitted parameters

Ref. S. Wang and T. Guhr, arXiv:1609.04890, 2016

A price impact model—setup

The price is changed by the stock *i* itself and another stock *j*

$$\begin{aligned}\log m_i(t) &= \sum_{t'<t}\Big[G_{ii}(t-t')f\big(v_i(t')\big)\varepsilon_i(t') + \eta_{ii}(t')\Big]\\ &+ \sum_{t'<t}\Big[G_{ij}(t-t')g\big(v_j(t')\big)\varepsilon_j(t') + \eta_{ij}(t')\Big]\\ &+ \log m_i(-\infty)\end{aligned}$$



Assume the impact function

temporary impact

$$G(\tau) = \frac{\Gamma_0}{\left[1+\left(\frac{\tau}{\tau_0}\right)^2\right]^{\beta/2}} + \Gamma$$

permanent impact

The properties of impact function, e.g.

- positive or negative impact
- temporary or permanent impact

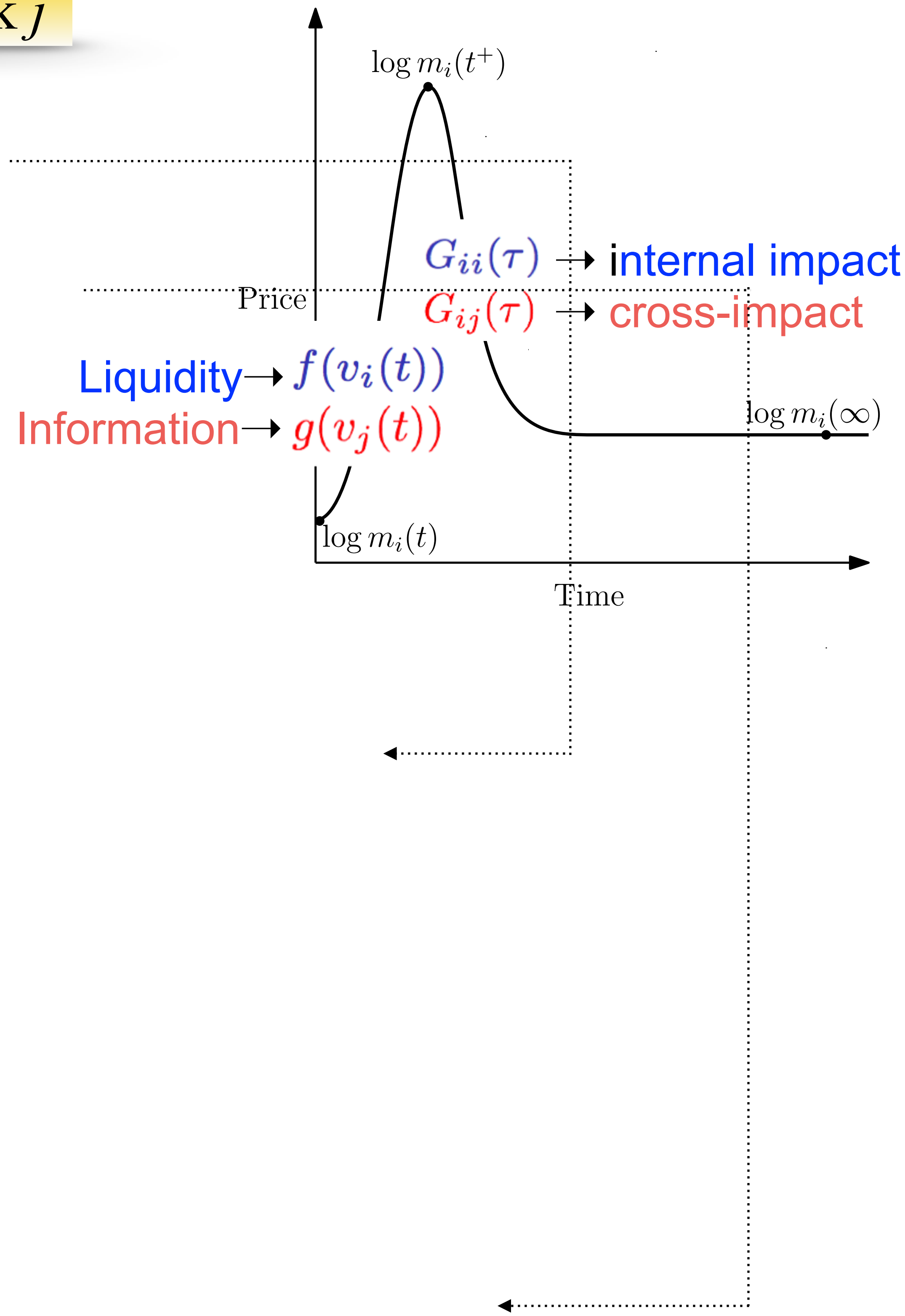
are decided by the fitted parameters

Ref. S. Wang and T. Guhr, arXiv:1609.04890, 2016

A price impact model—setup

The price is changed by the stock *i* itself and another stock *j*

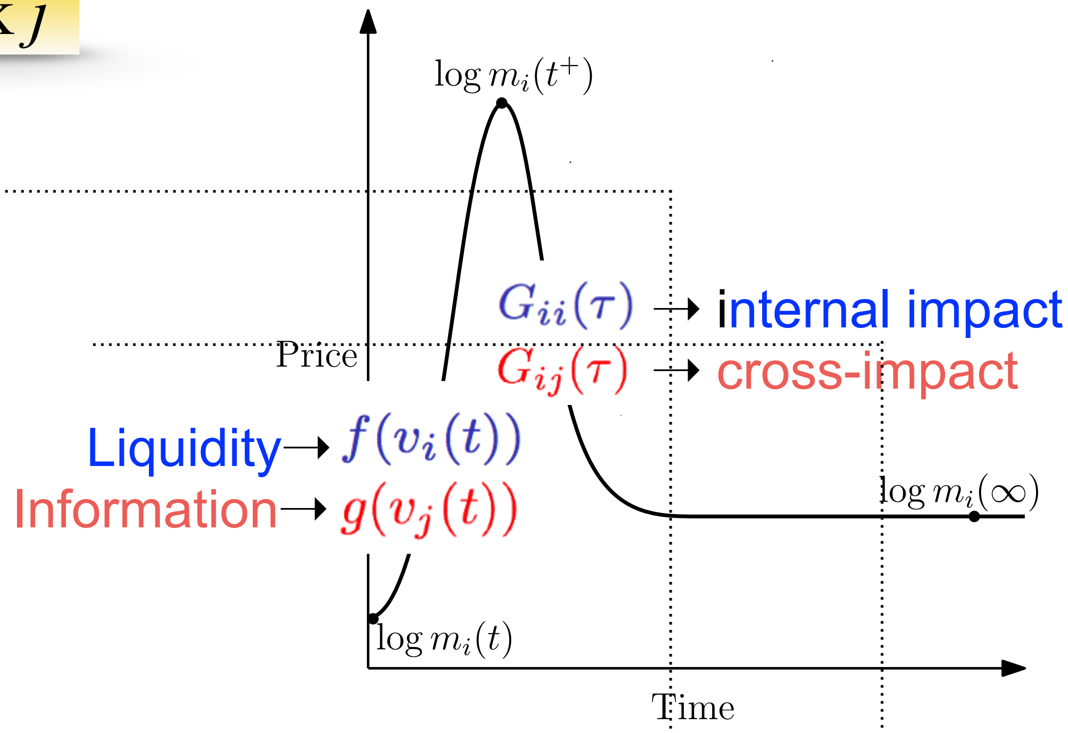
$$\begin{aligned}\log m_i(t) &= \sum_{t'<t}\Big[G_{ii}(t-t')f\big(v_i(t')\big)\varepsilon_i(t')+\eta_{ii}(t')\Big]\\ &+\sum_{t'<t}\Big[G_{ij}(t-t')g\big(v_j(t')\big)\varepsilon_j(t')+\eta_{ij}(t')\Big]\\ &+\log m_i(-\infty)\end{aligned}$$



A price impact model—setup

The price is changed by the stock *i* itself and another stock *j*

$$
\begin{aligned}
\log m_i(t) &= \sum_{t'<t} \left[G_{ii}(t-t') f\left(v_i(t')\right) \varepsilon_i(t') + \eta_{ii}(t') \right] \\
&+ \sum_{t'<t} \left[G_{ij}(t-t') g\left(v_j(t')\right) \varepsilon_j(t') + \eta_{ij}(t') \right] \\
&+ \log m_i(-\infty)
\end{aligned}
$$

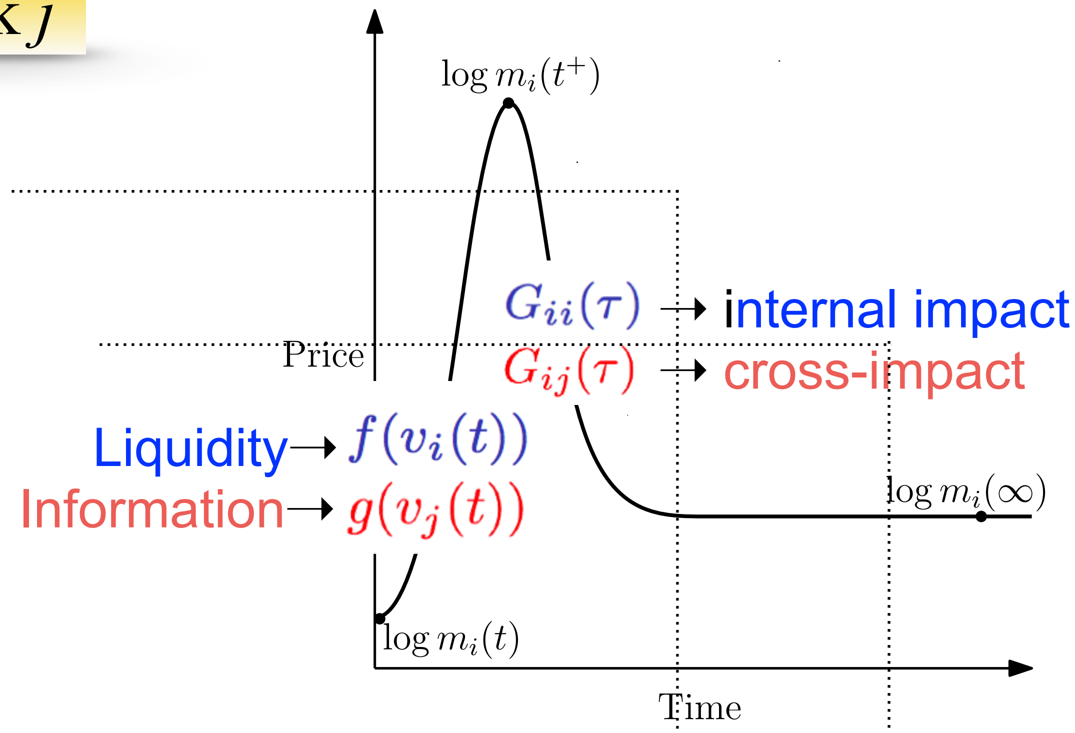


$$
\begin{aligned}
R_{ij}^{(C)}(\tau) &= \left\langle r_{ii}^{(L)}(t,\tau) \varepsilon_j(t) \right\rangle_t \\
&= \sum_{t \leq t' < t+\tau} G_{ii}(t+\tau-t') \left\langle f\left(v_i(t')\right) \right\rangle_t \Theta_{ij}(t'-t) \\
&+ \sum_{t'<t} \left[G_{ii}(t+\tau-t') - G_{ii}(t-t') \right] \left\langle f\left(v_i(t')\right) \right\rangle_t \Theta_{ji}(t-t')
\end{aligned}
$$

A price impact model—setup

The price is changed by the stock *i* itself and another stock *j*

$$\begin{aligned}\log m_i(t) &= \sum_{t'<t}\Big[G_{ii}(t-t')f\big(v_i(t')\big)\varepsilon_i(t') + \eta_{ii}(t')\Big]\\ &+ \sum_{t'<t}\Big[G_{ij}(t-t')g\big(v_j(t')\big)\varepsilon_j(t') + \eta_{ij}(t')\Big]\\ &+ \log m_i(-\infty)\end{aligned}$$



$$\begin{aligned}R_{ij}^{(C)}(\tau) &= \left\langle r_{ii}^{(L)}(t,\tau)\varepsilon_j(t)\right\rangle_t\\ &= \sum_{t\le t'<t+\tau} G_{ii}(t+\tau-t')\left\langle f\big(v_i(t')\big)\right\rangle_t \Theta_{ij}(t'-t)\\ &+ \sum_{t'<t}\Big[G_{ii}(t+\tau-t') - G_{ii}(t-t')\Big]\left\langle f\big(v_i(t')\big)\right\rangle_t \Theta_{ji}(t-t')\end{aligned}$$

$$\begin{aligned}R_{ij}^{(S)}(\tau) &= \left\langle r_{ij}^{(I)}(t,\tau)\varepsilon_j(t)\right\rangle_t\\ &= \sum_{t\le t'<t+\tau} G_{ij}(t+\tau-t')\left\langle g\big(v_j(t')\big)\right\rangle_t \Theta_{jj}(t'-t)\\ &+ \sum_{t'<t}\Big[G_{ij}(t+\tau-t') - G_{ij}(t-t')\Big]\left\langle g\big(v_j(t')\big)\right\rangle_t \Theta_{jj}(t-t')\end{aligned}$$

A price impact model—setup

The cross-response functions

$$R_{ij}(\tau) = R_{ij}^{(C)}(\tau) + R_{ij}^{(S)}(\tau)$$

The passive and active average cross-response functions

$$R_i^{(p)}(\tau) = R_{i,0}^{(p,C)}(\tau)\left\langle f_i^{(p)}(v_i)\right\rangle + R_{i,0}^{(p,S)}(\tau)\left\langle g_i^{(p)}(v_j)\right\rangle_j$$

$$R_i^{(a)}(\tau) = R_{i,0}^{(a,C)}(\tau)\left\langle f_i^{(a)}(v_j)\right\rangle_j + R_{i,0}^{(a,S)}(\tau)\left\langle g_i^{(a)}(v_i)\right\rangle$$

Definitions of average cross-responses and sign correlators

$$R_i^{(p)}(\tau) \; = \; \langle R_{ij}(\tau)\rangle_j$$

$$R_i^{(a)}(\tau) \; = \; \langle R_{ji}(\tau)\rangle_j$$

$$\Theta_i^{(p)}(\tau) \; = \; \langle \Theta_{ij}(\tau)\rangle_j$$

$$\Theta_i^{(a)}(\tau) \; = \; \langle \Theta_{ji}(\tau)\rangle_j$$

According to the empirical analysis,

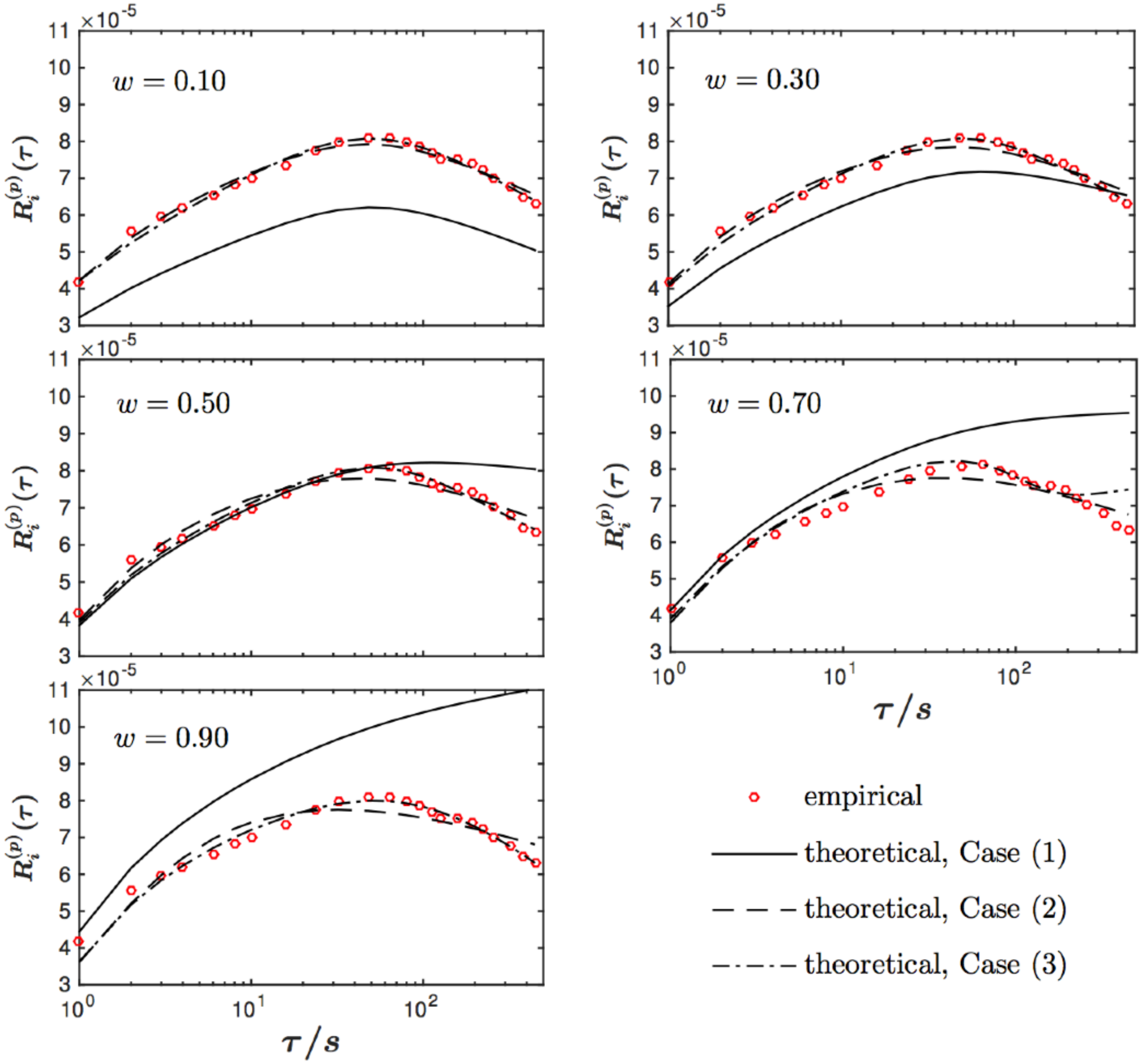
$$\langle f(v_i(t))\rangle_t = v_i^{\delta_i}(t) \;\; , \; \langle g(v_j(t))\rangle_t = v_j^{\delta_j}(t)$$

$$\langle f_i^{(p)}(v_i(t))\rangle_t,\; \langle f_i^{(a)}(v_i(t))\rangle_{t,j},\; \langle g_i^{(p)}(v_j(t))\rangle_{t,j},\; \langle g_i^{(a)}(v_i(t))\rangle_t \rightarrow \text{constant}$$

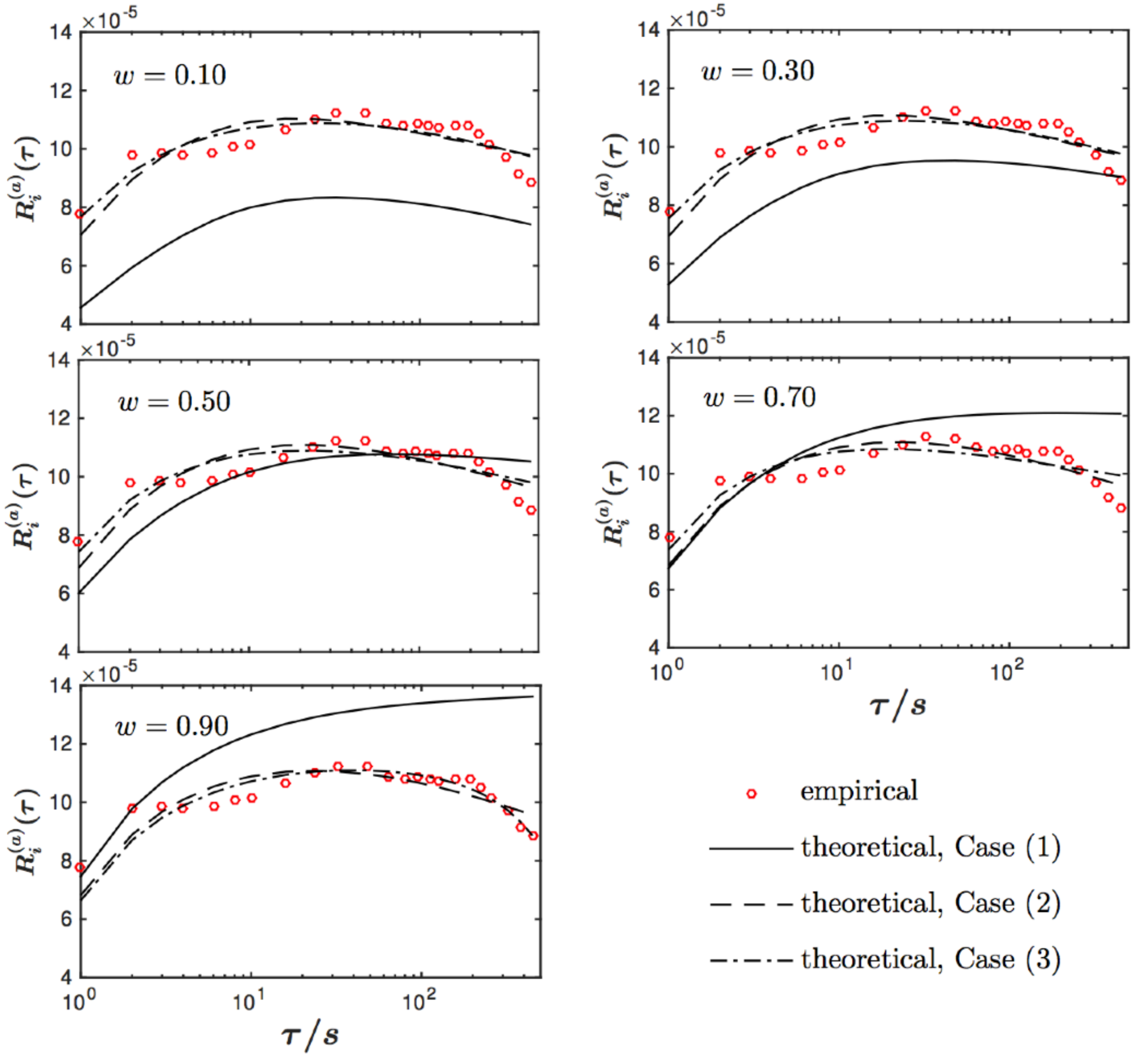
$$\frac{\langle R_{ij}(\tau)\rangle_{i \text{ or } j}}{\text{corresponding average impact of traded volumes}} \rightarrow R_{i,0}^{(p,C)},\; R_{i,0}^{(p,S)},\; R_{i,0}^{(a,C)},\; R_{i,0}^{(a,S)}$$

A price impact model—simulations of responses

Passive average cross-responses



Active average cross-responses

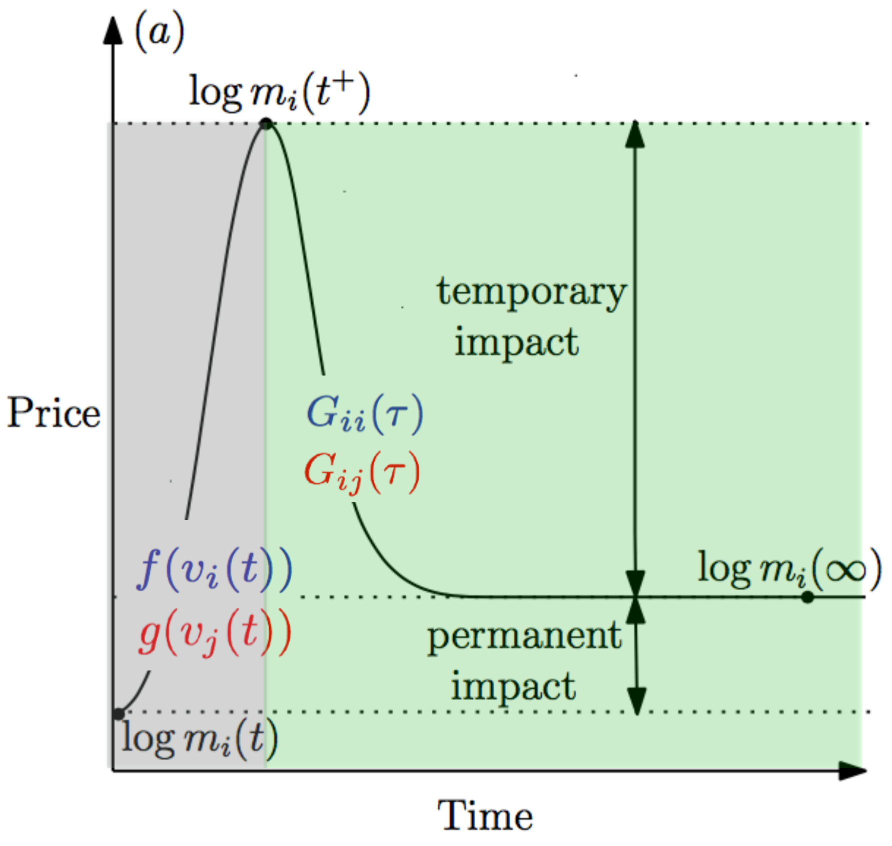


Ref. S. Wang and T. Guhr, arXiv:1609.04890, 2016

The stock i is MSFT in 2008, and the pairwise stocks j are other 30 stocks with the largest average number of daily trades in S&P 500 index of 2008.

A price impact model—impact functions

Sketch of price impacts



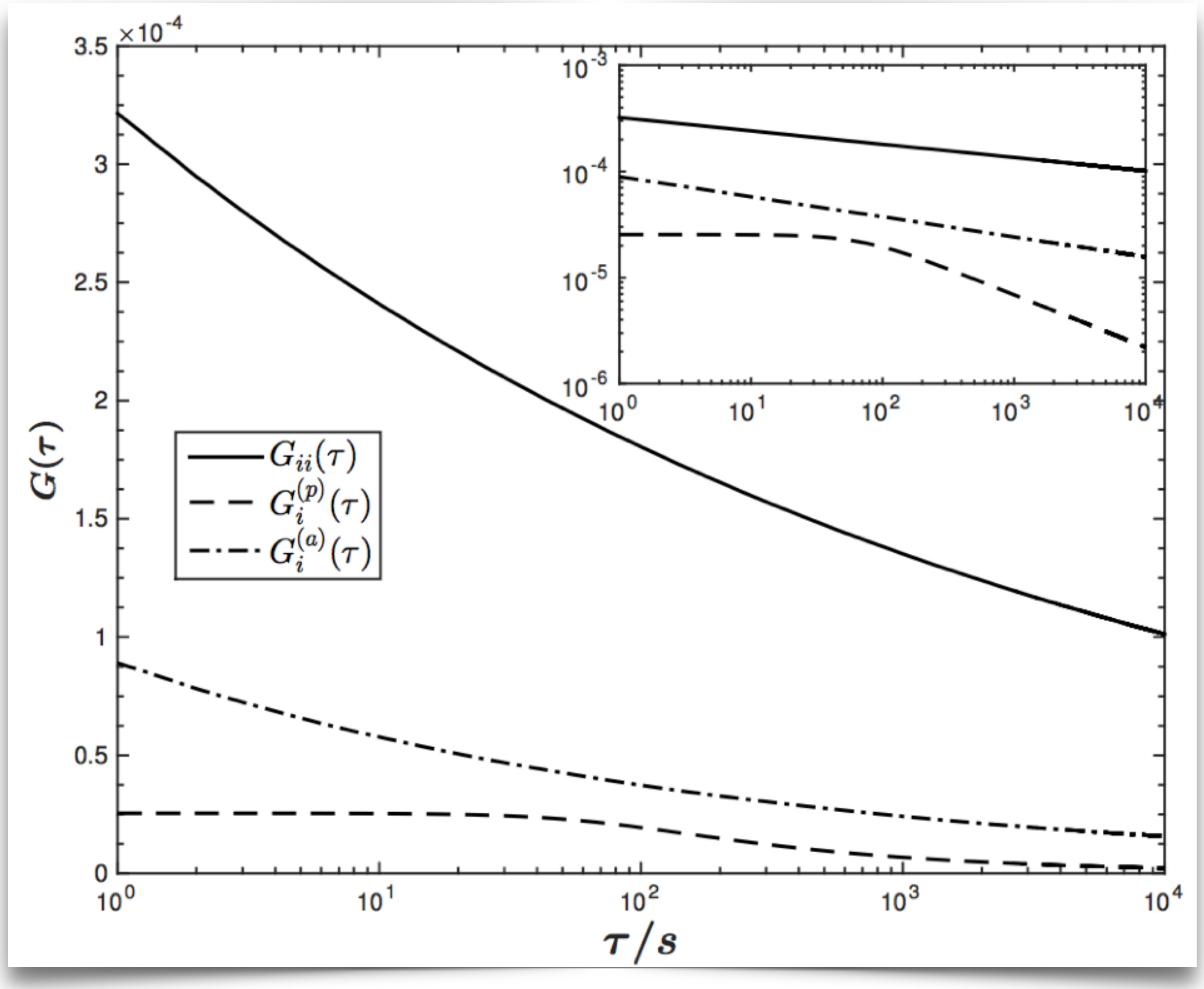
After averaging,

$$G_{ij}(\tau) \rightarrow G_i^{(p)}(\tau),\ G_i^{(a)}(\tau)$$

The simulated impact function

$$G(\tau) = \frac{\Gamma_0}{\left[1+\left(\frac{\tau}{\tau_0}\right)^2\right]^{\beta/2}} + \Gamma$$

Simulations of impact functions



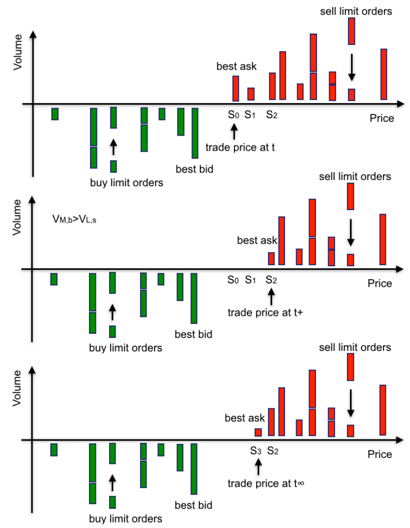
The stock *i* is MSFT in 2008

impact function	Γ $(\times 10^{-10})$	Γ_0 $(\times 10^{-4})$	τ_0 [s]	β
$G_{ii}(\tau)$	0.5	5.12	0.025	0.13
$G_i^{(p)}(\tau)$	0	0.25	70.873	0.49
$G_i^{(a)}(\tau)$	0	2.57	0.004	0.19

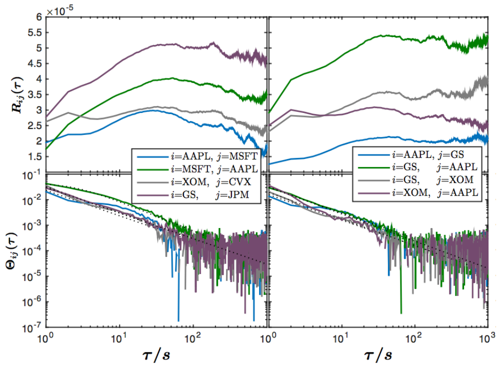
Ref. S. Wang and T. Guhr, arXiv:1609.04890, 2016

Summary

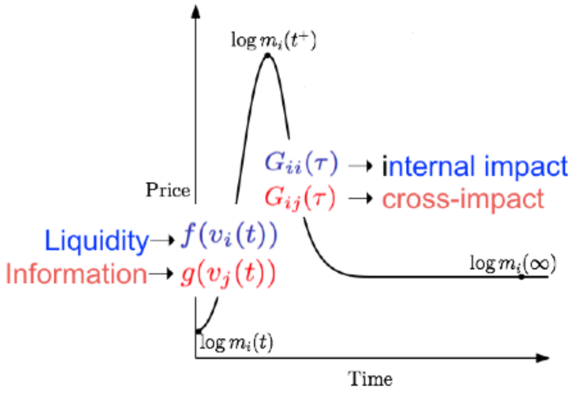
- The price formation:
  - due to the interaction of market orders and limit orders, where the liquidity plays an important role.



- Empirical results:
  - for cross-responses of stock pairs
  - for average cross-responses
  - the influencing and influenced stocks.



- A price impact model:
  - an internal and a cross-impact function
  - two response components related to the cross- and the self-correlators, respectively
  - the comparison of empirical and simulated results
  - the internal, active and passive impact functions.



References

[1] Jean-Philippe Bouchaud, Yuval Gefen, Marc Potters, and Matthieu Wyart. Fluctuations and response in financial markets: the subtle nature of ‘random’ price changes. *Quantitative Finance* **4**, 176 (2004).

[2] Shanshan Wang, Rudi Schäfer, and Thomas Guhr. Cross-response in correlated financial markets: individual stocks. *The European Physical Journal B* **89**, 105 (2016)

[3] Shanshan Wang, Rudi Schäfer, and Thomas Guhr. Average cross-responses in correlated financial market. *The European Physical Journal B* **89**, 207 (2016)

[4] Shanshan Wang and Thomas Guhr. Microscopic understanding of cross-responses between stocks: a two-component price impact model. *arXiv preprint arXiv:1609.04890*, 2016

Thank you for your attention!
Any questions?