Many-body quantum chaos in dual models - from coupled cat maps to kicked spin chains

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Outline of the talk

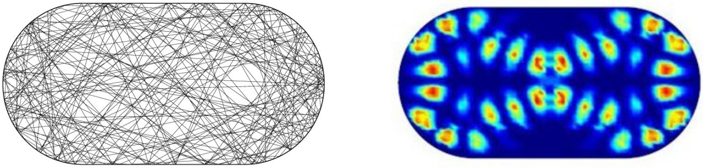
– Model for many-body quantum chaos: Coupled cat map lattices

– Classical duality

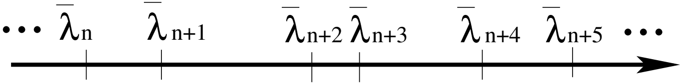
– Quantum duality

– Minimal realization: Quantum $1/2$-spin chain

"Single-particle" Quantum Chaos



Quantum: $-\Delta\varphi_n = \lambda_n\varphi_n, \ \varphi_n|_{\partial\Omega} = 0 \qquad \varphi_n \in L^2(\Omega)$



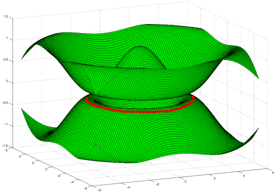
What is the statistics of $\bar\lambda_n = \lambda_n/\Delta$? ($\Delta$-mean level spacing)

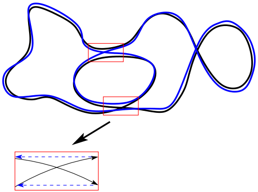
Classical Chaos $\Longleftrightarrow$ Universal (RMT) spectral statistics

From temporal to spacial-temporal chaos

A PDE generating spatial-temporal chaotic dynamics

$$" \partial_t^2\phi(x,t) - \partial_x^2\phi(x,t) + V'(\phi(x,t)) = 0, \quad \phi(x+L,t) = \phi(x,t) "$$





1) **PO** -are **2D toric surfaces**, rather than 1D lines

2) **Partner 2D tori**, rather than 1D **partner orbits**

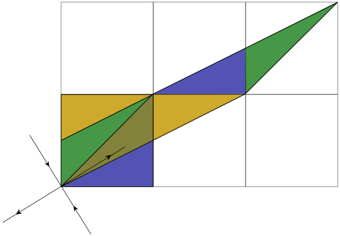
Can we make sense out of it?

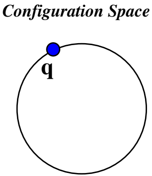
Model: Coupled cat map lattice

B.G, V. Osipov (2016), B.G., L Han, R. Jafari, A. K. Saremi, P. Cvitanović (2019)

Single Cat Map: $\mathbb{T}^2 \to \mathbb{T}^2$

Phase space: $q_t, p_t \in [0,1)$, **windings** $\mathrm{m}_t = (m_t^q, m_t^p) \in \mathbb{Z}$



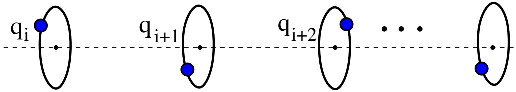


$$\begin{pmatrix} q_{t+1} \\ p_{t+1} \end{pmatrix} = \begin{pmatrix} a & 1 \\ ab-1 & b \end{pmatrix} \begin{pmatrix} q_t \\ p_t \end{pmatrix} - \begin{pmatrix} \mathrm{m}_t^q \\ \mathrm{m}_t^p \end{pmatrix},$$

$a, b \in \mathbb{Z}$. Chaos if $s \equiv |a+b| > 2$

Newton form: $\Delta q_t \equiv q_{t+1} - 2q_t + q_{t-1} = (s-2)q_t - \mathrm{m}_t$

Coupled-Cat Maps: $\mathbb{T}^{2N} \to \mathbb{T}^{2N}$



$$S(\mathsf{q}_t, \mathsf{q}_{t+1}) = S_0(\mathsf{q}_t, \mathsf{q}_{t+1}) + S_{\mathrm{int}}(\mathsf{q}_t), \; \mathsf{q}_t = (q_{1,t}, q_{2,t} \dots q_{N,t})$$

N Interacting cat maps, $q_{n,t}, p_{n,t} \in [0,1)$:

$$S_0 = \sum_{n=1}^{N} S_{\mathrm{cat}}(q_{n,t}, q_{n,t+1}) + V(q_{n,t}); \qquad S_{\mathrm{int}} = \underbrace{-\sum_{n=1}^{N} q_{n,t} q_{1+n,t}}_{\textit{interactions}}$$

Equations of motion:

$$p_{n,t} = -\frac{\partial S}{\partial q_{n,t}} \qquad p_{n,t+1} = \frac{\partial S}{\partial q_{n,t+1}}$$

Coupled-Cat Maps $V = 0$

$$Z_{t+1} = \mathcal{B}_N Z_t \mod 1, \qquad Z_t = (q_{1,t}, p_{1,t}, \ldots q_{N,t}, p_{N,t})^\intercal,$$

with $2N \times 2N$ matrix $\mathcal{B}_N$ given by:

$$\mathcal{B}_N = \begin{pmatrix} A & B & \mathbf{0} & \ldots & \mathbf{0} & B \\ B & A & B & \ldots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & B & A & \ldots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & A & B \\ B & \mathbf{0} & \mathbf{0} & \ldots & B & A \end{pmatrix}, A = \begin{pmatrix} a & 1 \\ ab-1 & b \end{pmatrix}, B = -\begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix}$$

Lyapunov exponents:

$$\cosh \lambda_k = (a+b)/2 - \cos(2\pi k/N), \qquad k = 1, \ldots N$$

Full chaos: $|\mathrm{Re}\lambda_k| > 0$, i.e. $s \equiv |a+b| > 4$

Newtonian form of equations

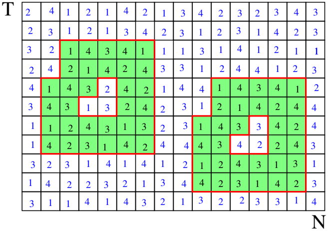
$$\boxed{\Delta q_{n,t} = (s-4)q_{n,t} + V'(q_{n,t}) - \mathrm{m}_{n,t}}$$

Discrete Laplacian $\Delta = \partial_n^2 + \partial_t^2$:

$$\Delta f_{n,t} \equiv f_{n+1,t} + f_{n-1,t} + f_{n,t+1} + f_{n,t-1} - 4f_{n+1,t}$$

Discretized field theory!

Array $\{\mathrm{m}_{n,t} | n = 1, \ldots, N, t = 1, \ldots T\}$ - provides symbolic encoding of periodic orbits i.e., tori



$$\mathbb{M}_\Gamma = \begin{pmatrix} \mathrm{m}_{1,1} & \mathrm{m}_{2,1} & \cdots & \mathrm{m}_{N,1} \\ \mathrm{m}_{1,2} & \mathrm{m}_{2,2} & \cdots & \mathrm{m}_{N,2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{m}_{1,T} & \mathrm{m}_{2,T} & \cdots & \mathrm{m}_{N,T} \end{pmatrix}$$

2D linear symbolic dynamics

$\surd$ **Small alphabet** $\mathrm{m}_{n,t} \in \mathcal{A} = \{-3, \ldots, s-1\}$ (independ. of N)

– Splitting of the alphabet: $\mathcal{A} = \mathcal{A}_{\text{in}} \cup \mathcal{A}_{\text{ex}}$,

$\mathcal{A}_{\text{in}} = \{0, \ldots, s-4\}, \quad \mathcal{A}_{\text{ex}} = \{-3, -2, -1\} \cup \{s-1, s-2, s-3\}$

– Trivial grammar rules for $\mathbb{M}_{\text{in}} = \{\mathrm{m}_{n,t} \in \mathcal{A}_{\text{in}}\}$ i.e., anything is allowed if all $\mathrm{m}_{n,t} \in \mathcal{A}_{\text{in}}$!

$\surd$ **Uniqueness** + $\underline{\Gamma \text{ can be easily restored from } \mathbb{M}_\Gamma}$

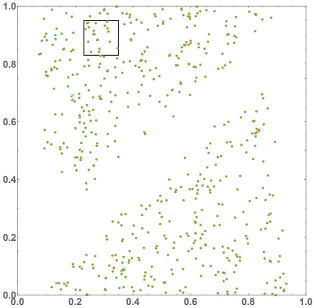
$\surd$ **Locality** ($p \times p$ square of symbols around (n, t) defines approx. position of the n'th particle at the time t)

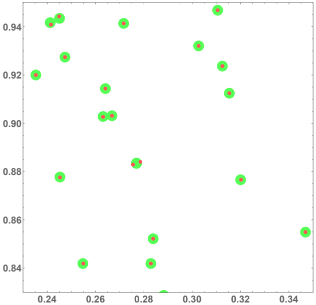
B.G., L Han, R. Jafari, A. K. Saremi, P. Cvitanović (2019)

2D Partner orbits ("Sieber-Richter pairs")

$$\Gamma \quad \begin{pmatrix}
9&1&1&8&4&7&7&0&5&8&1&7&1&0&6&2&1&2&9&6&7\\
4&7&1&0&2&4&4&1&6&5&4&1&3&2&3&3&9&6&9&0&7\\
4&8&3&8&8&6&9&7&1&5&6&6&6&3&3&6&4&6&9&4&0\\
9&0&4&9&5&0&1&9&3&0&8&1&8&0&1&4&7&3&6&3&4\\
2&9&4&9&9&4&1&9&0&1&1&7&4&7&6&3&4&6&8&2&2\\
4&6&3&7&3&5&6&5&1&5&2&6&3&3&0&7&2&2&4&5&6\\
8&1&2&6&6&4&6&2&7&8&3&3&0&0&1&4&3&4&4&1&8\\
2&6&3&1&1&1&0&3&1&7&3&3&6&7&6&6&5&3&2&4&1\\
4&5&6&6&9&5&3&2&5&0&8&4&3&6&1&2&7&9&0&2&9\\
5&7&3&6&1&1&1&0&2&3&9&5&3&7&5&0&5&9&0&8&7\\
5&7&6&3&9&5&5&3&3&4&8&1&6&5&3&9&6&5&4&1&3\\
1&6&3&7&6&5&9&3&2&3&1&2&9&0&3&8&7&3&3&0&6\\
1&3&4&3&0&2&8&0&6&2&1&7&9&1&5&7&5&1&2&2&0\\
0&6&3&7&8&2&1&1&1&0&8&3&8&8&6&9&7&1&4&0&4\\
0&5&3&4&9&7&4&7&7&9&0&4&9&5&0&1&9&3&2&3&6\\
3&5&2&9&8&3&9&0&7&9&9&4&9&9&4&1&9&0&0&6&8\\
1&5&2&7&4&1&6&8&5&4&6&3&7&2&7&6&5&1&8&8&7\\
9&4&1&6&6&7&1&1&8&9&1&2&6&1&7&6&2&7&2&2&5\\
5&7&1&0&3&3&3&0&6&9&6&3&1&9&2&0&3&1&9&3&2\\
6&1&2&1&1&0&5&0&6&6&5&6&6&9&5&3&2&5&6&4&4\\
1&9&9&2&0&8&6&7&5&9&7&3&6&1&1&1&0&2&8&2&1\\
3&1&8&3&4&3&6&1&9&8&7&6&3&9&5&5&3&3&6&2&3\\
6&2&1&7&6&7&7&3&5&9&4&7&5&5&1&2&9&0&8&6&9\\
7&8&8&9&6&1&1&4&9&3&8&5&4&8&4&2&1&6&3&7&9\\
5&8&7&5&3&8&7&1&8&0&8&5&4&3&0&4&0&3&4&1&7
\end{pmatrix}$$

$$\Gamma' \quad \begin{pmatrix}
9&1&1&8&4&7&7&0&5&8&1&7&1&0&6&2&1&2&9&6&7\\
4&7&1&0&2&4&4&1&6&5&4&1&3&2&3&3&9&6&9&0&7\\
4&8&3&8&8&6&9&7&1&5&6&6&6&3&3&6&4&6&9&4&0\\
9&0&4&9&5&0&1&9&3&0&8&1&8&0&1&4&7&3&6&3&4\\
2&9&4&9&9&4&1&9&0&1&1&7&4&7&6&3&4&6&8&2&2\\
4&6&3&7&2&7&6&5&1&5&2&6&3&3&0&7&2&2&4&5&6\\
8&1&2&6&1&7&6&2&7&8&3&3&0&0&1&4&3&4&4&1&8\\
2&6&3&1&9&2&0&3&1&7&3&3&6&7&6&6&5&3&2&4&1\\
4&5&6&6&9&5&3&2&5&0&8&4&3&6&1&2&7&9&0&2&9\\
5&7&3&6&1&1&1&0&2&3&9&5&3&7&5&0&5&9&0&8&7\\
5&7&6&3&9&5&5&3&3&4&8&1&6&5&3&9&6&5&4&1&3\\
1&6&3&7&6&5&9&3&2&3&1&2&9&0&3&8&7&3&3&0&6\\
1&3&4&3&0&2&8&0&6&2&1&7&9&1&5&7&5&1&2&2&0\\
0&6&3&7&8&2&1&1&1&0&8&3&8&8&6&9&7&1&4&0&4\\
0&5&3&4&9&7&4&7&7&9&0&4&9&5&0&1&9&3&2&3&6\\
3&5&2&9&8&3&9&0&7&9&9&4&9&9&4&1&9&0&0&6&8\\
1&5&2&7&4&1&6&8&5&4&6&3&7&3&5&6&5&1&8&8&7\\
9&4&1&6&6&7&1&1&8&9&1&2&6&6&4&6&2&7&2&2&5\\
5&7&1&0&3&3&3&0&6&9&6&3&1&1&1&0&3&1&9&3&2\\
6&1&2&1&1&0&5&0&6&6&5&6&6&9&5&3&2&5&6&4&4\\
1&9&9&2&0&8&6&7&5&9&7&3&6&1&1&1&0&2&8&2&1\\
3&1&8&3&4&3&6&1&9&8&7&6&3&9&5&5&3&3&6&2&3\\
6&2&1&7&6&7&7&3&5&9&4&7&5&5&1&2&9&0&8&6&9\\
7&8&8&9&6&1&1&4&9&3&8&5&4&8&4&2&1&6&3&7&9\\
5&8&7&5&3&8&7&1&8&0&8&5&4&3&0&4&0&3&4&1&7
\end{pmatrix}$$





Classical duality

Space-time Duality

$$\Delta q_{n,t} = (s-4)q_{n,t} + V'(q_{n,t}) - \mathrm{m}_{n,t}$$

Space-time symmetry: $t \longleftrightarrow n \qquad \Longrightarrow$

N-particle POs $\{\Gamma\}$ of period T $\Longleftrightarrow$ T-particle POs $\{\Gamma'\}$ of period N

Actions: $S(\Gamma) = S(\Gamma')$, Stabilities: $A_\Gamma = A_{\Gamma'}$

Corollary:
$$\underbrace{T^2 \prod_{k=1}^{N-1} 4\sin^2\left(\frac{\pi k T}{N}\right)}_{\#\text{ of } N\text{-particle PO with period } T} = \underbrace{N^2 \prod_{m=1}^{T-1} 4\sin^2\left(\frac{\pi m N}{T}\right)}_{\#\text{ of } T\text{-particle PO with period } N}$$

Space-time Duality implications. Correlators

Pair of local operators, $< Q_1 > = < Q_2 > = 0$**:**

$$Q_1(0,0) = Q_1(q_{0,0}, p_{0,0}), \qquad Q_2(n,t) = Q_2(q_{n,t}, p_{n,t})$$

Finite speed of propagation $\Longrightarrow$

$$< Q_1(0,0) Q_2(n,t) > = 0, \quad \text{for} \quad n > t$$

Duality $\Longrightarrow$

$$< Q_1(0,0) Q_2(n,t) > = 0, \quad \text{for} \quad n < t$$

Non-trivial correlations are possible only at $n = t$ **!**

Quantum duality

Quantum duality. Semiclassical approach

Evolution U_N is $L^N \times L^N$ unitary matrix, $2\pi L = \hbar_{eff}^{-1}$
Hannay, Berry (1980); Keating (1991); Rivas, et. al. (2000)

Trace formula:

$$\mathrm{Tr}\,(U_N)^T = \sum_{\Gamma \in \mathrm{PO}} A_\Gamma \exp(-iS_\Gamma/\hbar_{eff}).$$

All entries are symmetric under exchange $N \leftrightarrow T \Longrightarrow$

Quantum duality:

$$\mathrm{Tr}\,(U_N)^T = \mathrm{Tr}\,(U_T)^N$$

Whether it exact or semiclassical approximation?

Kicked representation

Step I

$$\begin{pmatrix} q_{n,t+1} \\ p_{n,t+1} \end{pmatrix} = \begin{pmatrix} a & 1 \\ ab-1 & b \end{pmatrix} \begin{pmatrix} q_{n,t} \\ p_{n,t} \end{pmatrix} - \mod \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

Step II

$$\begin{cases} x_{n,t} = x_{n,t+1} \\ p_{n,t+1} = p_{n,t} - (x_{n-1,t} + x_{n+1,t}) - V'(x_{n,t}) \mod 1 \end{cases}$$

Kicked Hamiltonian:

$$H = H_I + H_K \sum_{\tau=-\infty}^{\infty} \delta(t-\tau)$$

$$H_I = -\sum_{n=1}^{N} q_{n,t} q_{n+1,t} - V(q_{n,t}), \qquad H_K = \sum_{n=1}^{N} H_{cat}(q_{n,t}, p_{n,t})$$

Kicked representation

Quantum evolution

$$U = U_I U_K, \quad U_I = e^{-iL2\pi H_I}, U_K = e^{-iL2\pi H_K}$$

$2\pi L = \hbar^{-1}$ the inverse Planck's constant

Interaction part:

$$\langle \mathbf{q} | U_I | \mathbf{q}' \rangle = \underbrace{e^{\frac{i2\pi}{L} \sum_{n=1}^{N} -q_n q_{n+1} + LV(q_n/L)} \delta(\mathbf{q} - \mathbf{q}')}_{\text{Diagonal}}$$

Kicked part:

$$U_K = \bigotimes_{n=1}^{N} u_K, \qquad \langle q | u_K | q' \rangle = \underbrace{\frac{i}{\sqrt{L}} e^{\frac{i\pi}{L}(aq^2 - 2qq' + bq'^2)}}_{\text{Single cat evolution}}$$

Generalization: $L^N \times L^N$ time evolution $U = U_I U_K$

$$\langle\vec{\sigma}|U_I|\vec{\sigma}'\rangle = \exp\left(i\sum_{n=1}^{N} f_I(\sigma_n,\sigma_{n+1})\right)\delta_{\boldsymbol{\sigma},\boldsymbol{\sigma'}}\,,$$

$$U_K = \bigotimes_{n=1}^{N} u_K \qquad \text{with} \quad \langle\sigma|u_K|\sigma'\rangle = \frac{1}{\sqrt{L}}e^{if_K(\sigma,\sigma')}\,.$$

$L^T \times L^T$ Dual evolution $\widetilde{U} = \widetilde{U}_I\widetilde{U}_K$, $f_I \leftrightarrow f_K$:

$$\langle\vec{\sigma}|\widetilde{U}_I|\vec{\sigma}'\rangle = \exp\left(i\sum_{t=1}^{T} f_K(\sigma_t,\sigma_{t+1})\right)\delta_{\boldsymbol{\sigma},\boldsymbol{\sigma'}}$$

$$\widetilde{U}_K = \bigotimes_{t=1}^{T} \tilde{u}_K \quad \text{with} \quad \langle\sigma|\tilde{u}_K|\sigma'\rangle = \frac{1}{\sqrt{L}}e^{if_I(\sigma,\sigma')}$$

Duality: $\mathrm{Tr}\,U^T = \mathrm{Tr}\,\widetilde{U}^N$

Dual models

In general $\tilde{U}_T$ is **non-unitary!**
M. Akila, D. Waltner, B. G., P. Braun, T. Guhr, (2016)

If both f_I, f_K are real i.e.,

$$\langle\sigma|u_K|\sigma'\rangle = \frac{1}{\sqrt{L}}e^{if_K(\sigma,\sigma')}, \qquad \langle\sigma|\tilde{u}_K|\sigma'\rangle = \frac{1}{\sqrt{L}}e^{if_I(\sigma,\sigma')}$$

are complex Hadamard matrices, then $\tilde{U}_T$ is **unitary!**

a) If $U \neq \tilde{U}_T$ – **Unitary dual**

b) If, in addition $\tilde{U} = U$ (e.g., $f_I = f_K$) – **Unitary self-dual**

Example: Coupled cat lattice model

Dual models – Maximum of chaos

– Wigner-Dyson statistics for typical parameters

Resilient to many body localisation
Bertini, Kos, Prosen (2018)
M. Akila, D. Waltner, B. G., P. Braun, T. Guhr (2019)

– Linear growth of entanglement entropies with time.
Bertini, Kos, Prosen (2018)

Flat entanglement spectrum
Gopalakrishnan, S. and Lamacraft (2019)

– Exponentially decaying correlators
Bertini, Kos, Prosen (2019)

In spite of full chaos amenable to the exact analytical treatment.

2-Point correlators. Strictly local operators.

For traceless $Q_1(0)$, $Q_2(n)$

$$C_{1,2} = L^{-N}\mathrm{Tr}\left(U^T Q_1 U^{-T} Q_2\right)$$

Dual representation:

$$C_{1,2} = \mathrm{Tr}\big(W^{N-n-1}\overline{W}_{q_1} W^{n-1} W_{q_2}\big),$$

For $N > n + T$,

$$W^{N-n-1} = |\Psi_0\rangle\langle\Psi_0| \qquad \Longrightarrow \qquad C_{1,2} = 0$$

In general

$$< Q_1(z_1)Q_2(z_2) > = < Q_1 >< Q_2 >$$

2-Point correlators. Non-local operators.

$$C(n,T) = L^{-N}\mathrm{Tr}\, U^T \Sigma_0 U^{-T} \Sigma_n$$

For non-local operators

$$\Sigma_0 = Q_1(0)Q_2(1), \qquad \Sigma_n = Q_3(n)Q_4(n+1)$$

$$C(n,T) = \langle \bar{\Phi}_{\Sigma_0} | \mathbb{T}^{T-2} | \Phi_{\Sigma_n} \rangle \delta(n-T),$$

$\mathbb{T}$ is $2L \times 2L$ doubly stochastic matrix $\Longrightarrow$

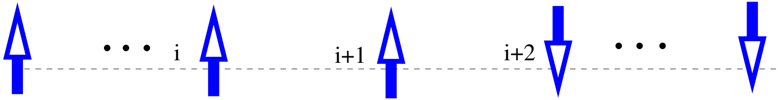
$$\mathrm{Spec}(\mathbb{T}): \quad \mu_1 = 1 \geq \mu_2 \geq \cdots \geq \mu_{2L}$$

Generically, for traceless Q_i's $C(n,T)$ decays exponentially!

Minimal realization $L = 2$:

Kicked spin-1/2 chain

Kicked Spin-1/2 Chain



$$H = H_I + H_K \sum_{\tau=-\infty}^{\infty} \delta(t-\tau)$$

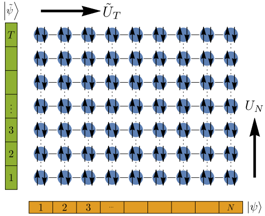
$$H_I = \sum_{n=1}^{N} J\, \hat{\sigma}_n^z \hat{\sigma}_{n+1}^z + h \hat{\sigma}_n^z, \qquad H_K = b \sum_{n=1}^{N} \hat{\sigma}_n^x$$

Cyclic boundary cond. $\hat{\sigma}_n^\alpha = \hat{\sigma}_{n+N}^\alpha$

$$U_N(J,b,h) = e^{-iH_I} e^{-iH_K}$$

Dynamics depend on b, J, h, vary from integrable to chaotic
T. Prosen (2000, 2001)

Dual Kicked Spin-1/2 Chain



$$Z(N,T) = \mathrm{Tr}\,\underbrace{[U_N(J,b,h)]^T}_{\text{time evolution}} = \mathrm{Tr}\,\underbrace{[U_T(\tilde{J},\tilde{b},h)]^N}_{\text{space evolution}}$$

Model is self-dual ($J = \tilde{J}, b = \tilde{b}$) if $J = b = \pi/4$, arbitrary h

$$u_I = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i(\pi/4+h)} & e^{i\pi/4} \\ e^{i\pi/4} & e^{-i(\pi/4-h)} \end{pmatrix}, u_K = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

Spectrum is Wigner-Dyson, except

fully integrable: $h = 0, h = \pi/2$, "mystery": $h = \pi/3$

Correlators in Spin-1/2 Chain

$$\mathbb{T} = \frac{1}{2}\begin{pmatrix} a & b & b & a \\ b & a & a & b \\ b & a & a & b \\ a & b & b & a \end{pmatrix}, \qquad a = \cos^2 h, b = \sin^2 h$$

$$\text{Spec}(\mathbb{T}) : \mu_1 = 1, \mu_2 = \cos 2h, \mu_3 = \mu_4 = 0$$

$$< \sigma_0^\alpha(0)\sigma_1^\beta(0)\sigma_n^\gamma(t)\sigma_{n+1}^\delta(t) >= \delta(n-t)\ \mathcal{C}_{\alpha\beta}^{\delta\gamma}\ \cos^n 2h,$$

$\mathcal{C}_{yz}^{yz} = 1$, $\mathcal{C}_{yx}^{xz} = \tan^2 2h$, $\mathcal{C}_{yx}^{yz} = \mathcal{C}_{yz}^{xz} = -\tan 2h$

For two integrable points $h = 0, \pi/2$, $|\cos 2h| = 1$. No decay of correlations.

Summary: Self-dual models

☐ **Spatial propagation = Time propagation**

$$\text{PO(N,T)} \Longleftrightarrow \text{PO(T,N)} \qquad \mathrm{tr}\,(U_N)^T = \mathrm{tr}\,(U_T)^N$$

☐ **Classified by complex Hadamard matrices**

Perturbed coupled cat maps provide generic model.
Spin 1/2 chain - minimal realization

☐ **Exactly solvable**

Local correlators can be calculated exactly

$$< Q_1(0,0)Q_2(n,t) >= 0$$

$$< Q_1(0,0)Q_2(1,0)Q_3(n,t)Q_4(n+1,t) >= C(t)\delta(n-t)$$

Summary: Order of limits

Generically, finite gap $\Delta\mu = 1 - \mu_1$ in Spec($\mathbb{T}$) for a fixed L

□ If $N \to \infty$, L is fixed

$C(t) \sim e^{-\Delta\mu t}$ decays exponentially independently of classical limit

$\Longrightarrow$ **Quantum chaos**

□ If $L \to \infty$, N is fixed

the gap $\Delta\mu$ closes in the absence of classical chaos !

$\Longrightarrow$ **No chaos**

Everything depends on the order of limits: $N \to \infty$, $L \to \infty$