UNIVERSITÄT DUISBURG ESSEN

*Open-Minded*

Faculty of Physics

# Influences of large local fluctuations on copula-based dependence of demands between stocks

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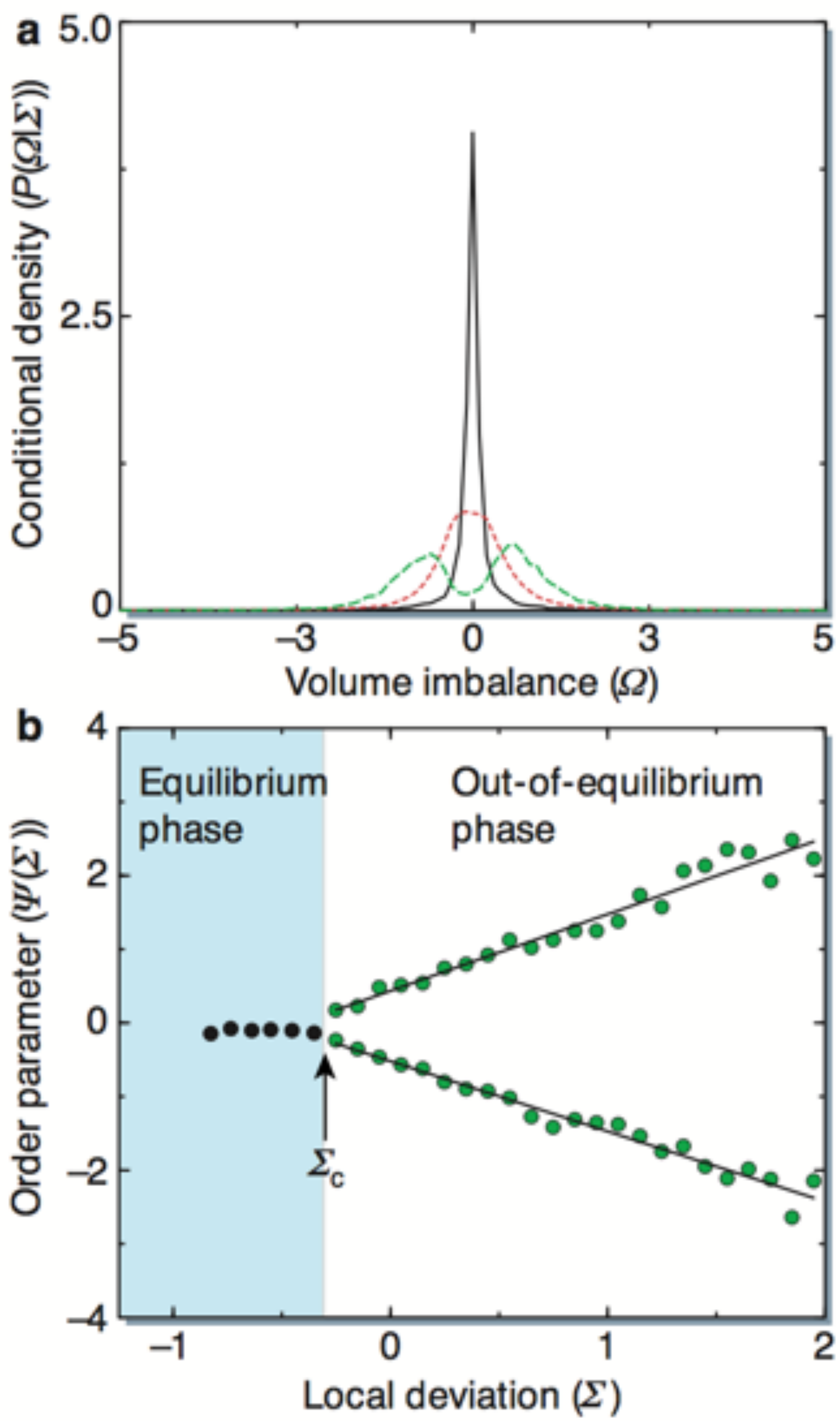
# Contents

# 1 Introduction—two-phase behaviour






**Volume imbalance** $\Omega(t) \equiv Q_B - Q_S = \sum_{i=1}^{N} q_i a_i$

**Local noise intensity** $\Sigma(t) \equiv \langle | q_i a_i - \langle q_i a_i \rangle | \rangle$

$\langle \ldots \rangle$ denotes the local expectation value

**Order parameter**

$$\Psi(\Sigma) = \begin{cases} 0 & [\Sigma < \Sigma_c] \\ \Sigma - \Sigma_c & [\Sigma >> \Sigma_c] \end{cases}$$

displays positions of the maxima of the distribution

$$\begin{cases} \Sigma < \Sigma_c\ , & P(\Omega|\Sigma) \text{ has a single peak (a single maximum)} \\ \Sigma \approx \Sigma_c\ , & P(\Omega|\Sigma) \text{ flattens near to the origin} \\ \Sigma > \Sigma_c\ , & P(\Omega|\Sigma) \text{ has two peaks (two maxima)} \end{cases}$$

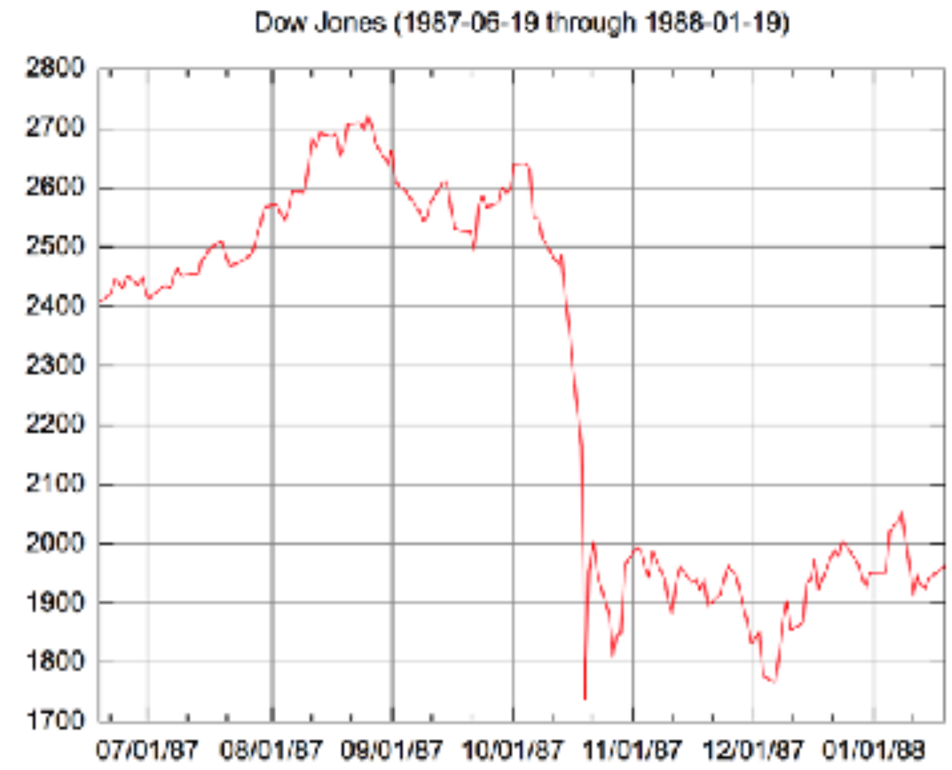
Ref. Plerou, Gopikrishnan, and Stanley, Nature, 421 (2003)

# 1 Introduction—market collective behavior

**stock market crash:**
Black Monday (1987)

On Monday, October 19, 1987, the Dow Jones Industrial Average (DJIA) fell exactly 508 points to 1,738.74 (22.61%)

figure from: https://upload.wikimedia.org/wikipedia/commons/a/af/Black_Monday_Dow_Jones.svg

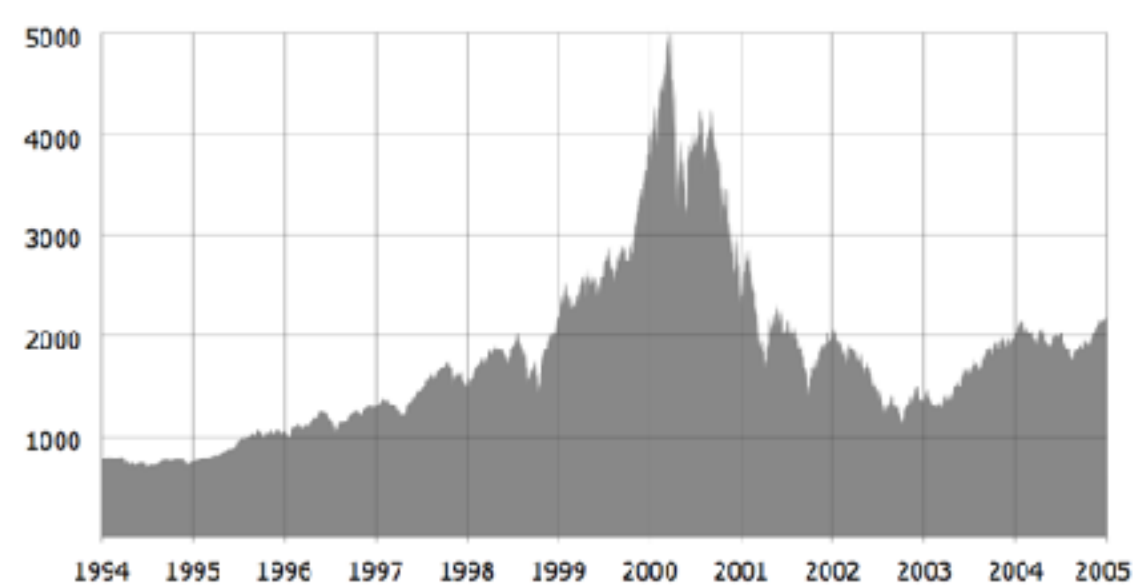



**stock market bubble:**
Dot-com bubble

The NASDAQ Composite index spiked in the late 1990s and then fell sharply as a result of the dot-com bubble.

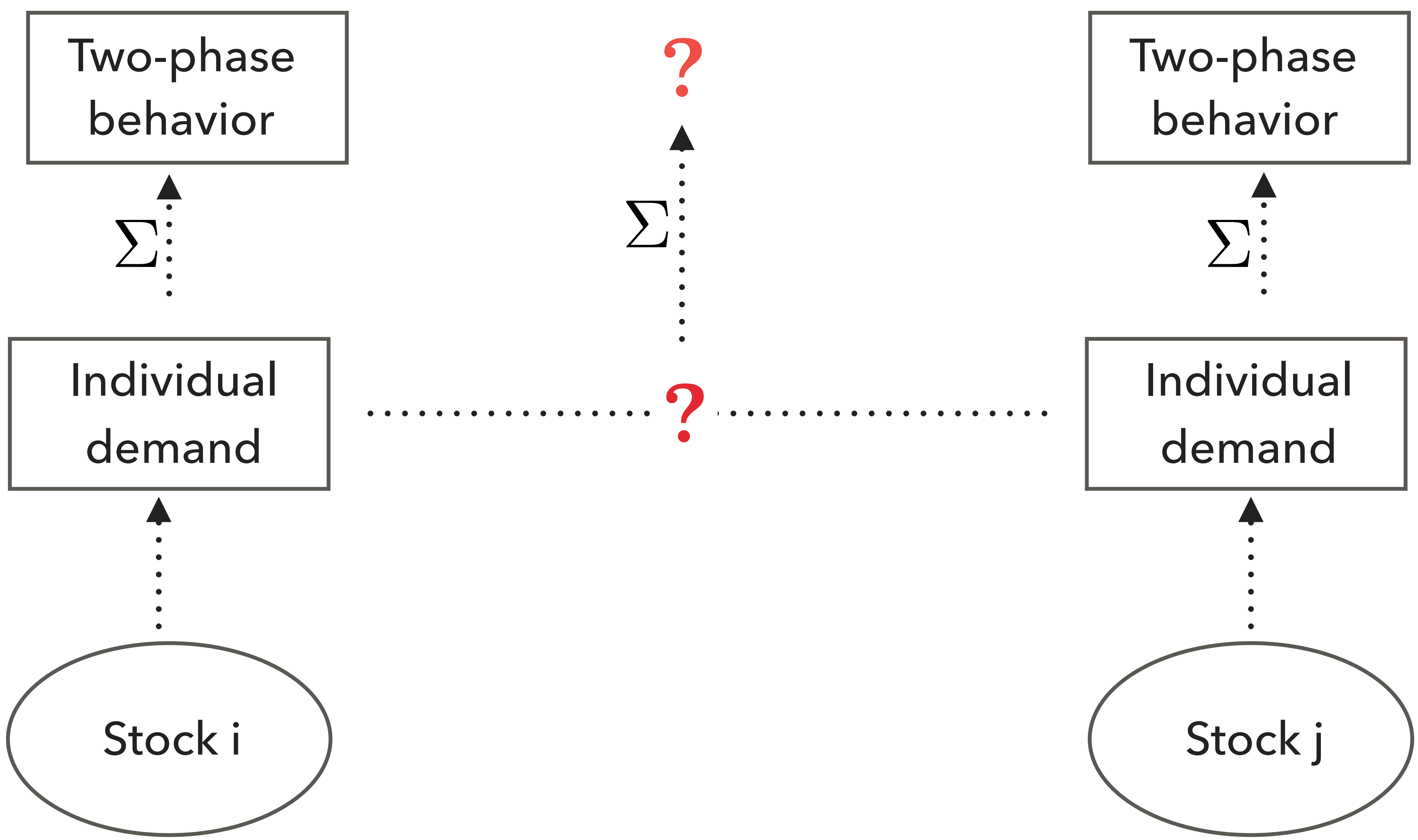
figure from: https://upload.wikimedia.org/wikipedia/commons/8/84/Nasdaq_Composite_dot-com_bubble.svg

NASDAQ Composite index



# 1 Introduction—two issues to be addressed





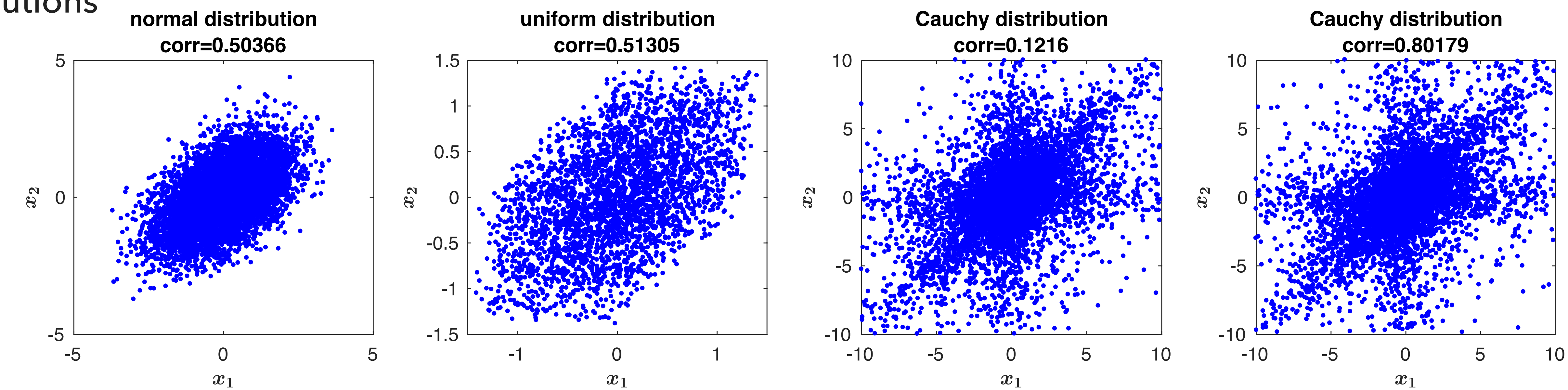
# 1 Introduction—why we use copulas instead of joint distributions?



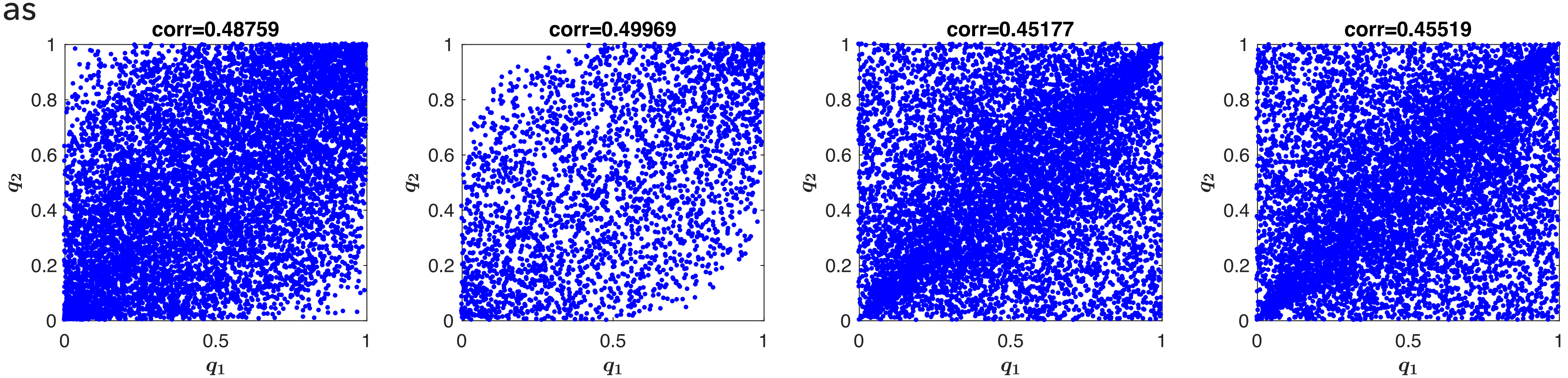
$$x_1 = \sqrt{c}\eta + \sqrt{1-c}\xi_1 \qquad c = 0.5$$

$$x_2 = \sqrt{c}\eta + \sqrt{1-c}\xi_2 \qquad \eta,\ \xi_1 \text{ and } \xi_2 \text{ are random variables with following distributions}$$

Joint distributions




Copulas




**Idea behind the copula** is to map all marginal distributions to uniform distributions and then to measure the joint distribution density as function of the corresponding quantities.

# 2 Copula-based dependence of demands—data set



- NASDAQ stock market
- Trades and Quotes (TAQ) data set
- for conditional probability density distributions: 496 available stocks from S&P 500 index in 2008
- for copula densities: the first 100 stocks with the largest average number of daily trades among the 496 stocks.
- excluding the first and the last ten minutes during the intraday trading time to avoid the large fluctuations in the market opening and closing and the effect of overnight, so that total 370 minutes in each trading day are available
- considering the time interval of one minute to aggregate the volume imbalance and calculate the local fluctuation
- for each trading day and each stock, the length of available data points is 370

# 2 Copula-based dependence of demands—definitions

According to Sklar's theorem, there exists a copula satisfying

$$F_{kl}(x_1, x_2) = \mathrm{Cop}_{kl}\big(F_k(x_1), F_l(x_2)\big)$$

$F_{kl}(x_1, x_2)$ is a joint cumulative distribution

$F_k(x_1), F_l(x_2)$ are marginal cumulative distribution

$$F_k(x_1) = \int_{-\infty}^{x_1} f_k(s)ds$$

Using inverse cumulative distribution function, we have

$$q_1 = F_k(x_1) \quad \text{and} \quad x_1 = F_k^{-1}(q_1)$$

Copula can be expressed as the cumulative joint distribution of quantiles

$$\mathrm{Cop}_{kl}(q_1, q_2) = F_{kl}\big(F_k^{-1}(q_1), F_l^{-1}(q_2)\big)$$

Copula density is given by

$$\mathrm{cop}_{kl}(q_1, q_2) = \frac{\partial^2}{\partial q_1 \partial q_2}\mathrm{Cop}_{kl}(q_1, q_2)$$

# 2 Copula-based dependence of demands—empirical copula

**Demand: volume imbalance**

$$\nu_k(t) = \sum_{n=1}^{N_{\text{trades}}(t)} v_k(t;n)\varepsilon_k(t;n)$$

- $t$ : the index of time interval of **one minute**
- $v_k(t;n)$ : trade volume
- trade sign

$$\varepsilon_k(t;n) = \begin{cases} 1, & \text{for a buy trade} \\ -1, & \text{for a sell trade} \end{cases}$$

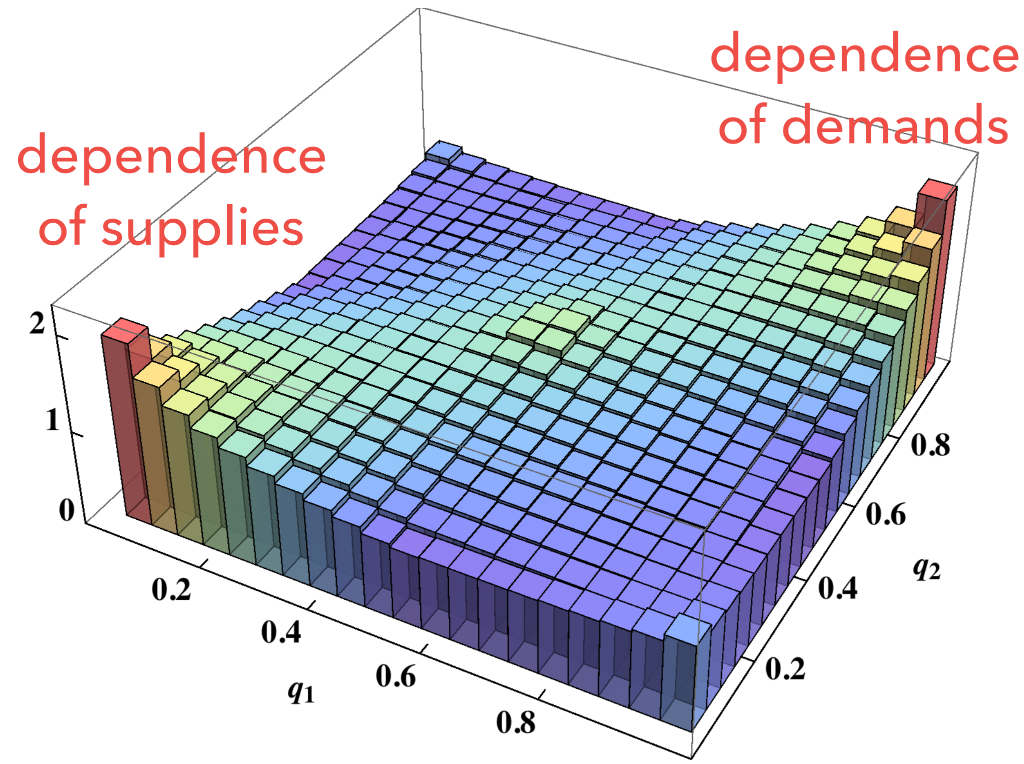
For example

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\nu_k(t)$ | 1.5 | −1.2 | 0.8 | 2.5 | −0.3 | 1.7 | 2 |
| $rank$ | 4 | 1 | 3 | 7 | 2 | 5 | 6 |
| $q(t)$ | $\frac{3.5}{7}$ | $\frac{0.5}{7}$ | $\frac{2.5}{7}$ | $\frac{6.5}{7}$ | $\frac{1.5}{7}$ | $\frac{4.5}{7}$ | $\frac{5.5}{7}$ |

$$q_1(t) = F_k(\nu_k(t)) = \frac{1}{T}\sum_{\tau=1}^{T} 1\{\nu_k(\tau) \le \nu_k(t)\} - \frac{1}{2T}$$

$$\text{cop}_{kl}(q_1, q_2) = \frac{\partial^2}{\partial q_1 \partial q_2}\text{Cop}_{kl}(q_1, q_2)$$

$$\text{cop}(q_1, q_2) = \frac{2}{K(K-1)}\sum_{k=1}^{K-1}\sum_{l=k+1}^{K} \text{cop}_{kl}(q_1, q_2)$$



# 2 Copula-based dependence of demands—bivariate K-copula density

Vector $\mathbf{r} = (\vec{r}_1(t), ..., \vec{r}_K(t))$

K distribution

$$\langle g\rangle(\mathbf{r}|C,N) = \frac{1}{2^{N/2+1}\Gamma(N/2)\sqrt{\det(2\pi C/N)}}\frac{\mathcal{K}_{(K-N)/2}\left(\sqrt{N\mathbf{r}^\dagger C^{-1}\mathbf{r}}\right)}{\sqrt{N\mathbf{r}^\dagger C^{-1}\mathbf{r}}^{(K-N)/2}}$$

$$= \frac{1}{(2\pi)^K\Gamma(N/2)\sqrt{\det C}}\int_0^\infty dz z^{\frac{N}{2}-1}e^{-z}\sqrt{\frac{\pi N}{z}}^K \exp\left(-\frac{N}{4z}\mathbf{r}^\dagger C^{-1}\mathbf{r}\right)$$

modified Bessel function of the second kind of order *(K-N)/2*

this distribution results from a random matrix average to model non-stationary, i.e., fluctuating covariance or correlation matrices with a mean value C

N measures the strength of these fluctuations, 1/N can be viewed as the corresponding variance

correlation matrices

Ref. Schmitt, Chetalova, Schäfer and Guhr, EPL, 103, 5 (2013); Wollschläger and Schäfer, arXiv: 1506.08054; Chetalova, Schäfer and Guhr, Journal of Statistical Mechanics, P01029 (2015)

$$K = 2$$ correlation matrix $$C = \begin{bmatrix} 1 & c \\ c & 1 \end{bmatrix}$$

The joint probability density of variables $x_1$ and $x_2$ is

$$f(x_1, x_2) = \frac{1}{\Gamma(N/2)} \int_0^\infty dz z^{\frac{N}{2}-1} e^{-z} \frac{N}{4\pi z} \frac{1}{\sqrt{1-c^2}} \exp\left(-\frac{N}{4z} \frac{x_1^2 - 2cx_1x_2 + x_2^2}{1-c^2}\right)$$

The marginal distribution density of variable $x_1$

$$f_k(x_1) = \int_{-\infty}^{\infty} dx_2 f(x_1, x_2)$$

The marginal cumulative distribution function

$$F_k(x_1) = \int_{-\infty}^{x_1} d\xi f_k(\xi)$$

Analogously for variable $x_2$. Bivariate K-copula density function

$$\mathrm{cop}^{\mathcal{K}}_{c,N}(q_1, q_2) = \frac{f\left(F_k^{-1}(q_1), F_l^{-1}(q_2)\right)}{f_k\left(F_k^{-1}(q_1)\right) f_l\left(F_l^{-1}(q_2)\right)}$$

Ref. Wollschläger and Schäfer, Journal of Risk, 19, 1(2016); Chetalova, Schäfer and Guhr, Journal of Statistical Mechanics, P01029 (2015)

# 2 Copula-based dependence of demands—Gaussian copula density

The bivariate cumulative normal distribution of variables $x_1$ and $x_2$ is given by

$$F(x_1, x_2) = \int_{-\infty}^{x_1} \int_{-\infty}^{x_2} \frac{1}{2\pi\sqrt{1-c^2}} \exp\left(-\frac{y_1^2 + y_2^2 - 2cy_1y_2}{2(1-c^2)}\right) dy_2 dy_1$$

The marginal cumulative normal distribution of variable $x_1$ is

$$F_k(x_1) = \int_{-\infty}^{x_1} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y_1^2}{2}\right) dy_1$$

Gaussian copula density

$$\begin{aligned} \mathrm{cop}_c^G(q_1, q_2) &= \frac{\partial^2}{\partial q_1 \partial q_2} F\left(F_k^{-1}(q_1), F_l^{-1}(q_2)\right) \\ &= \frac{1}{\sqrt{1-c^2}} \exp\left(-\frac{c^2 F_k^{-1}(q_1)^2 + c^2 F_l^{-1}(q_2)^2 - 2cF_k^{-1}(q_1)F_l^{-1}(q_2)}{2(1-c^2)}\right) \end{aligned}$$

# 2 Copula-based dependence of demands—results of fitting

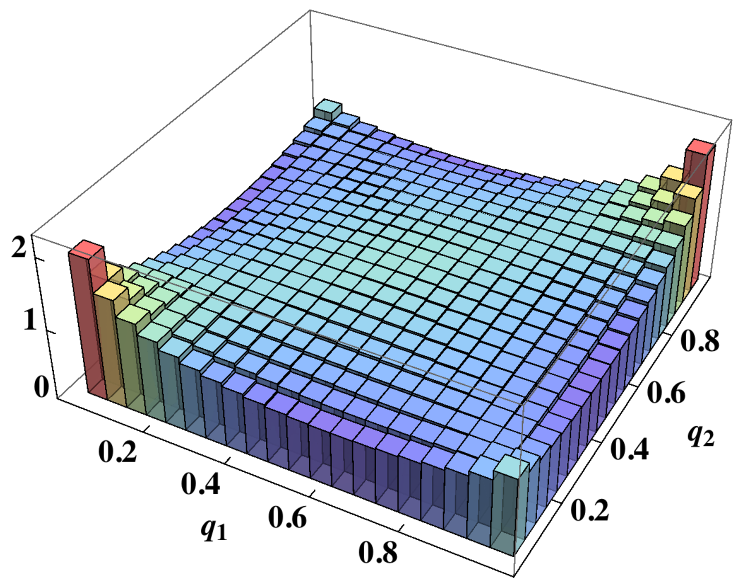


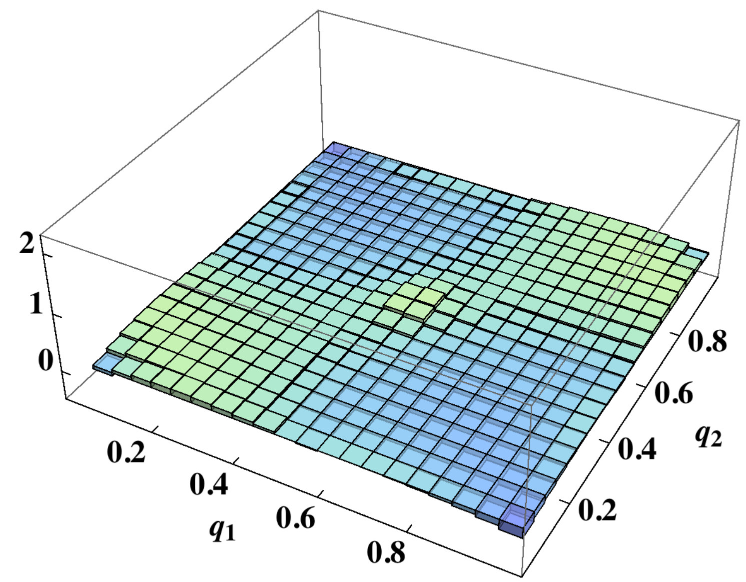
Fits

Errors

K-copula density

$\mathbf{cop}^{\mathcal{K}}_{\bar{c},N}(q_1, q_2)$

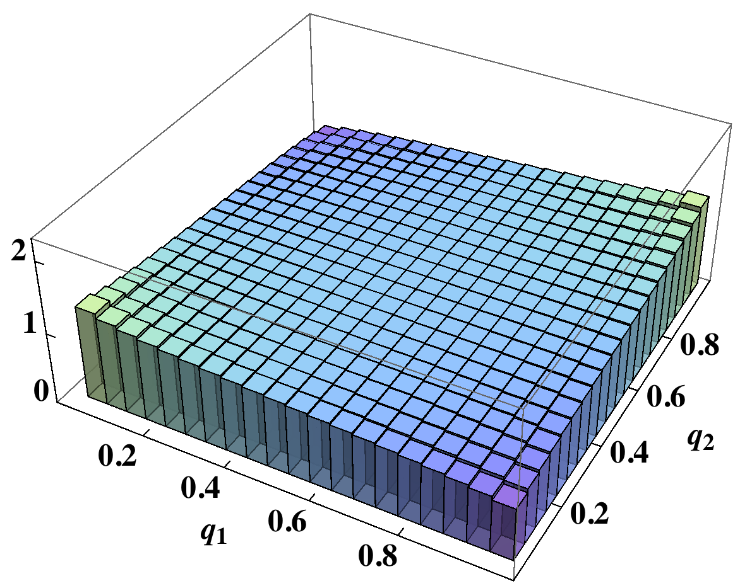


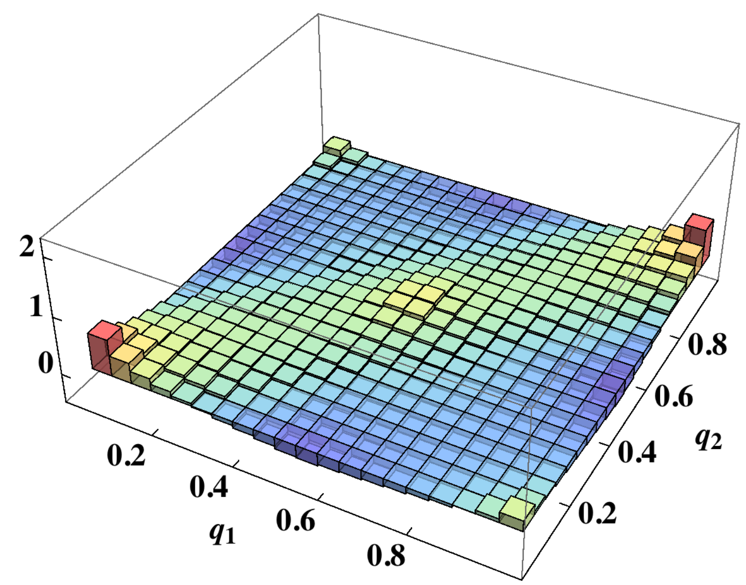




$\mathbf{cop}(q_1, q_2) - \mathbf{cop}^{\mathcal{K}}_{\bar{c},N}(q_1, q_2)$

Gaussian copula density

$\mathbf{cop}^{G}_{\bar{c}}(q_1, q_2)$




$\mathbf{cop}(q_1, q_2) - \mathbf{cop}^{G}_{\bar{c}}(q_1, q_2)$

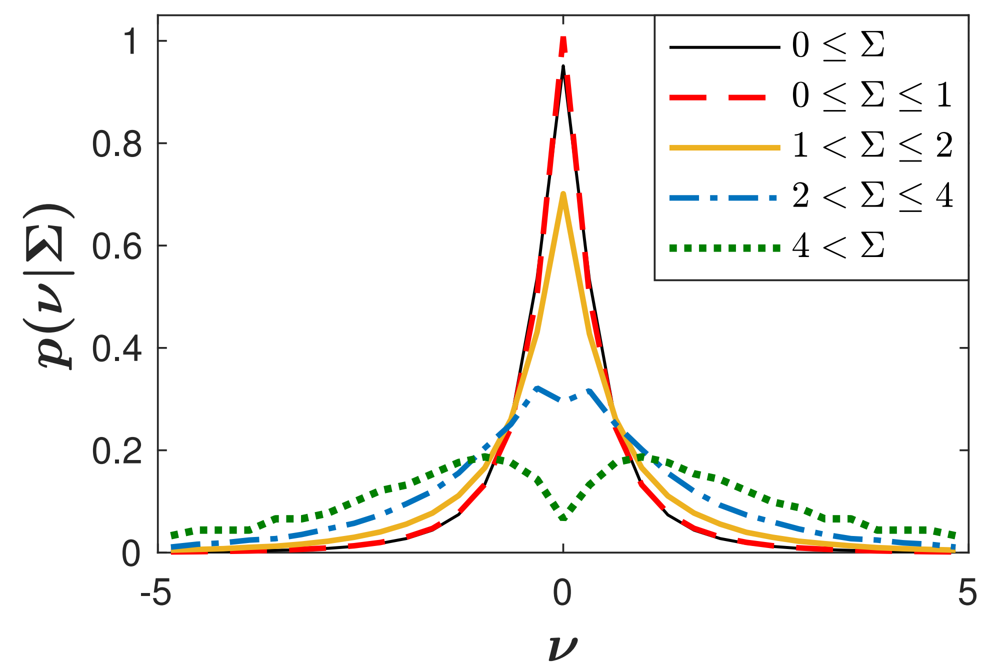
Conclusion 1: Empirical copula densities can be described well by a bivariate K-copula density function, especially for the dependencies in the tails

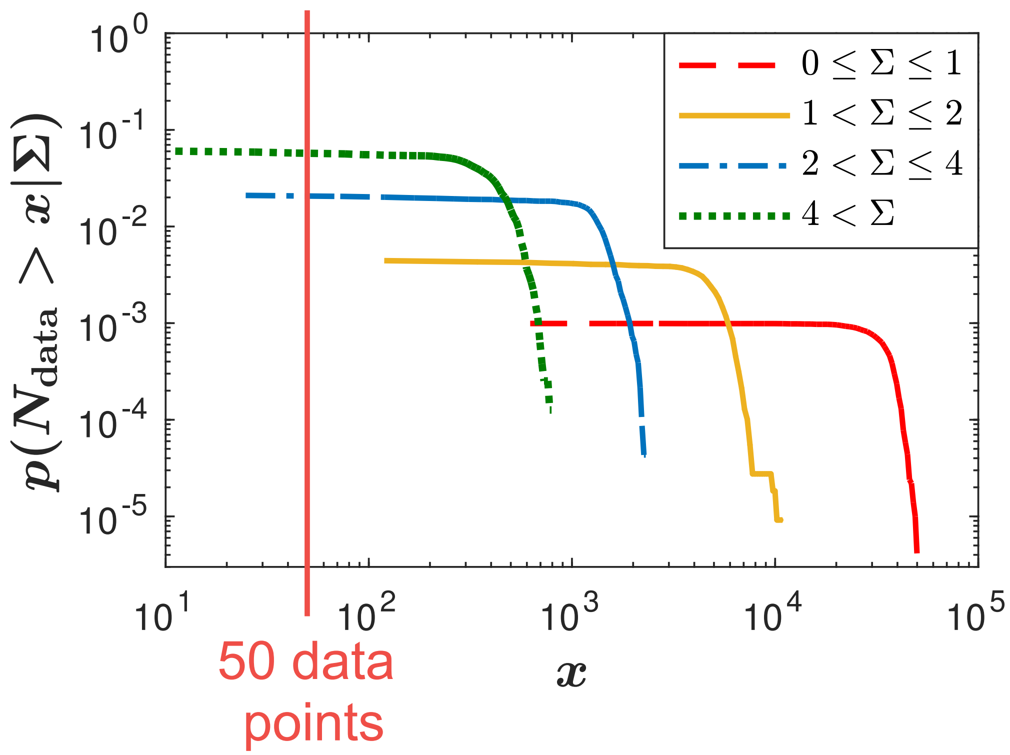
# 3 Influence of local fluctuations on dependencies



**Local fluctuations: local noise intensity**

$$\Sigma(t) = \left\langle \left| v(t;n)\varepsilon(t;n) - \left\langle v(t;n)\varepsilon(t;n) \right\rangle_n \right| \right\rangle_n$$




Influence of local fluctuations = Unconditional copula densities — Conditional copula densities

Conditional copula densities exclude 50 data points of the largest or smallest local fluctuations

# 3 Influence of local fluctuations on dependencies

## Conditional copula densities

$$\begin{aligned}
\mathbf{cop}^{(\mathrm{ss})}(q_1,q_2) &= \mathbf{cop}\left(q_1,q_2 \middle| \Sigma_k < \Sigma_{k,\mathrm{max}}, \Sigma_l < \Sigma_{l,\mathrm{max}}\right)\\
\mathbf{cop}^{(\mathrm{ll})}(q_1,q_2) &= \mathbf{cop}\left(q_1,q_2 \middle| \Sigma_k > \Sigma_{k,\mathrm{min}}, \Sigma_l > \Sigma_{l,\mathrm{min}}\right)\\
\mathbf{cop}^{(\mathrm{sl})}(q_1,q_2) &= \mathbf{cop}\left(q_1,q_2 \middle| \Sigma_k < \Sigma_{k,\mathrm{max}}, \Sigma_l > \Sigma_{l,\mathrm{min}}\right)\\
\mathbf{cop}^{(\mathrm{ls})}(q_1,q_2) &= \mathbf{cop}\left(q_1,q_2 \middle| \Sigma_k > \Sigma_{k,\mathrm{min}}, \Sigma_l < \Sigma_{l,\mathrm{max}}\right)
\end{aligned}$$

| Stock k | | Stock l | |
|---|---|---|---|
| $\nu_k$ | $\Sigma_k$ | $\nu_l$ | $\Sigma_l$ |
| 1.17 | 0.18 | 1.74 | 0.36 |
| -0.75 | 0.26 | -0.64 | 0.23 |
| -0.64 | 0.25 | 2.34 | 0.17 |
| ~~2.00~~ | ~~0.42~~ | -2.17 | 0.12 |
| ~~1.00~~ | ~~0.78~~ | ~~2.21~~ | ~~0.07~~ |
| -0.35 | 0.12 | 1.60 | 0.18 |
| 1.64 | 0.08 | ~~1.18~~ | ~~0.05~~ |

| Stock k | | Stock l | |
|---|---|---|---|
| $\nu_k$ | $\Sigma_k$ | $\nu_l$ | $\Sigma_l$ |
| 1.17 | 0.18 | 1.74 | 0.36 |
| -0.75 | 0.26 | -0.64 | 0.23 |
| -0.64 | 0.25 | 2.34 | 0.17 |
| -0.35 | 0.12 | -2.17 | 0.12 |
| 1.64 | 0.08 | 1.60 | 0.18 |
| | | | |
| | | | |

# 3 Influence of local fluctuations on dependencies



Conditional copula densities

$$\begin{aligned}
\mathrm{cop}^{(\mathrm{ss})}(q_1, q_2) &= \mathrm{cop}\left(q_1, q_2 \middle| \Sigma_k < \Sigma_{k,\mathrm{max}}, \Sigma_l < \Sigma_{l,\mathrm{max}}\right) \\
\mathrm{cop}^{(\mathrm{ll})}(q_1, q_2) &= \mathrm{cop}\left(q_1, q_2 \middle| \Sigma_k > \Sigma_{k,\mathrm{min}}, \Sigma_l > \Sigma_{l,\mathrm{min}}\right) \\
\mathrm{cop}^{(\mathrm{sl})}(q_1, q_2) &= \mathrm{cop}\left(q_1, q_2 \middle| \Sigma_k < \Sigma_{k,\mathrm{max}}, \Sigma_l > \Sigma_{l,\mathrm{min}}\right) \\
\mathrm{cop}^{(\mathrm{ls})}(q_1, q_2) &= \mathrm{cop}\left(q_1, q_2 \middle| \Sigma_k > \Sigma_{k,\mathrm{min}}, \Sigma_l < \Sigma_{l,\mathrm{max}}\right)
\end{aligned}$$

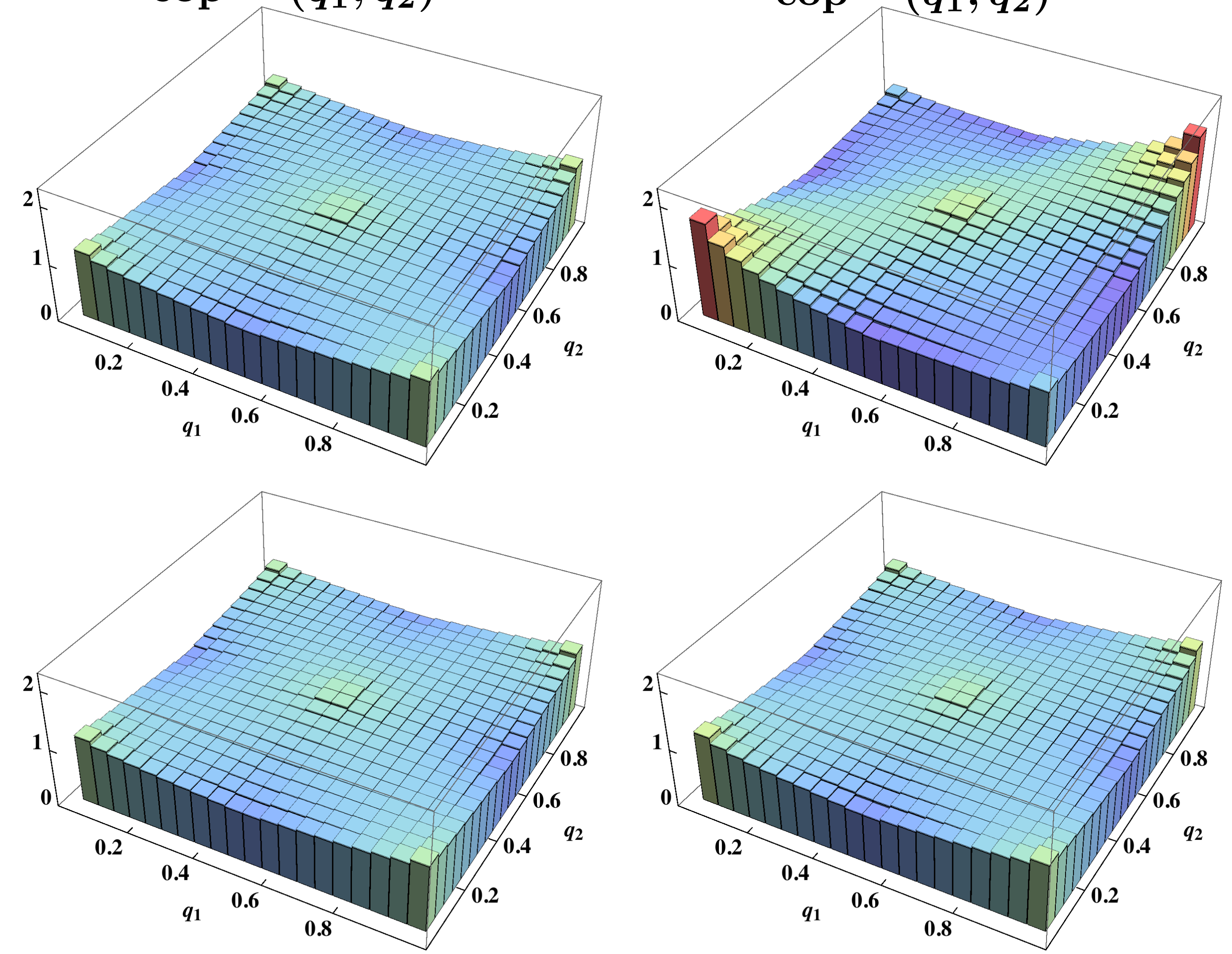
| Stock k | | | | Stock l | | | |
|---|---|---|---|---|---|---|---|
| $q_1$ | rank | $\nu_k$ | $\Sigma_k$ | $\nu_l$ | $\Sigma_l$ | rank | $q_2$ |
| 0.7 | 4 | 1.17 | 0.18 | 1.74 | 0.36 | 4 | 0.7 |
| 0.1 | 1 | -0.75 | 0.26 | -0.64 | 0.23 | 2 | 0.3 |
| 0.3 | 2 | -0.64 | 0.25 | 2.34 | 0.17 | 5 | 0.9 |
| 0.5 | 3 | -0.35 | 0.12 | -2.17 | 0.12 | 1 | 0.1 |
| 0.9 | 5 | 1.64 | 0.08 | 1.60 | 0.18 | 3 | 0.5 |

# 3 Influence of local fluctuations on dependencies

## Influence on the dependence structure

strong dependencies
in positive diagonal

$\mathrm{cop}^{(ss)}(q_1, q_2)$ $\mathrm{cop}^{(ll)}(q_1, q_2)$




$\mathrm{cop}^{(sl)}(q_1, q_2)$ $\mathrm{cop}^{(ls)}(q_1, q_2)$

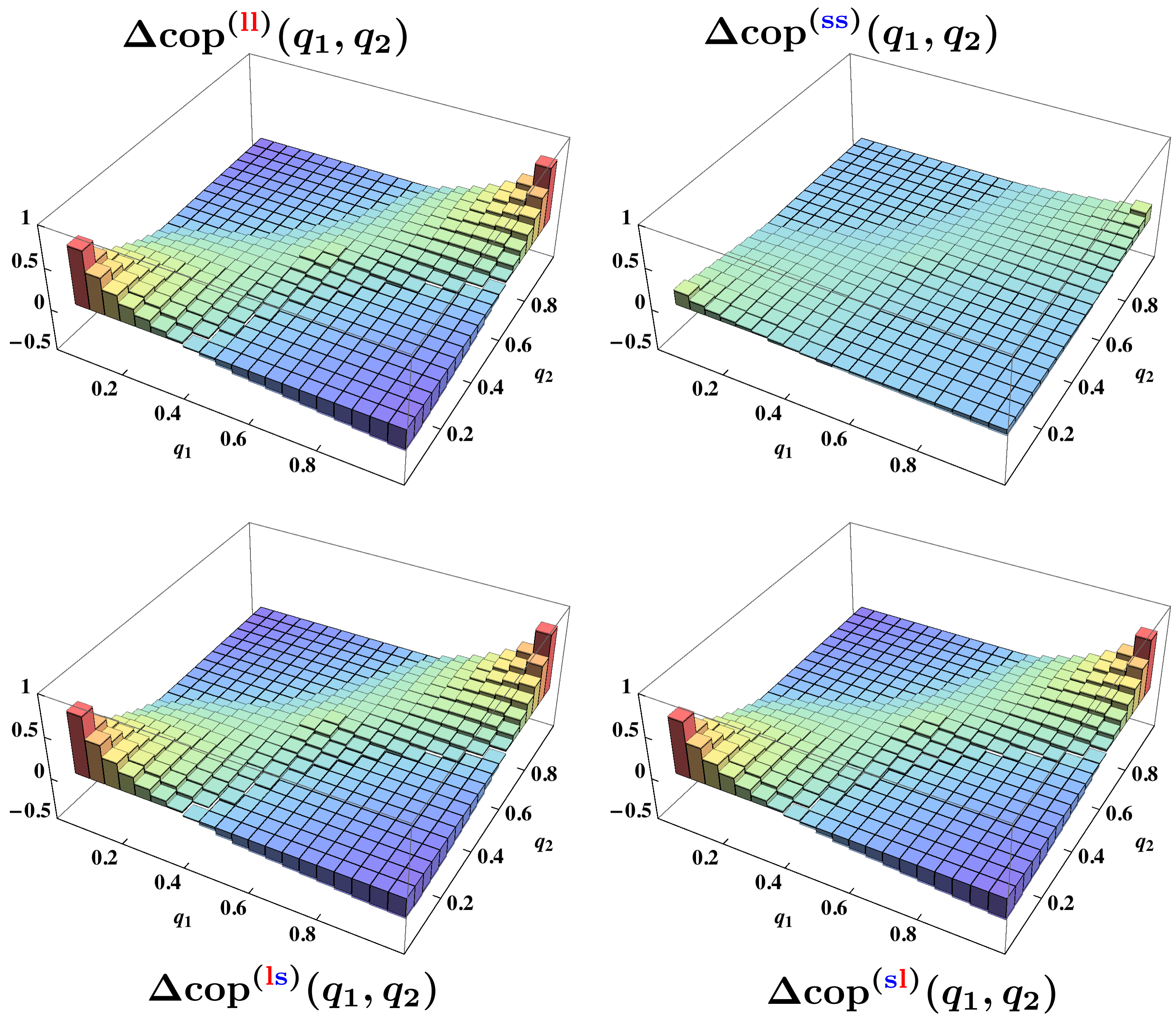
# 3 Influence of local fluctuations on dependencies

Influence of local fluctuations

$$
\begin{aligned}
\boldsymbol{\Delta}\mathbf{cop}^{(\mathbf{ll})}(q_1, q_2) &= \mathbf{cop}(q_1, q_2) - \mathbf{cop}^{(\mathbf{ss})}(q_1, q_2) \\
\boldsymbol{\Delta}\mathbf{cop}^{(\mathbf{ss})}(q_1, q_2) &= \mathbf{cop}(q_1, q_2) - \mathbf{cop}^{(\mathbf{ll})}(q_1, q_2) \\
\boldsymbol{\Delta}\mathbf{cop}^{(\mathbf{ls})}(q_1, q_2) &= \mathbf{cop}(q_1, q_2) - \mathbf{cop}^{(\mathbf{sl})}(q_1, q_2) \\
\boldsymbol{\Delta}\mathbf{cop}^{(\mathbf{sl})}(q_1, q_2) &= \mathbf{cop}(q_1, q_2) - \mathbf{cop}^{(\mathbf{ls})}(q_1, q_2)
\end{aligned}
$$

# 3 Influence of local fluctuations on dependencies

## Influence on the dependence structure




Conclusion 2: Large local fluctuations in either stock of a pair are important to cause the strong positive dependencies

# 3 Influence of local fluctuations on dependencies

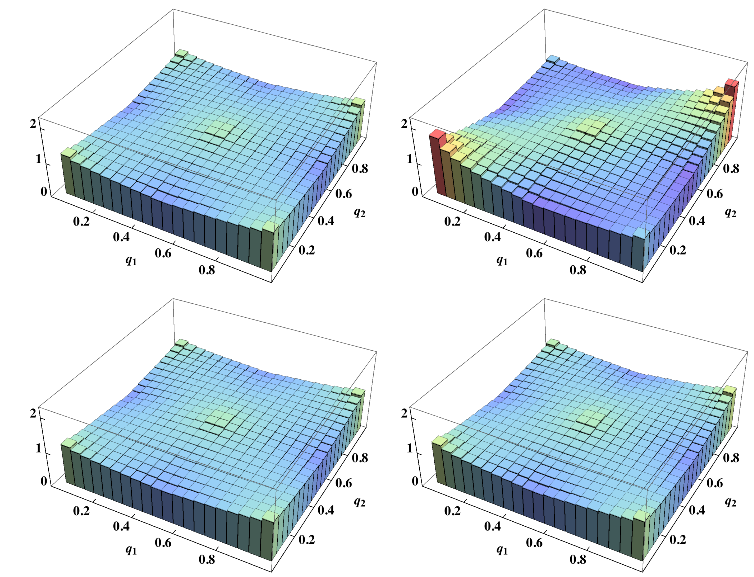


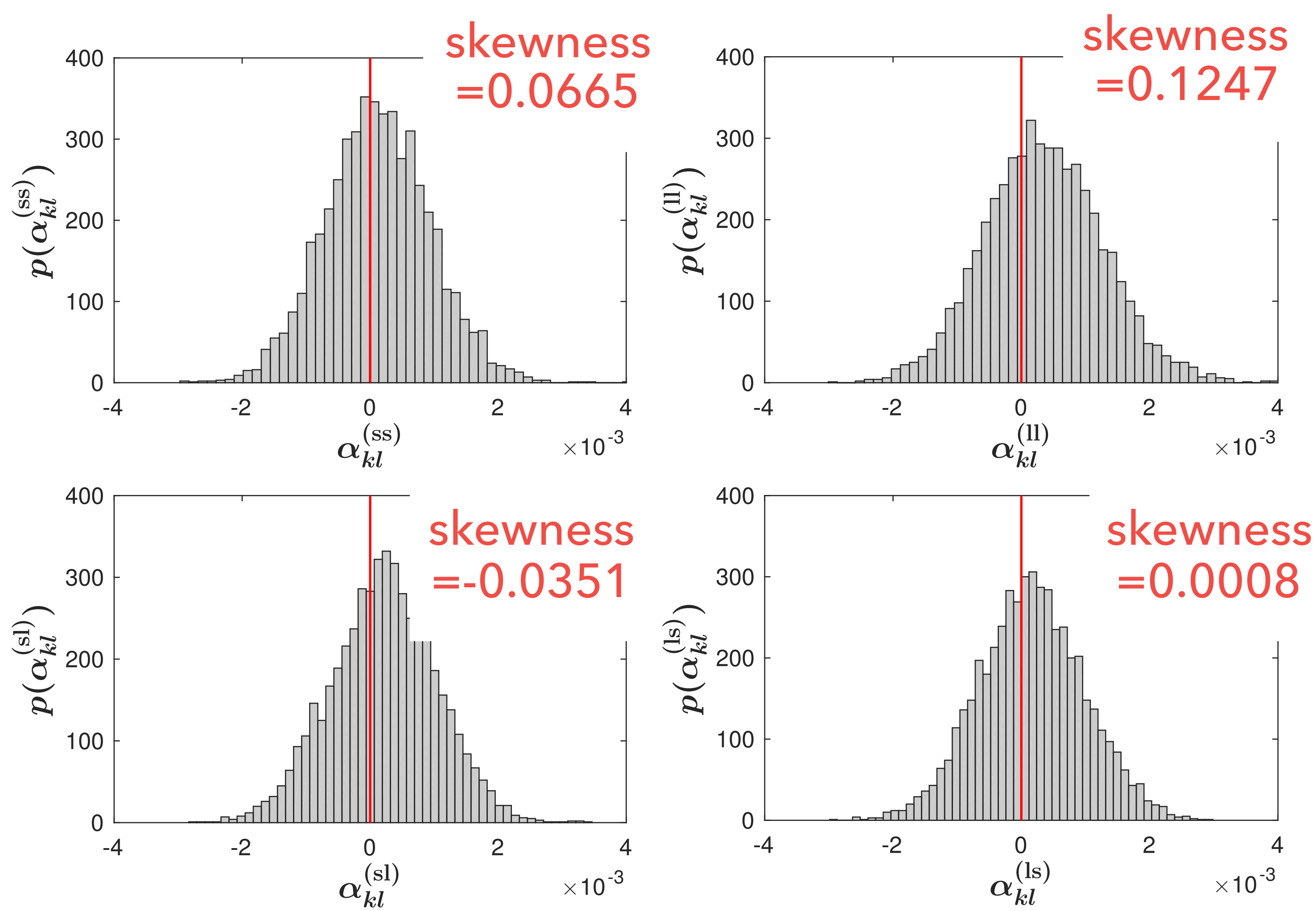
## Influence on the asymmetries of tail dependencies

1. Asymmetry of positive dependencies in tail:

$$\alpha_{kl} = \int_{0.8}^{1} dq_1 \int_{0.8}^{1} dq_2 \ \mathrm{cop}_{kl}(q_1, q_2) - \int_{0}^{0.2} dq_1 \int_{0}^{0.2} dq_2 \ \mathrm{cop}_{kl}(q_1, q_2)$$

dependence of demands — dependence of supplies






$$\mathrm{skewness} = \frac{E(x-\mu)^3}{\sigma^3}$$

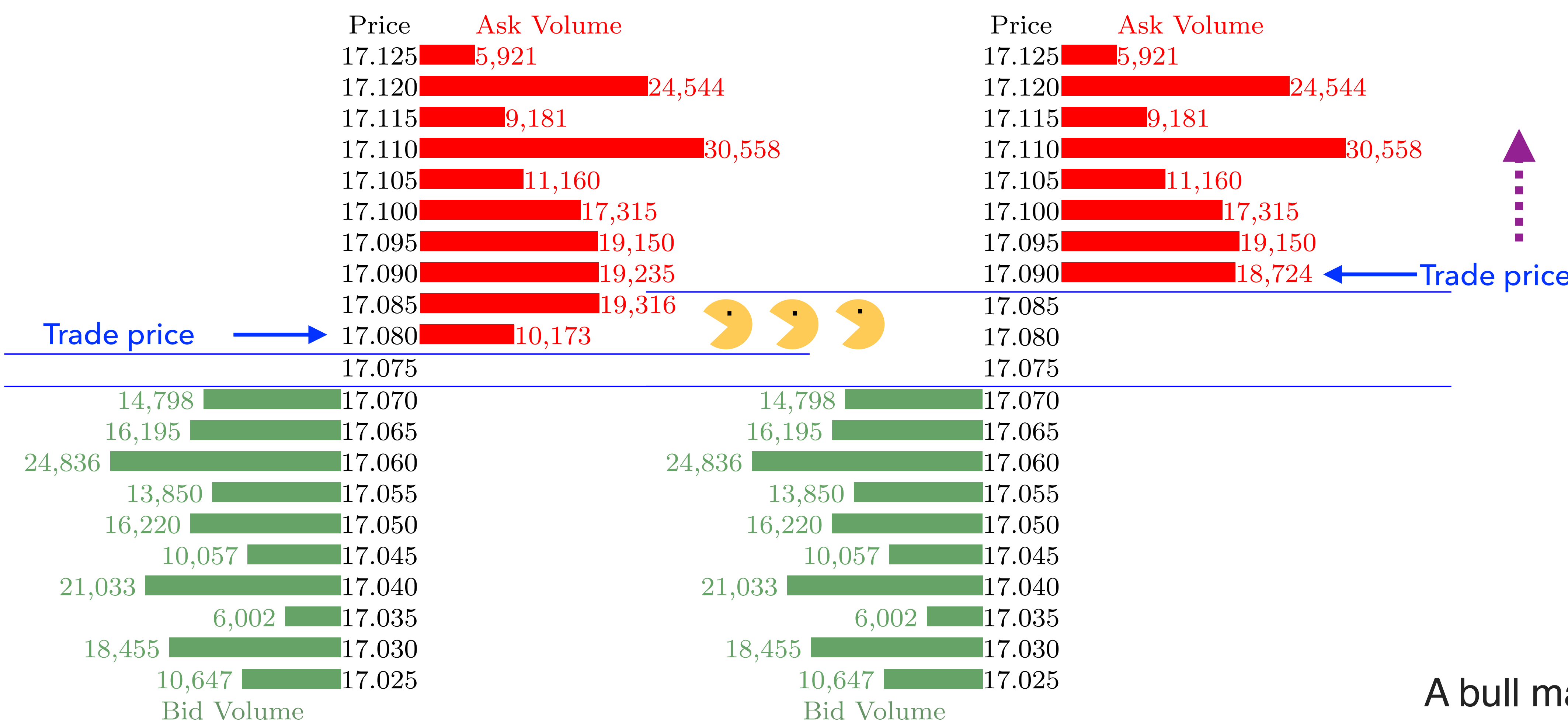
large local fluctuations

stronger dependence of demands than dependence of supplies

a large trade to buy one stock is more likely to find large trades to buy other stocks

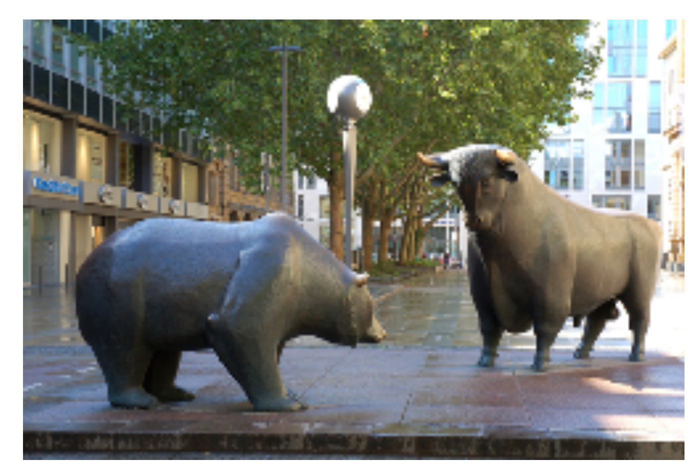
# 3 Influence of local fluctuations on dependencies

## Influence on the asymmetries of tail dependencies




Price change in the order book for market orders to buy.
Left: order book at t=0; right: order book at t=T

A bull market is a period of generally rising prices.



Conclusion 3: Large local fluctuations cause the stronger dependence of demands than the dependence of supplies, which implies price raising of most stocks, resulting in a bull market if this state persists.

# 3 Influence of local fluctuations on dependencies

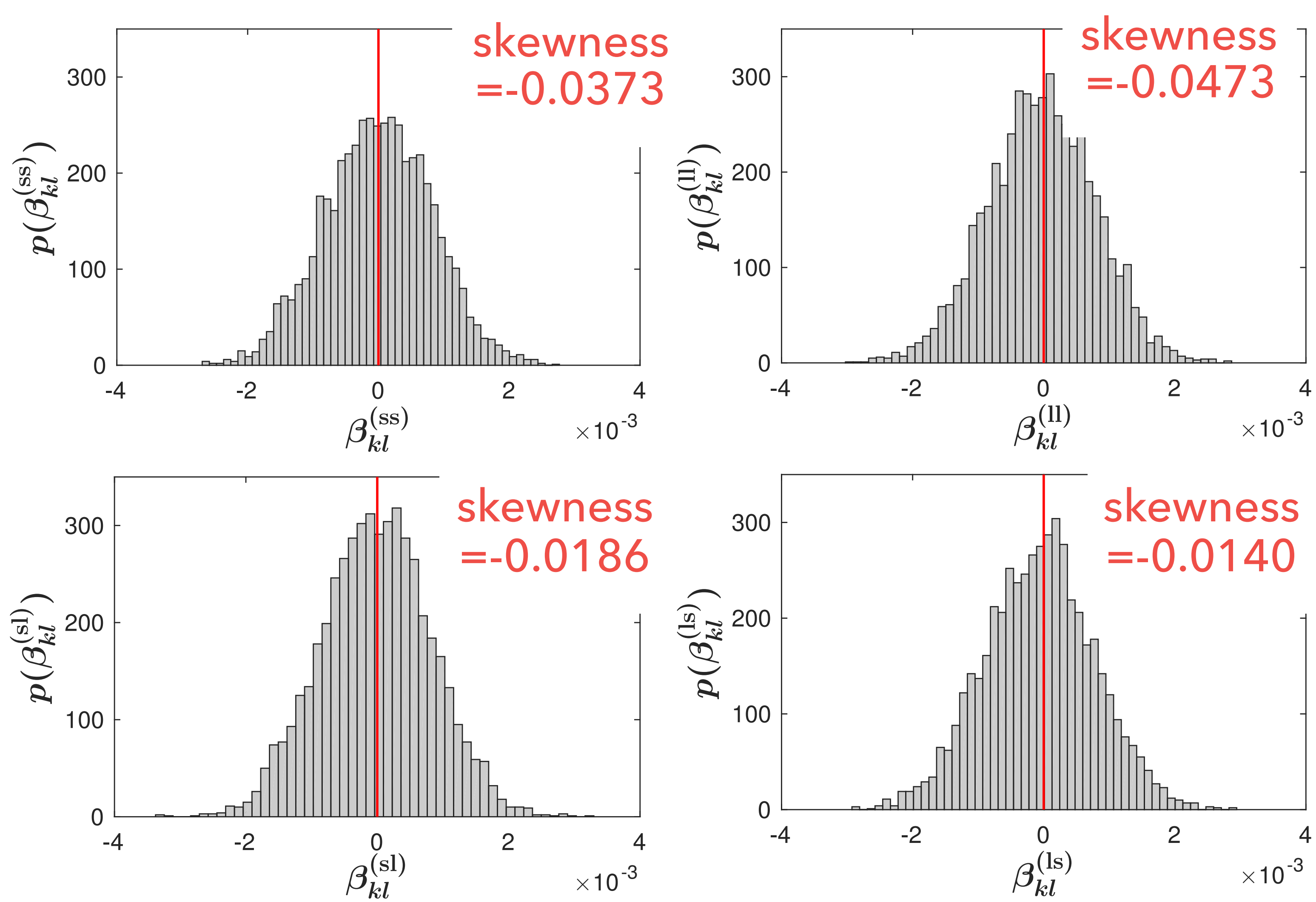


## Influence on the asymmetries of tail dependencies

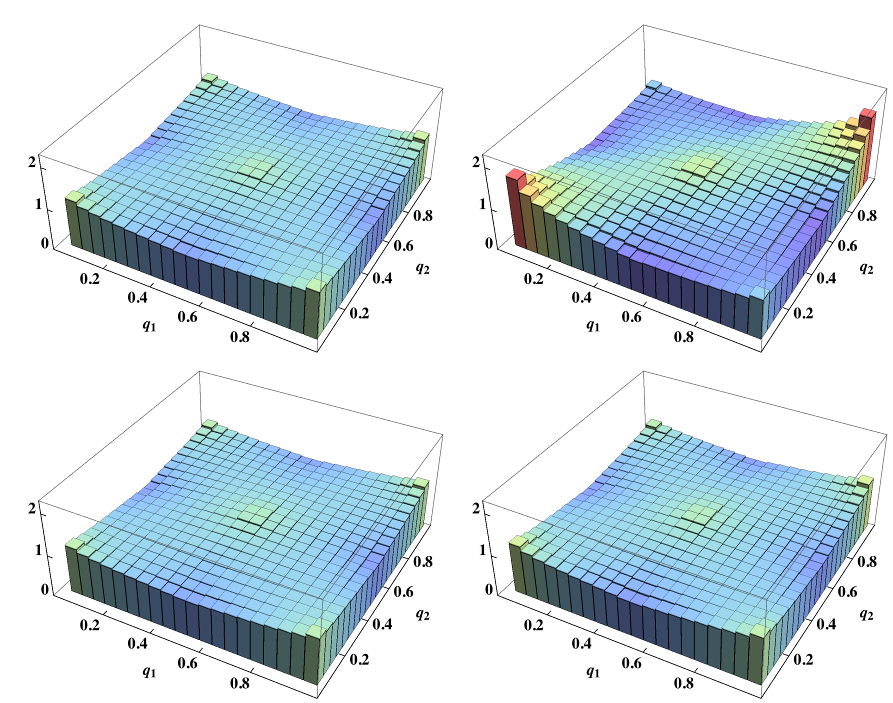
2. Asymmetry of negative dependencies in tail

$$\beta_{kl} = \int_0^{0.2} dq_1 \int_{0.8}^{1} dq_2 \ \mathrm{cop}_{kl}(q_1, q_2) - \int_{0.8}^{1} dq_1 \int_0^{0.2} dq_2 \ \mathrm{cop}_{kl}(q_1, q_2)$$

dependence of supply and demand — dependence of demand and supply

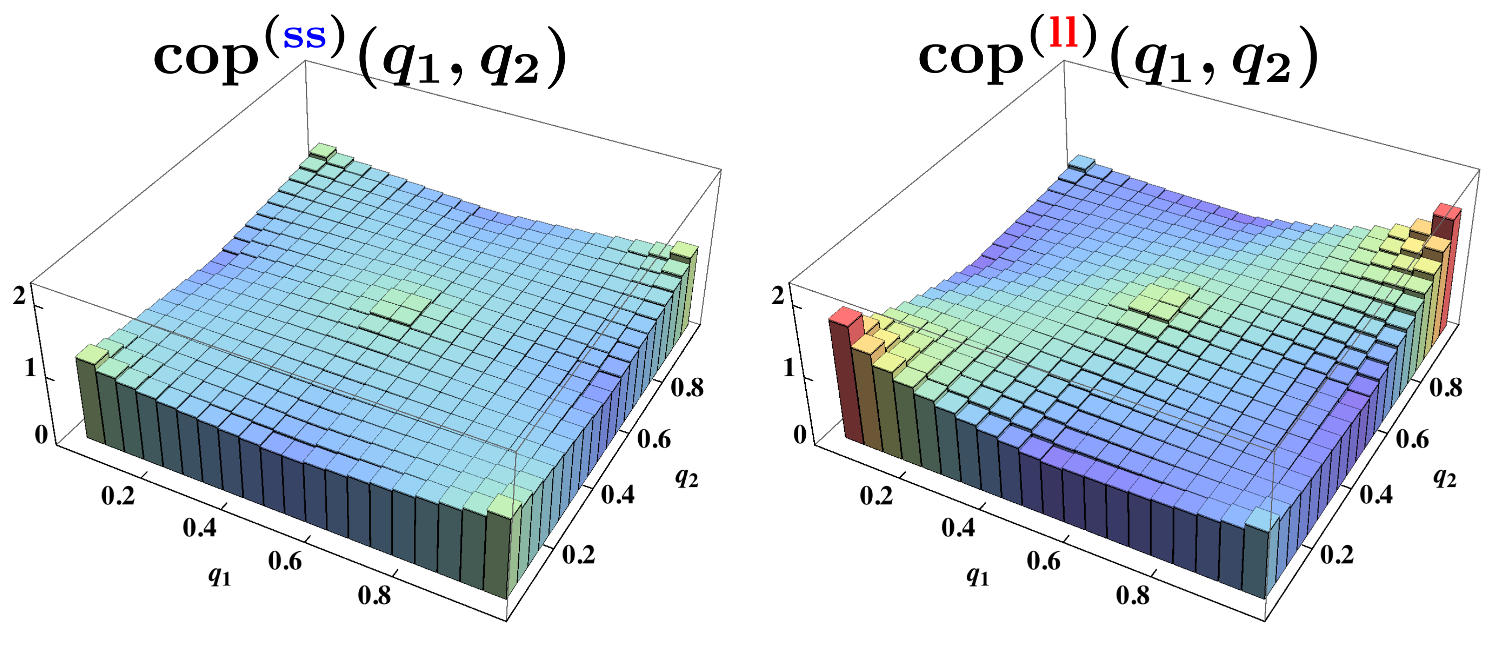



$$\text{skewness} = \frac{E(x-\mu)^3}{\sigma^3}$$



# 3 Influence of local fluctuations on dependencies

## Influence on the asymmetries of tail dependencies




3. Asymmetry of positive and negative dependencies in tail:

$$\begin{aligned}\gamma_{kl} &= \Big(\int_{0.8}^{1} dq_1 \int_{0.8}^{1} dq_2\ \mathrm{cop}_{kl}(q_1,q_2) + \int_{0}^{0.2} dq_1 \int_{0}^{0.2} dq_2\ \mathrm{cop}_{kl}(q_1,q_2)\Big) \\ &- \Big(\int_{0}^{0.2} dq_1 \int_{0.8}^{1} dq_2\ \mathrm{cop}_{kl}(q_1,q_2) + \int_{0.8}^{1} dq_1 \int_{0}^{0.2} dq_2\ \mathrm{cop}_{kl}(q_1,q_2)\Big)\end{aligned}$$

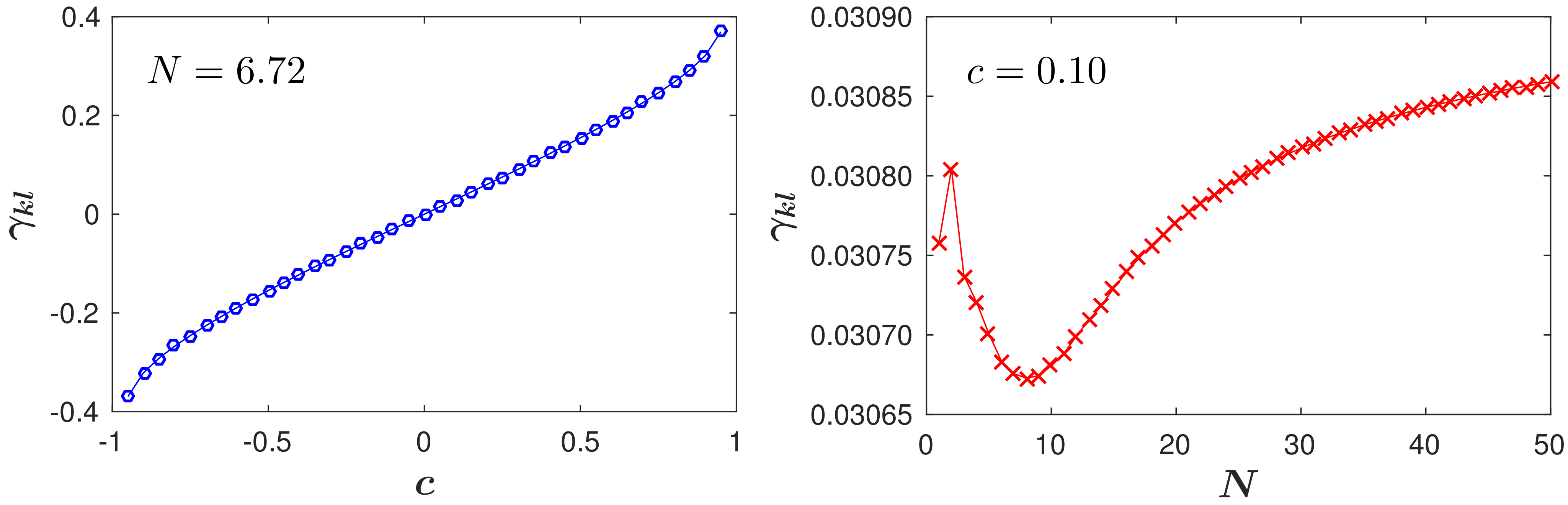
positive dependencies — negative dependencies

# 3 Influence of local fluctuations on dependencies



**K-copula density**

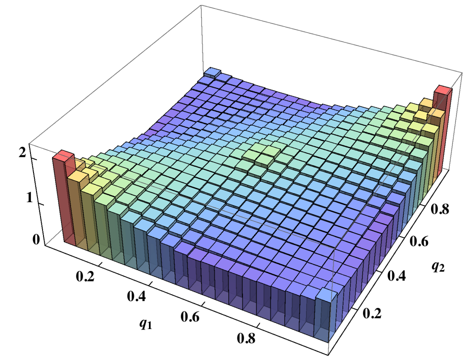
$$\mathrm{cop}^{\mathcal{K}}_{c,N}(q_1, q_2) = \frac{f\left(F_k^{-1}(q_1), F_l^{-1}(q_2)\right)}{f_k\left(F_k^{-1}(q_1)\right) f_l\left(F_l^{-1}(q_2)\right)}$$

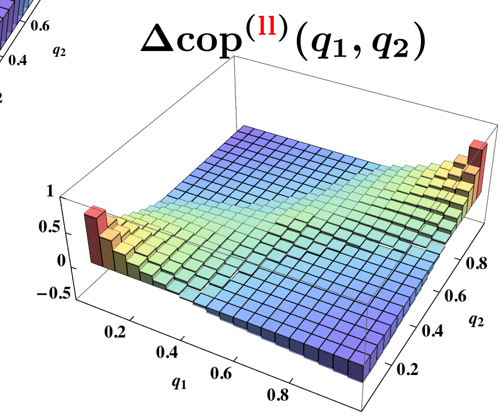


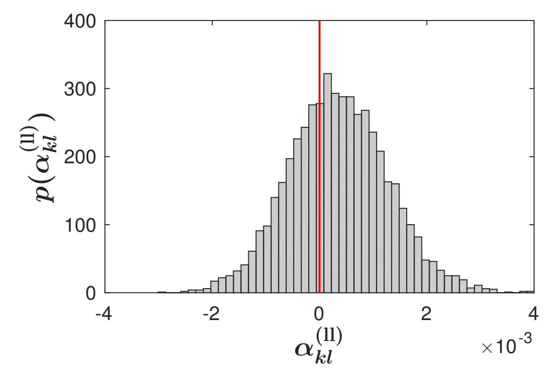

Conclusion 4: Large local fluctuations are more likely to induce the strong correlation, leading to the change of tailed dependencies, and further to the strong dependence of demands

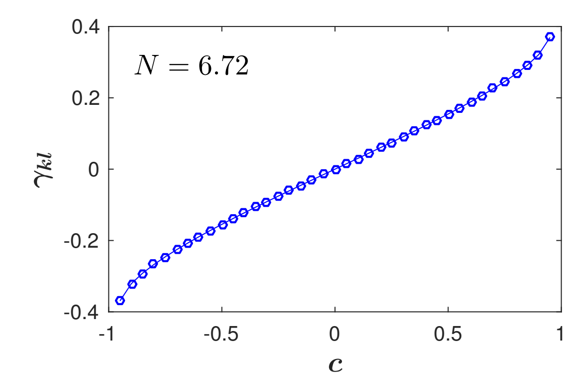
# 4 Summary

- Empirical copula densities can be described well by a bivariate K-copula density function
- Influence on dependence structures: large local fluctuations in either stock of a pair are important to cause the strong positive dependencies, including the dependence of supplies and the dependence of demands
- Influence on asymmetries: Large local fluctuations cause the stronger dependence of demands than the dependence of supplies, which implies price raising of most stocks, resulting in a bull market if this state persists.
- The mechanism of influences: Large local fluctuations are more likely to induce the strong correlation, leading to the change of tailed dependencies, and further to the strong dependence of demands



Thank you for your attention !

Q&A