Credit risk and the instability of the financial system: An ensemble approach

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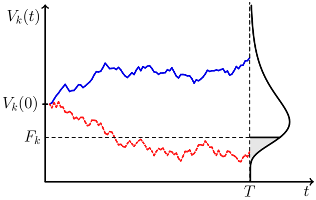
November 18th, 2016

Outline

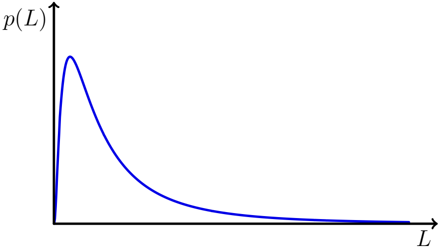
1. Introduction
2. Model
   - Ensemble approach
   - Average loss distribution
3. Results



Motivation

- Subprime crisis 2007 – 2009
- Studies pointed out the importance of better credit risk estimation
- Determine loss distribution for a portfolio of credit contracts
- In the heavy right tail of the loss distribution the very large losses occur
- Reducing this heavy tail would increase the stability of a financial system
- Is it possible to get rid of this heavy tail by diversification?
- Diversification here: Enlarging the number of obligors and credit contracts

Loss distribution

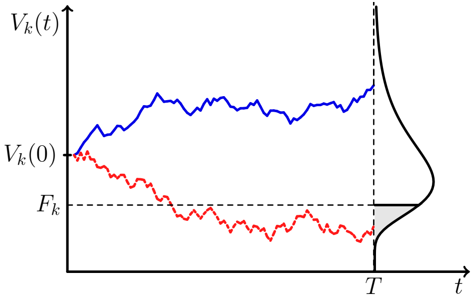


All grapics are taken from

T. Schmitt et al., *Europhysics Letters*, **105** (2014) 38004
T. Schmitt et al., *Europhysics Letters*, **103** (2013) 58003

- Asymmetry and heavy tail
- Correlations between asset values (obligors) have to be involved

Merton model



- K obligors (publicly traded companies)
- At maturity time T each obligor has to pay back the face value F_k
- Mertons idea: Use the stock price $S_k(t)$ as a proxy for the stochastic process $V_k(t)$
- Zero coupon bonds

Merton model

Normalized dimentionless loss

$$L_k = \frac{F_k - V_k(T)}{F_k} \Theta\left(F_k - V_k(T)\right)$$

Total portfolio loss

$$L = \sum_{k=1}^{K} L_k f_k \qquad \text{with} \qquad f_k = \frac{F_k}{\sum_{i=1}^{K} F_i}$$

Distribution of the portfolio loss

$$p(L) = \int d[V] g(V|\Sigma) \delta\left(L - \sum_{k=1}^{K} L_k f_k\right)$$

$g(V|\Sigma)$ distribution of the asset values at maturity time T
Σ stationary covariance matrix
$V = (V_1(T), \ldots, V_K(T))$

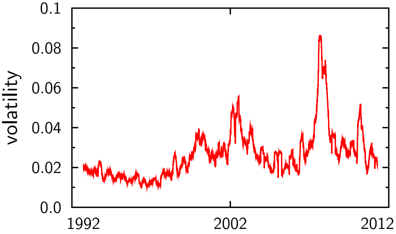
Ensemble approach

Requirements to $g(V|\Sigma)$

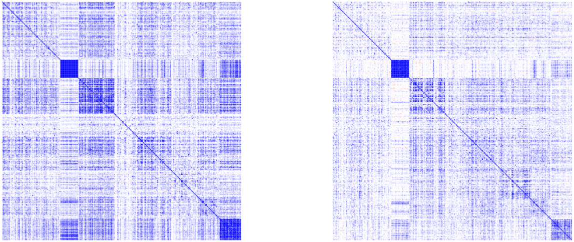
- has to describe empirical data well and address the non-stationarity of financial markets
- should be analytically tractable

Non Stationarity is studied for the returns

$$r_k(t) = \frac{S_k(t + \Delta t) - S_k(t)}{S_k(t)}$$



Ensemble approach



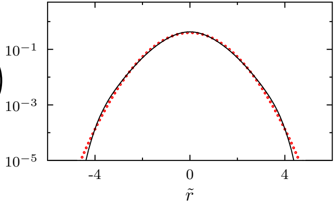
Correlation matrices of $K = 306$ companies for the fourth quarter of 2005 and the first quarter of 2006

- Correlation matrices measured at different times differ from each other
- As a consequence of non-stationarity the ensemble really exists
- We average over an ensemble of random matrices

Ensemble approach

- In short data intervalls Σ_S is assumed to be stationary
- Data can be described by a multivariate Gaussian

$$g(r|\Sigma_S) = \frac{1}{\sqrt{\det(2\pi\Sigma_S)}} \exp\left(-\frac{1}{2} r^\dagger \Sigma_S^{-1} r\right)$$



We are interested in large time intervals where sizable non-stationarity is present

Average over the correlation matrices by means of a random matrix approach

Ensemble average

- We average over an ensemble of Wishart distributed random matrices
- $\Rightarrow$ Drastic reduction in the number of degrees of freedom

$$\langle g \rangle (r|\Sigma, N) = \frac{\sqrt{N}^K}{\sqrt{2}^{N-2}\Gamma(N/2)\sqrt{\det(2\pi\Sigma)}} \frac{\mathcal{K}_{(K-N)/2}\left(\sqrt{N r^\dagger \Sigma^{-1} r}\right)}{\sqrt{N r^\dagger \Sigma^{-1} r}^{(K-N)/2}}$$

- N controlls the strength of the fluctuations around the average correlation matrix C with $\Sigma = \sigma C \sigma$
- N has to be determined from the data

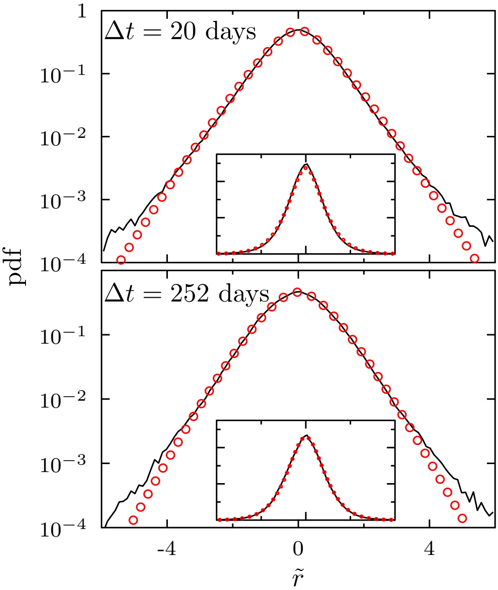
Ensemble average

We achieve analytical tractability by assuming a homogeneous average correlation matrix $\Sigma = \sigma C \sigma$

$$C = (1-c)\mathbb{1}_K + cee^\dagger = \begin{pmatrix} 1 & c & c & \dots \\ c & 1 & c & \dots \\ c & c & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Two parameters describe the distribution

- N controlls the strength of the fluctuations around the average correlation matrix C
- c is the average correlation level


$\Delta t = 20$ days
$\Delta t = 252$ days
pdf
1
10^{-1}
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10^{-3}
10^{-4}
-4
0
4
$\tilde{r}$

Merton model

Asset values follow a geometric Brownian motion

$$r_k = \ln \frac{V_k(T)}{V_k(0)} - \mu_k T - \frac{\sigma_k^2}{2}$$

μ_k drift

σ_k standard deviation

Both parameters have to be determined from data

Average loss distribution

Loss distribution

$$L = \sum_{k=1}^{K} L_k f_k \qquad \text{with} \qquad f_k = \frac{F_k}{\sum_{i=1}^{K} F_i}$$

$$\langle p \rangle (L) = \int d[V] \langle g \rangle (V|\Sigma) \delta \left(L - \sum_{k=1}^{K} L_k f_k \right)$$

- All face values are of the same order $\Rightarrow f_k \approx \frac{1}{K}$
- Carry out a second-order approximation in f_k

Average loss distribution

$$\langle p\rangle(L|c,N) = \frac{1}{\sqrt{2\pi}2^{N/2}\Gamma(N/2)} \int_0^{\infty} dz\ z^{N/2-1} e^{-z/2} \sqrt{\frac{N}{2\pi}}$$
$$\times \int_{-\infty}^{+\infty} du \exp\left(-\frac{N}{2}u^2\right) \frac{1}{\sqrt{M_2(z,u)}} \exp\left(-\frac{(L-M_1(z,u))^2}{2M_2(z,u)}\right)$$

$$M_1(z,u) = \sum_{k=1}^{K} f_k m_{1k}(z,u) \quad M_2(z,u) = \sum_{k=1}^{K} f_k^2 \left(m_{2k}(z,u) - m_{1k}(z,u)^2\right)$$

$$m_{jk}(z,u) = \frac{\sqrt{NT}}{\sigma_k\sqrt{2\pi T(1-c)}} \int_{-\infty}^{\hat{F}_k} d\hat{V}_k \left(1 - \frac{V_{k0}}{F_k} \exp\left(\sqrt{z}\hat{V}_k + \mu_k T - \frac{\sigma_k^2}{2}\right)\right)^j$$
$$\times \exp\left(-\frac{\left(\hat{V}_k + \sqrt{c}u\sigma_k\right)^2}{2(1-c)\sigma_k^2/N}\right)$$

Homogeneous portfolio

All parameters are the same

$\mu_k = \mu_0,\ \sigma_k = \sigma_0,\ F_k = F_0 \Rightarrow f_k = \frac{1}{K}$

Limiting distribution

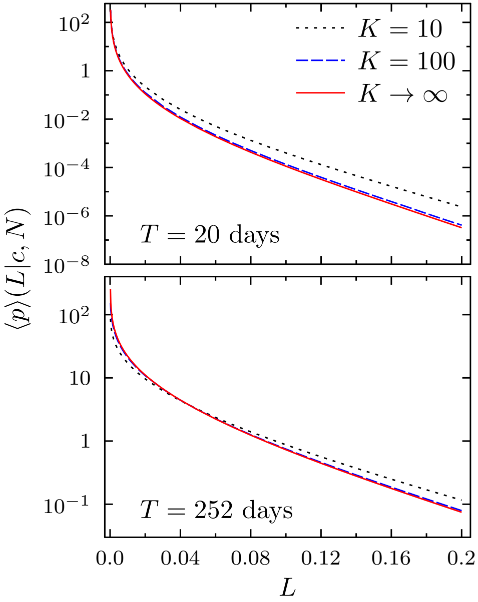
$$\lim_{K\to\infty} \langle p \rangle (L|c, N) = \frac{1}{2^{N/2}\Gamma(N/2)} \sqrt{\frac{N}{2\pi}} \int_0^\infty dz\, z^{N/2-1} e^{-z/2} \times \exp\left(-\frac{N}{2} u_0^2\right) \frac{1}{\left|\partial m_{10}(z,u)/\partial u\right|_{z,u_0}}$$

where u_0 is such that

$$L - m_{10}(z, u_0) = 0$$

$c = 0.26$
$\mu_0 = 0.013\,\text{month}^{-1}$
$\rho_0 = 0.1\,\text{month}^{-1/2}$
$N = 4.2$

$c = 0.28$
$\mu_0 = 0.17\,\text{year}^{-1}$
$\rho_0 = 0.35\,\text{year}^{-1/2}$
$N = 6$



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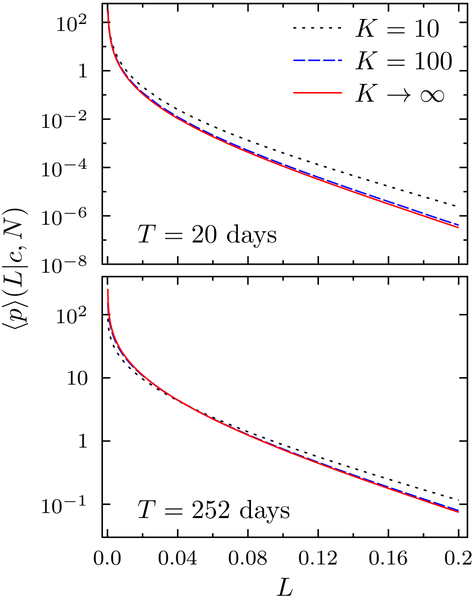
Diversification does not work!



Subordinated model

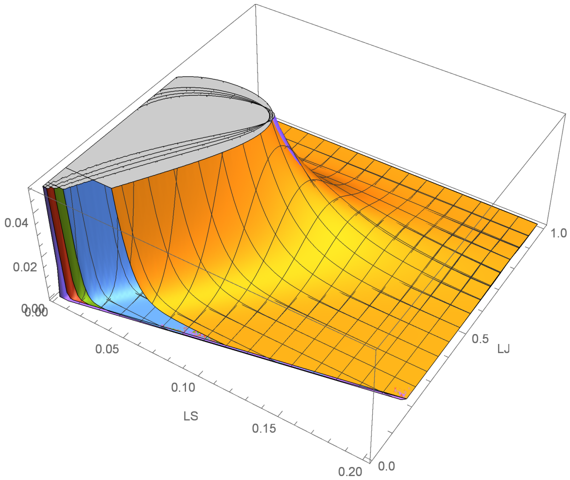
- Two or more creditors
- Subordinated debt: Senior creditor is paid out first
- Junior subordinated creditor is only paid out if the senior creditor regained the full promised payment

Difficulties

- Long mathematical expressions
- Solving the two dimensional integral

References

- Thilo A. Schmitt and Desislava Chetalova and Rudi Schäfer and Thomas Guhr, Credit risk and the instability of the financial system: An ensemble approach, *Europhysics Letters*, **105** (2014) 38004
- Thilo A. Schmitt, Desislava Chetalova, Rudi Schäfer, and Thomas Guhr, Non-Stationarity in Financial Time Series and Generic Features, *Europhysics Letters*, **103** (2013) 58003
- Thilo A. Schmitt, Rudi Schäfer, and Thomas Guhr, Credit risk: taking fluctuating asset correlations into account, *Journal of Credit Risk*, **11** (2015) 3
- Robert C. Merton, On the pricing of corporate debt: the risk structure of interest rates, *Journal of Finance*, **29** (1974) 2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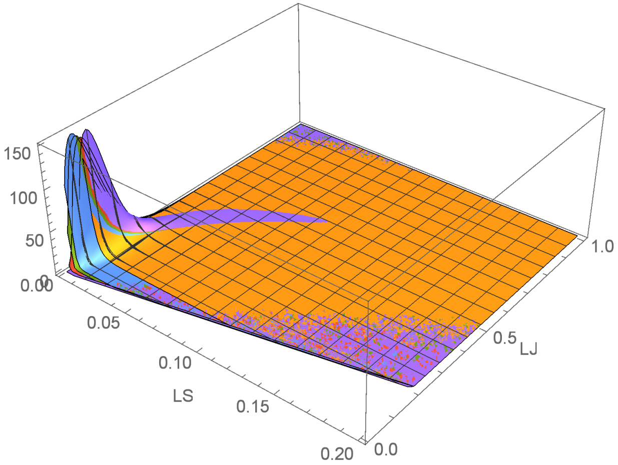
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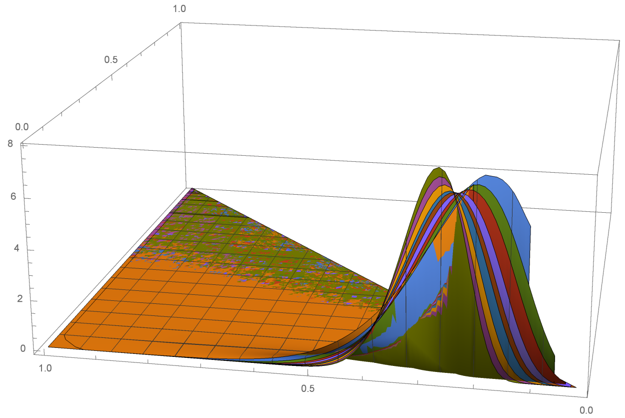
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