FACULTY OF PHYSICS

UNIVERSITÄT DUISBURG ESSEN

Open-Minded

# Kicked spin chain: dual operator method

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# Outline

I. Regular kicked Ising chain of large spins: quantum to classical correspondence

- Classical orbit actions from quantum spectra. Dual approach
- Spectral effects of periodic orbit manifold

II. Disordered Ising chain of spin $1/2$: self-dual, close and not so close to self-duality

- “New” RMT universality classes in self-dual chain
- Small deviations from self-duality: effect on form factor
- Transition from ergodicity to localization

# Regular chain, large spins: quantum to classical correspondence

# Kicked Spin Chain

$N$-particle quantum system: kicked spin chain consisting of $N$ coupled spin-$s$-particles:

$$\hat{H}(t) = \underbrace{\sum_{n=1}^{N} \frac{4J\hat{s}^z_{n+1}\hat{s}^z_n}{(s+1/2)^2}}_{\text{nearest neighbor Ising interaction}} + \underbrace{\frac{2}{s+1/2}\sum_{n=1}^{N} \mathbf{B}\cdot\hat{\mathbf{s}}_n}_{\text{local kick part}} \sum_{\tau=-\infty}^{\infty}\delta(t-\tau)$$

Periodic boundary conditions: $\hat{\mathbf{s}}_{N+1} = \hat{\mathbf{s}}_1$

Evolution per period operator

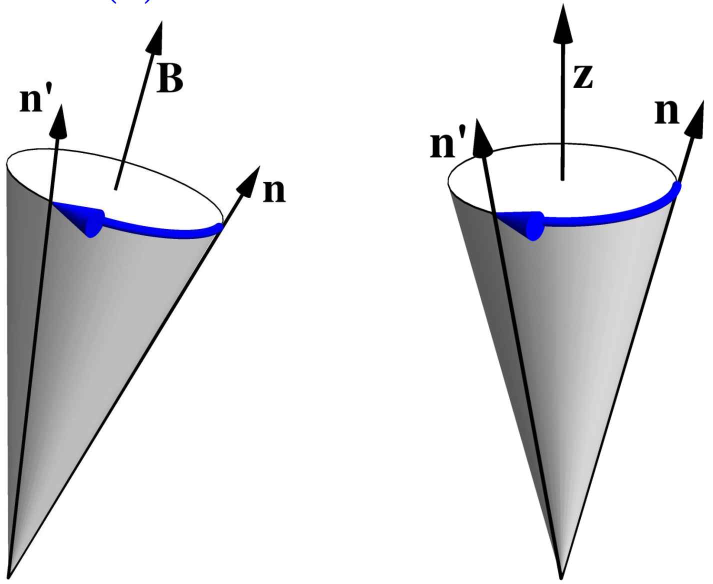
$$\hat{U} = e^{-i\hat{H}_\mathrm{I}} e^{-i\hat{H}_\mathrm{k}}$$

# Classical Dynamics

Classical $m$-th spin represented as unit vector $\mathbf{n}_m(t)$ on the Bloch sphere with dynamics:

Rotation: $$\mathbf{n}_m(t+T) = \left(R_{\mathbf{z}}(4J\chi_m)R_{\mathbf{B}}(2|\mathbf{B}|)\right)^T \mathbf{n}_m(t)$$

Angle: $$\chi_m = n^z_{m-1} + n^z_{m+1}$$



# Semiclassics

Trace formula for isolated orbits:

$$\mathrm{Tr}\hat{U}^T \sim \sum_{\gamma(T)} A_\gamma \mathrm{e}^{isS_\gamma}$$

with periodic orbits $\gamma(T)$, stability prefactor $A_\gamma$, action $S_\gamma$

Limit: Large spin quantum number $s \gg 1$

Fourier-transform yields action spectrum:

$$\rho(S) \propto \sum_{s=1}^{s_{\mathrm{cut}}} \mathrm{e}^{-isS}\mathrm{Tr}\hat{U}^T \sim \sum_{\gamma(T)} A_\gamma \delta(S - S_\gamma)$$

For a single kicked spin: Kuś, Haake, Delande, 1993

# Duality Relation

Problem specific for many-body system:

$$\rho(S) \propto \sum_{s=1}^{s_{\rm cut}} \mathrm{e}^{-isS} \mathrm{Tr} \underbrace{\hat{U}^T}_{(2s+1)^N} \quad \sim \quad \sum_{\gamma(T)} A_\gamma \delta(S - S_\gamma)$$

-dimensional

Example: $s_{\rm cut} = 10$, $N = 20 \to (2s+1)^N = 2.8 \cdot 10^{26}$

Solution: Duality

$$\mathrm{Tr}\, \hat{U}^T = \mathrm{Tr}\, \hat{W}^N, \quad \dim \hat{W} = (2s+1)^T \times (2s+1)^T$$

$\Rightarrow$ Dimensional reduction achieved for short times

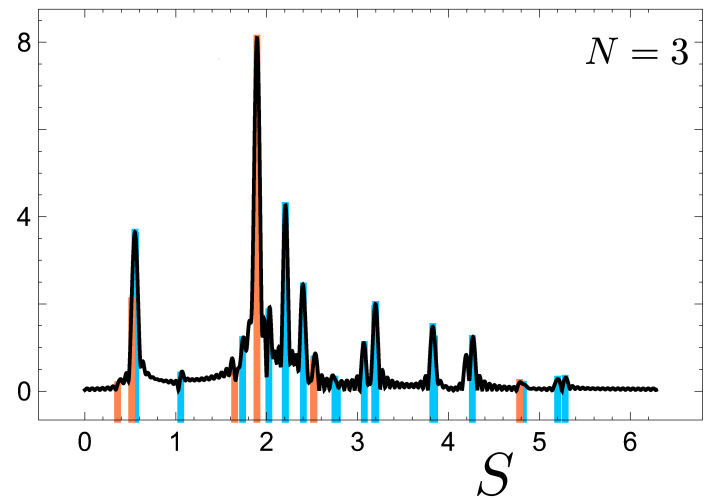
Gutkin, Osipov, 2016; Akila, Waltner, Gutkin, Guhr 2016)

# Action Spectrum. $N = 4m$-peculiarity

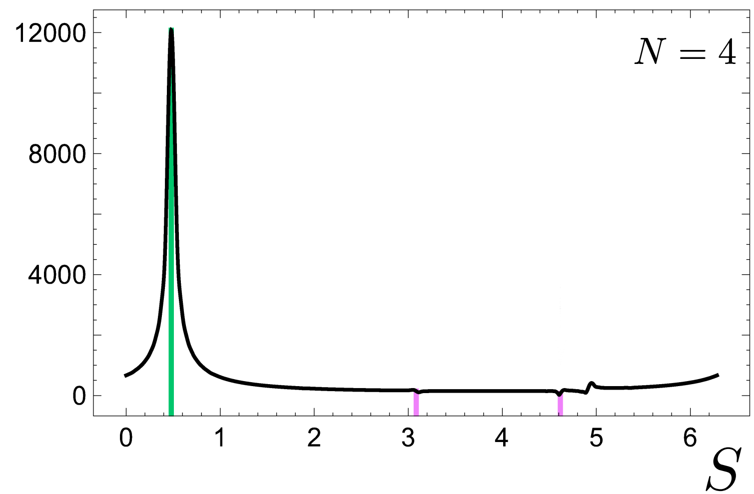
Resolution depends on $s_{\text{cut}}$. Peak height is independent of $s_{\text{cut}}$ with orbit isolated, or grows like $s_{\text{cut}}^{\alpha}$ (bifurcation), or exponentially decays (ghost)

Parameters: $T = 2$, $J = 0.7$, $B^x = B^z = 0.9$, $s_{\text{cut}} = 114$

$N = 3$




$N = 4$

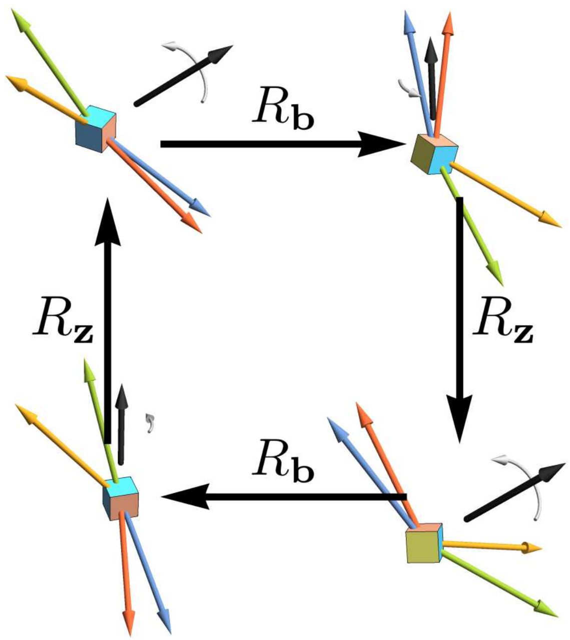



For $N = 4m$ the action spectrum is dominated by a single peak

Akila, Waltner, Gutkin, Braun, Guhr, 2017

# Classical Collective Dynamics

4-dimensional manifold of (nonisolated) periodic orbits for $N = 4$ with equal actions: spins perform solid body rotation




Peak height scales like $s_{\rm cut}^{\alpha_0 N}, \quad \alpha_0 \approx 0, 2$, instead of naive $s_{\rm cut}^2$

Akila, Waltner, Gutkin, Braun, Guhr, 2017.

# Disordered chain of spin $1/2$

# Kicked disordered Ising chain, spin $1/2$

Setup:

$$\hat{H}(t) = \hat{H}_I + \hat{H}_b \sum_{\tau=-\infty}^{\infty} \delta(t-\tau)$$

$$\hat{H}_I = \sum_{n=1}^{N} \left( J_n \hat{\sigma}_n^z \hat{\sigma}_{n+1}^z + b_n^z \hat{\sigma}_n^z \right), \qquad \hat{H}_b = b^x \sum_{n=1}^{N} \hat{\sigma}_n^x$$

- Spin chain with ring topology, nearest neighbor interaction and disordered local magnetic fields
- Periodic strong and short pulses of transverse magnetic field

# Time evolution. Duality

Time evolution operator:

$$\hat{U}_N = \hat{U}_I \hat{U}_b, \quad \hat{U}_I = \exp(-i\hat{H}_I), \quad \hat{U}_b = \exp(-i\hat{H}_b)$$

Dual approach, interchange of evolution in time and in spin number.

$$\mathrm{Tr}\left(\hat{U}_N\right)^T \qquad = \qquad \mathrm{Tr}\,\hat{W}$$

Dimensions: $\quad 2^N \times 2^N \qquad 2^T \times 2^T$

Dual operator for disordered chain:

$$\hat{W} = \hat{W}_N \hat{W}_{N-1} \ldots \hat{W}_1$$

Operators $W_n$ depend on local $J_n, b_n^z$

# Self-duality

Dual operator is unitary if

$$\text{Ising constants } J_n = \frac{\pi}{4};$$

$$\text{Transversal magnetic fields } b^x = \frac{\pi}{4};$$

Longitudinal magnetic fields $b_n^z$: arbitrary

Physical chain, $N$ spins: disordered; pulses regular in time;

Dual chain, $T$ spins: regular; disordered sequence of $N$ pulses periodically repeated in “time”

Akila, Waltner, Gutkin, Guhr, 2016; Bertini, Kos, Prosen 2018

# Symmetry, general case

- Evolution operator $\hat{U}$ is time reversal covariant with $\hat{T}^2 = 1$;
- Dual spin chain is symmetric with respect to cyclic shifts and reflection. The dual operators $\hat{W}_n$ fall into symmetry blocks $\hat{W}_n^K$ with $K = (0\pm), 1, \ldots, T-1$.
- Connection between operators with $J = \pi/4 - \Delta J$ (primed) and $J = \pi/4 + \Delta J$ (non-primed):

$$\hat{U}' = (-i)^N \hat{\Sigma}_y^N \hat{U}^* \hat{\Sigma}_y^N \qquad \hat{W}_n' = (-i)^T \hat{\Sigma}_y^T \hat{W}_n^* \hat{\Sigma}_y^T,$$

$$\hat{\Sigma}_y^N = \otimes_{n=1}^N \hat{\sigma}_n^y, \qquad \hat{\Sigma}_y^T = \otimes_{t=1}^T \hat{\sigma}_t^y.$$

Form factor: symmetric around self-dual case.

# Symmetry of self-dual chain, $\Delta J = 0$

Anti-unitary "charge conjugation" operator ($\hat{K}$ is c.c. in standard basis)

$$\hat{C}^{N,T} = \hat{\Sigma}_y^{N,T} \hat{K},$$
$$\left[\hat{C}^{N,T}\right]^2 = (-1)^{N,T}.$$

Unitary $\hat{U}_M := \hat{U} e^{iN\pi/4}, \quad \hat{W}_{n,M} := \hat{W}_n e^{iT\pi/4}$ satisfy

$$\left[\hat{C}^N, \hat{U}_M\right] = 0, \qquad \left[\hat{C}^T, \hat{W}_{n,M}\right] = 0,$$

Consequences:

- Matrices $\hat{U}_M, W_{n,M}$ are unitary symplectic in an appropriate basis set if $N$ resp. $T$ is odd, and orthogonal if $N$ resp. $T$ is even;
- Eigenvalues of $\hat{U}_M, \hat{W}_{n,M}$ exist in complex conjugate pairs, eigenphases $\phi = 0, \pi$ singled out.

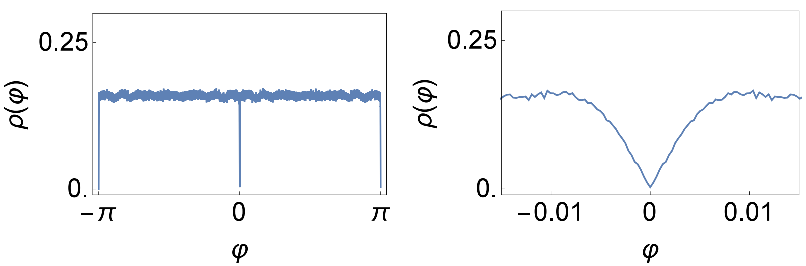
# Symmetry of self-dual chain, $\Delta J = 0$

Evolution operators $\hat{U}_M$ and symmetry blocks of dual operators $\hat{W}_M = \prod_{n=1}^{N} \hat{W}_{n,M}$ belong to one of "new" universality classes depending on parity of $N, T$,

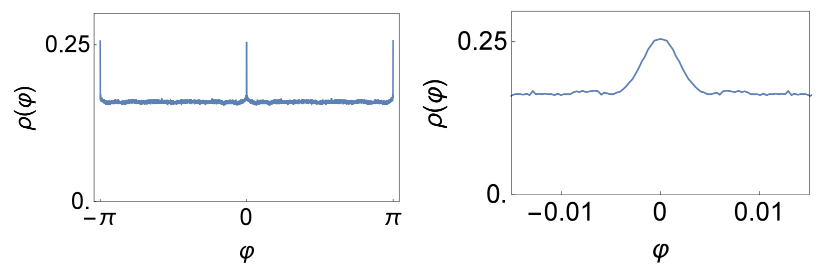
(Verbaarschot, Zahed, 1993; Altland, Zirnbauer, 1997)

Eigenphase density non-uniform:

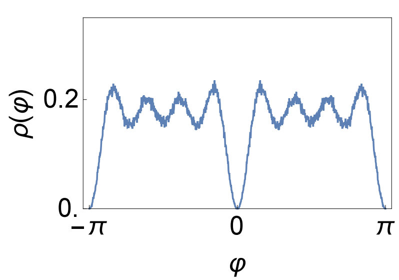
$\hat{U}_M, \quad N = 9$



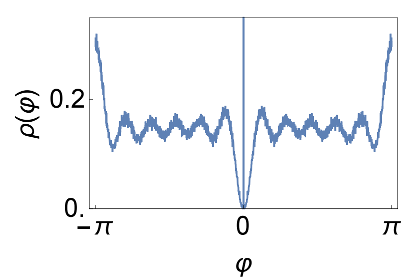

$\hat{U}_M, \quad N = 10$




$\hat{W}_M^{0+}, \quad T = 5, N = 13$




$\hat{W}_M^{0+}, \quad T = 6, N = 13$



# Analytic averaging over disorder

From Bertini, Kos, Prosen=BKP, 2018:

Analytic averaging over disorder of $|\mathrm{Tr}\, W|^2$ gives form factor as

$$K(T) = \mathrm{Tr}\mathcal{A}(T)^N$$

where $\mathcal{A}$ is $2^{2T} \times 2^{2T}$. In the limit of strong disorder

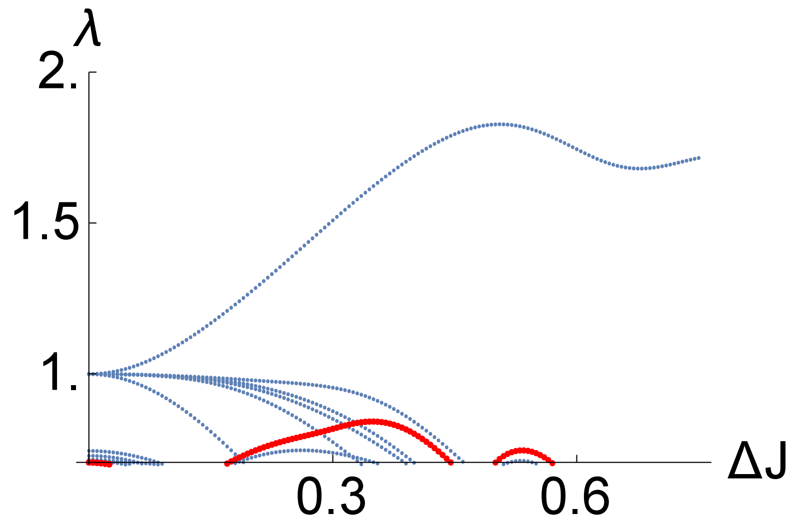
$$\mathcal{A} = \mathcal{P}\left(\bar{W} \times \bar{W}^*\right)\mathcal{P}$$

with $\mathcal{P}$ a projector.

In self-dual case $\mathcal{A}$ has $2T$ or $2T+1$ eigenvalues $1$; all other eigenvalues have modulus smaller than $1$. Taking to power $N \gg 1$ leads to form factor $2T$ resp. $2T+1$ similar to COE at small times.

# Small deviations from self-duality

Degenerate eigenvalue $\lambda = 1$ of $\mathcal{A}$ splits into a fan of levels when $J = \pi/4 + \Delta J, \quad \Delta J \neq 0$. Example, $T = 9$:




For level $\lambda_K$ associated with symmetry block $\hat{W}^K$ with size $m_K$ we have

$$\lambda_K(T, \Delta J) = 1 + B_K (\Delta J)^2 + O\left((\Delta J)^4\right),$$

$$B_K(T) = \frac{\partial^2 \mathrm{Tr}\left[\hat{W}^K \left(\hat{W}^K\right)^\dagger\right]}{2 m_K \partial J^2}$$

# Perturbation theory, Form Factor

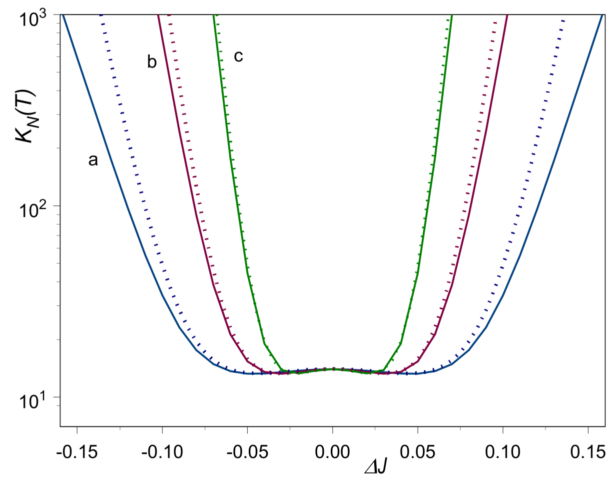
For $T$ prime, analytic $B_K$ found; all of them but $B_{0+}$ are negative

$\Longrightarrow$ only level $\lambda_{0+}$ grows with $\Delta J$

Form factor in limit $\Delta J \to 0, \quad N \to \infty, \quad x = (\Delta J)^2 N = \mathrm{const}$:

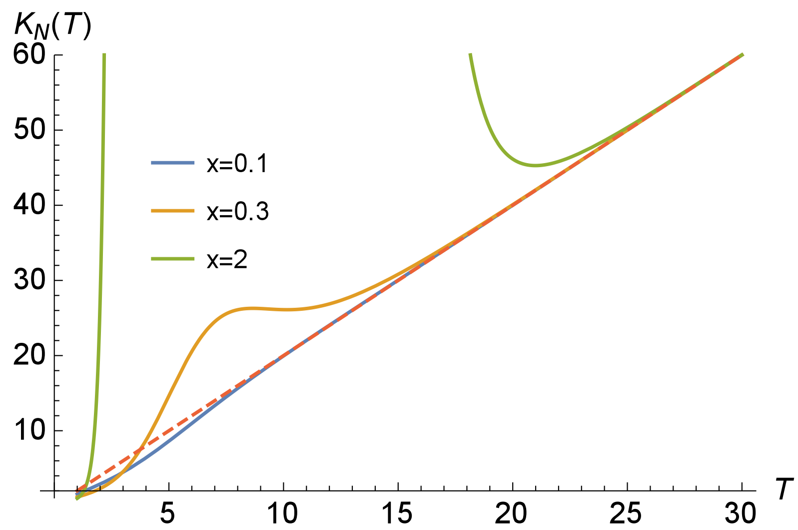
$$K_N(T, \Delta J) = e^{xB_{0+}} + e^{xB_{0-}} + 2(T-1)e^{xB_1}$$

Example: fixed time $T = 7$, function of $\Delta J$; $N = 40$ (a), $80$ (b) and $160$ (c); dots numerical




Braun, Waltner, Akila, Gutkin, Guhr, 2019

# Perturbation theory, Thouless Time

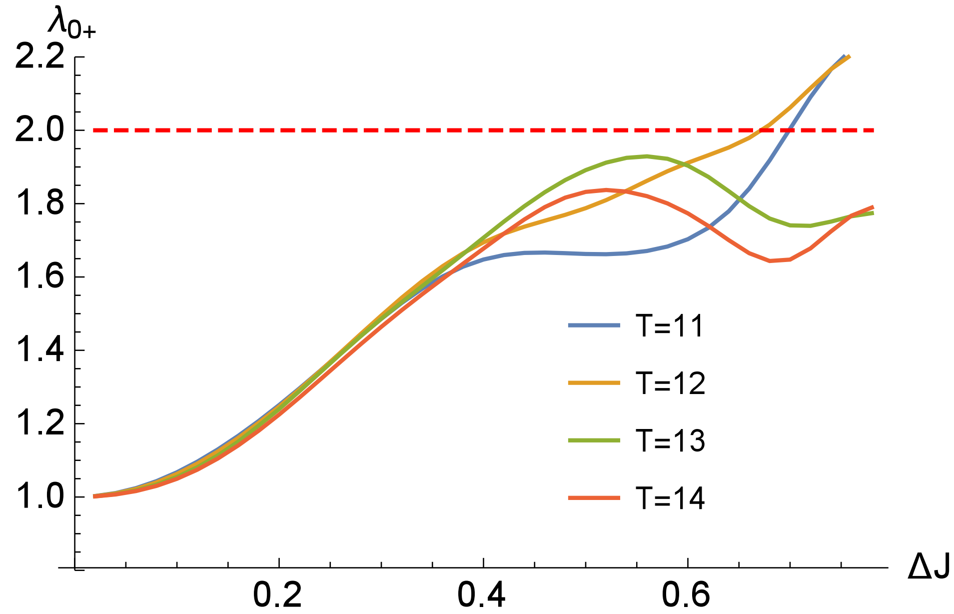



For fixed $x = N(\Delta J)^2$ and time growing, $K_N(T, \Delta J)$ first undergoes strong non-monotonic changes and then stabilizes. Demanding that its difference from $2T$ is much smaller than $2T$ we get the Thouless time $T_{Th}$. If $x \gtrsim 0.3$ the condition simplifies to,

$$\frac{\log 2T}{B_{0+}(T)} \gtrsim N\,\Delta J^2, \qquad B_{0+}(T) = \frac{2T(T-1)}{2^{(T-1)/2} - 1}$$

# Large deviations from self-duality

Senior eigenvalue $\lambda_{0+}$ of $\mathcal{A}(\Delta J, T)$ changes with $T$ approximately periodically with period 4

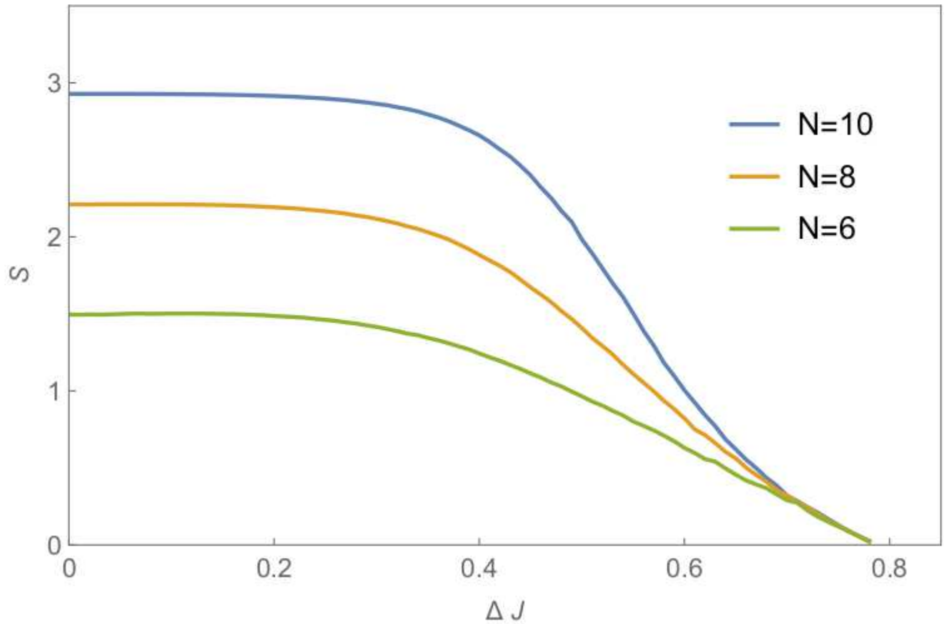



Curves form doublets with partner swapping close to $\Delta J = 0.6$ where transition to localization is suspected

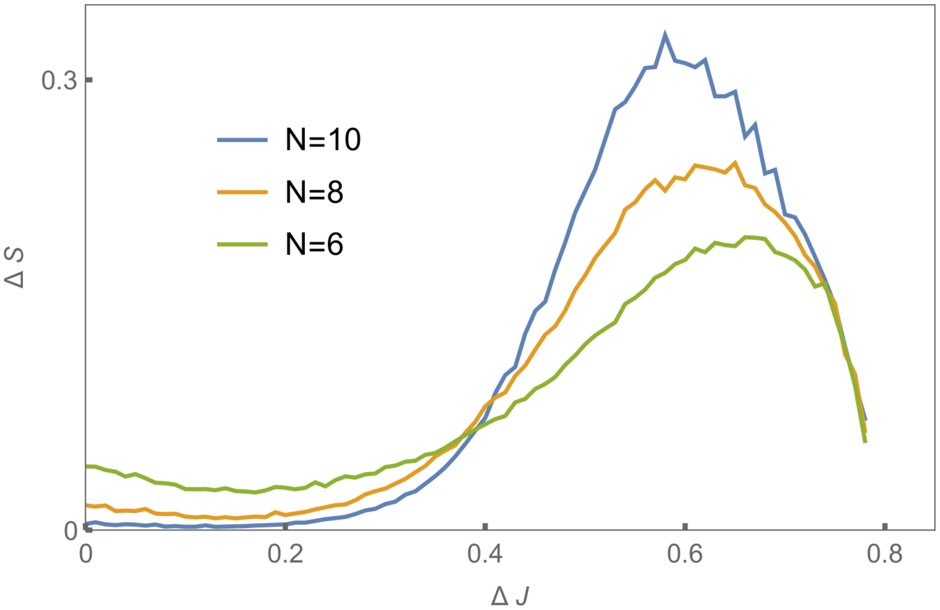
# Entanglement entropy

Symmetric splitting of the chain into subsystems $A$ and $B$, entanglement entropy

$$S(\Delta J) = -\mathrm{Tr}\rho_A \ln \rho_A$$




Average entropy

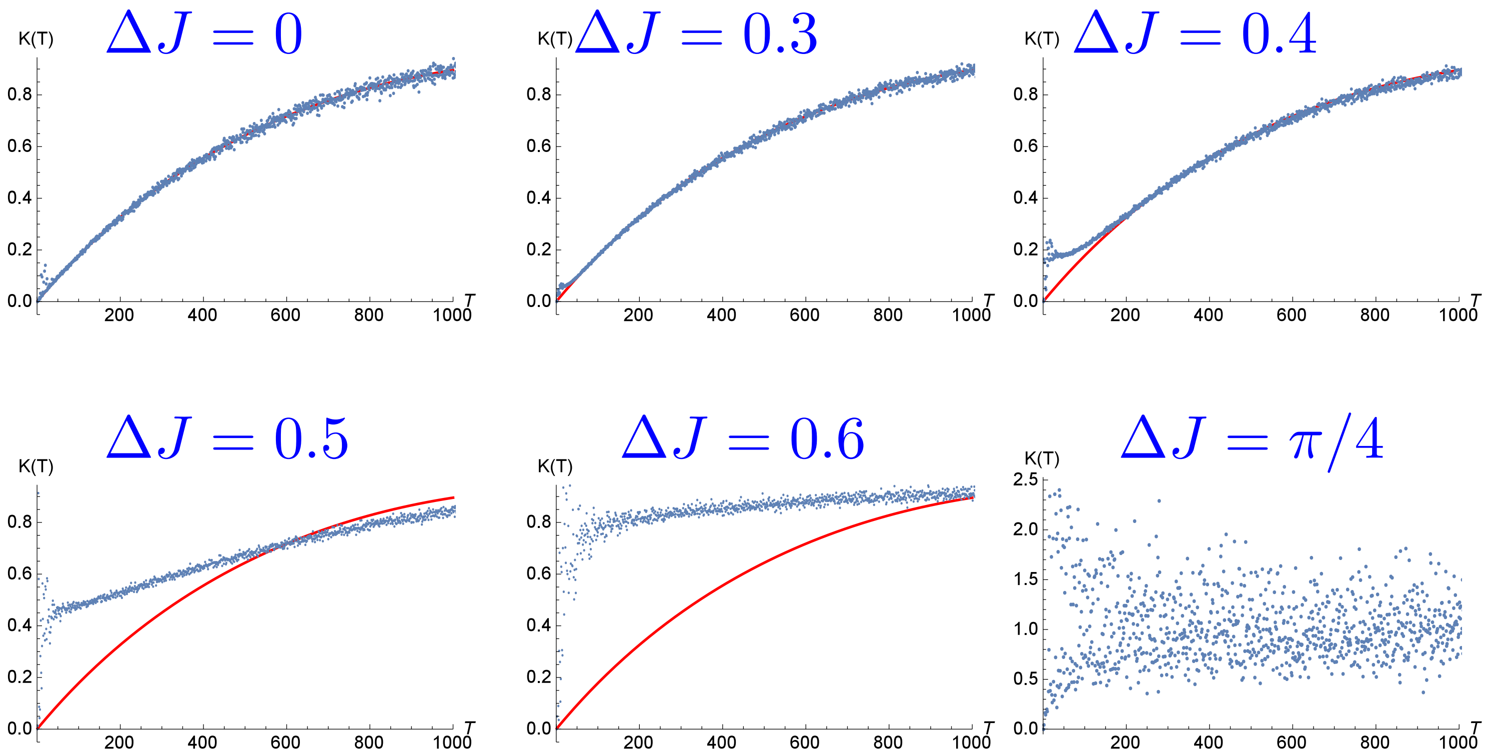



Entropy mean deviation

Transition to localization at $\Delta J \approx 0.6$

# Form factor, large deviations from self-duality

$10$ spins, $T_H = 1024$, $5000$ disorder realizations; Red: $K_{COE}$

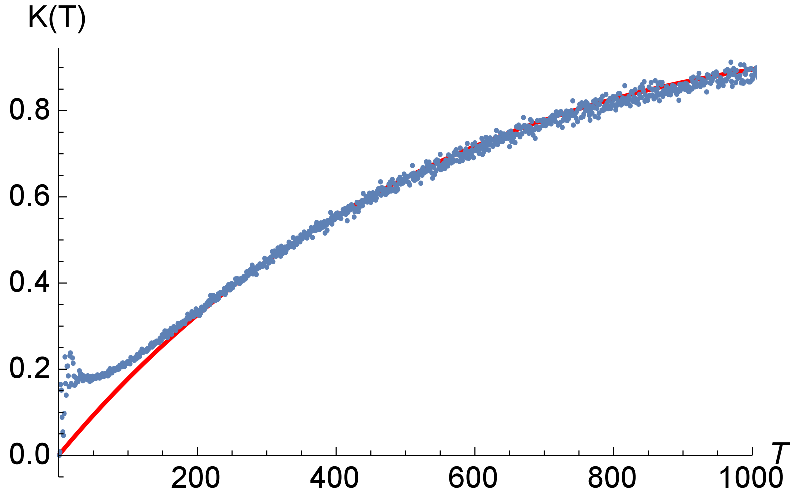


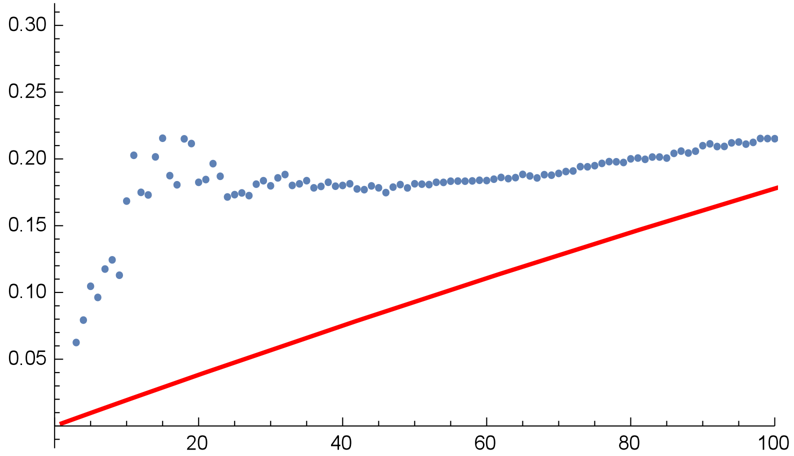

Form factor extrapolated to $T/T_H = 0_+$ takes values $\eta \in \{0, 1\}$

(Wigner-Dyson: $\eta = 0$, Poisson: $\eta = 1$)

# Senior BPK eigenvalue at start of localization

Example: form factor at $N = 10, \Delta J = 0.4$




Critical times

$T > T_1$, form factor smooth, $T_1 \sim N$

$T > T_2$, form factor coincides with COE (Thouless time), $0 < T_2 < 2^N$

At $T \geq T_1$, for given proportion of Poisson statistics $\eta$:

$$\eta = K_{T/T_H \to 0} = \frac{\langle |\mathrm{Tr} U^T|^2 \rangle}{T_H} = \left( \frac{\lambda_{0+}}{2} \right)^N$$

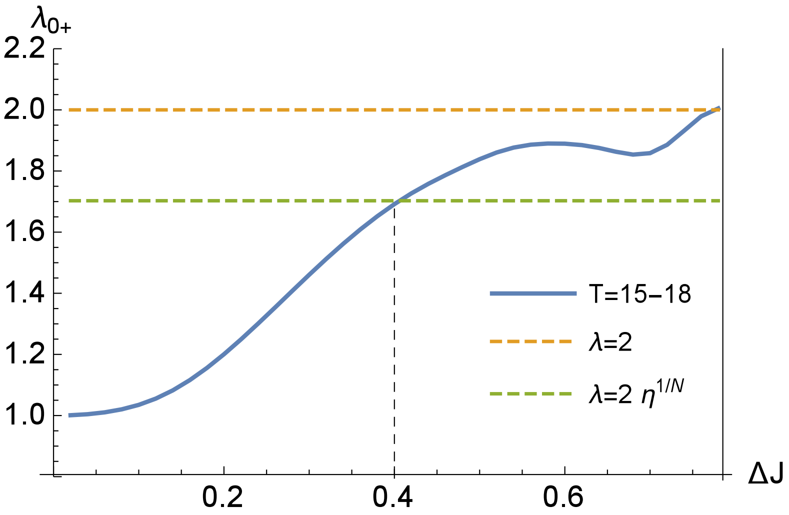
# Senior BPK eigenvalue at start of localization

Criterion of localization

$$\lambda_{0+}(T \geq T_1) = 2\eta^{1/N}$$

In the example $T_1 \sim 16 - 20, \quad \eta \sim 0.2 \Longrightarrow \lambda_{0+} \sim 1.7$

$\lambda_{0+}(T)$, average over quadruplet $T = 15, 16, 17, 18$:



# Conclusion

- Classic orbits distilled from quantum spectra of many-body system
- “New” symmetry classes observed for kicked Ising spin-$1/2$ chain
- Form factor explosion at small times caused by deviation from self-duality
- Transition from ergodicity to localization witnessed by senior eigenvalue of BKP operator $\mathcal{A}$