Correlation Dynamics: Linear Drag and Potential Functions of the Market States

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Content

Data Set

- Stocks of **S&P500** for 21 years (1992 - incl. 2012)
- KxK correlation matrices, with $K = 307$
- Averaged over T = 42 td (2 trading months)
- Got rid of local trends and variable volatility keeping cross-correlations: **local normalisation**[1]

[1] **R. Schäfer and T. Guhr**, *Local normalization: Uncovering correlations in non-stationary financial time series,* Physica A **389**, 3856 (2010)

Market States

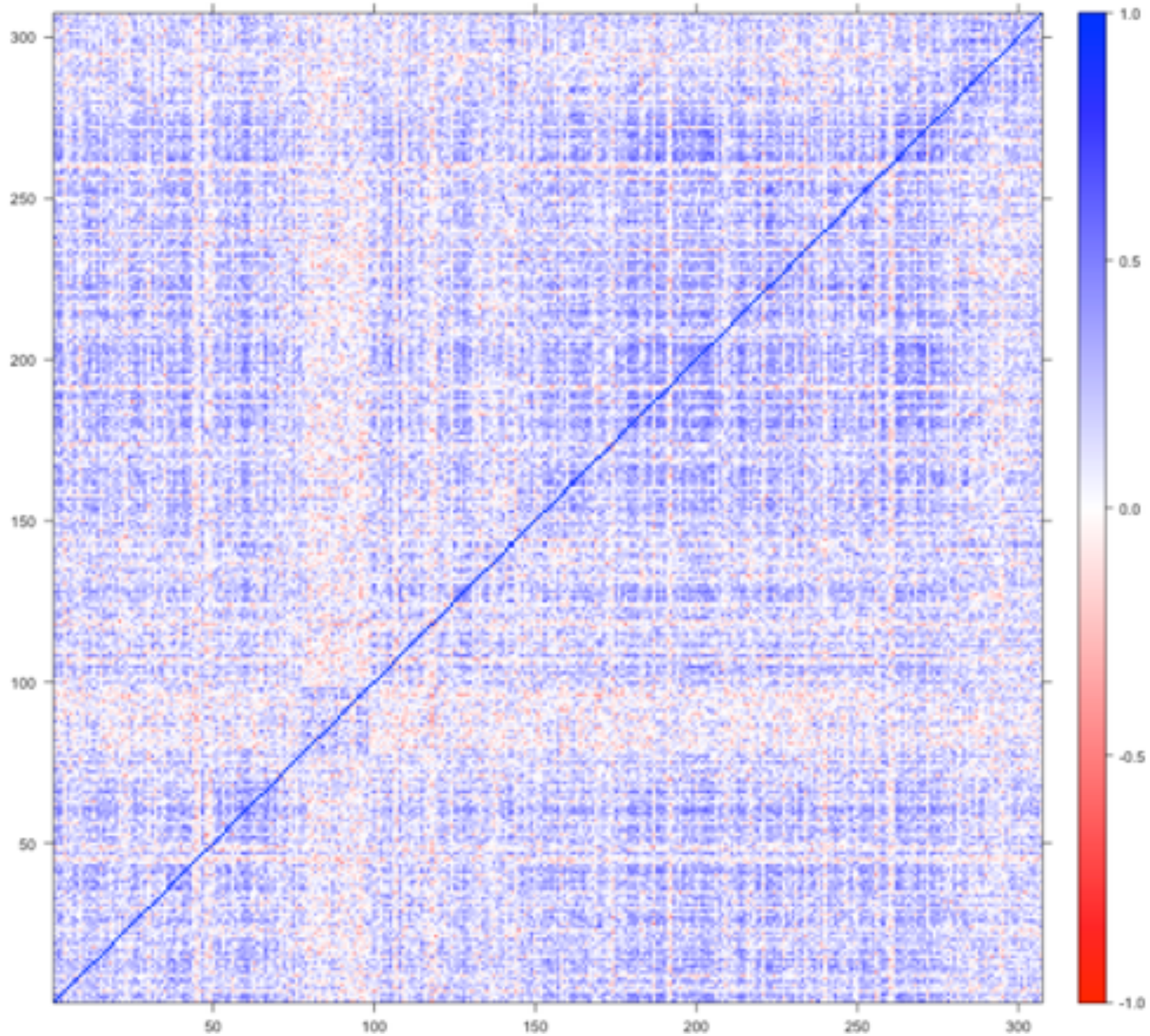
- We have n = 5169 correlation matrices $\underline{C}(t) \Leftrightarrow \vec{C}(t) \in \mathbb{R}^d$

  with $d = \frac{K^2-K}{2} = 46971$.

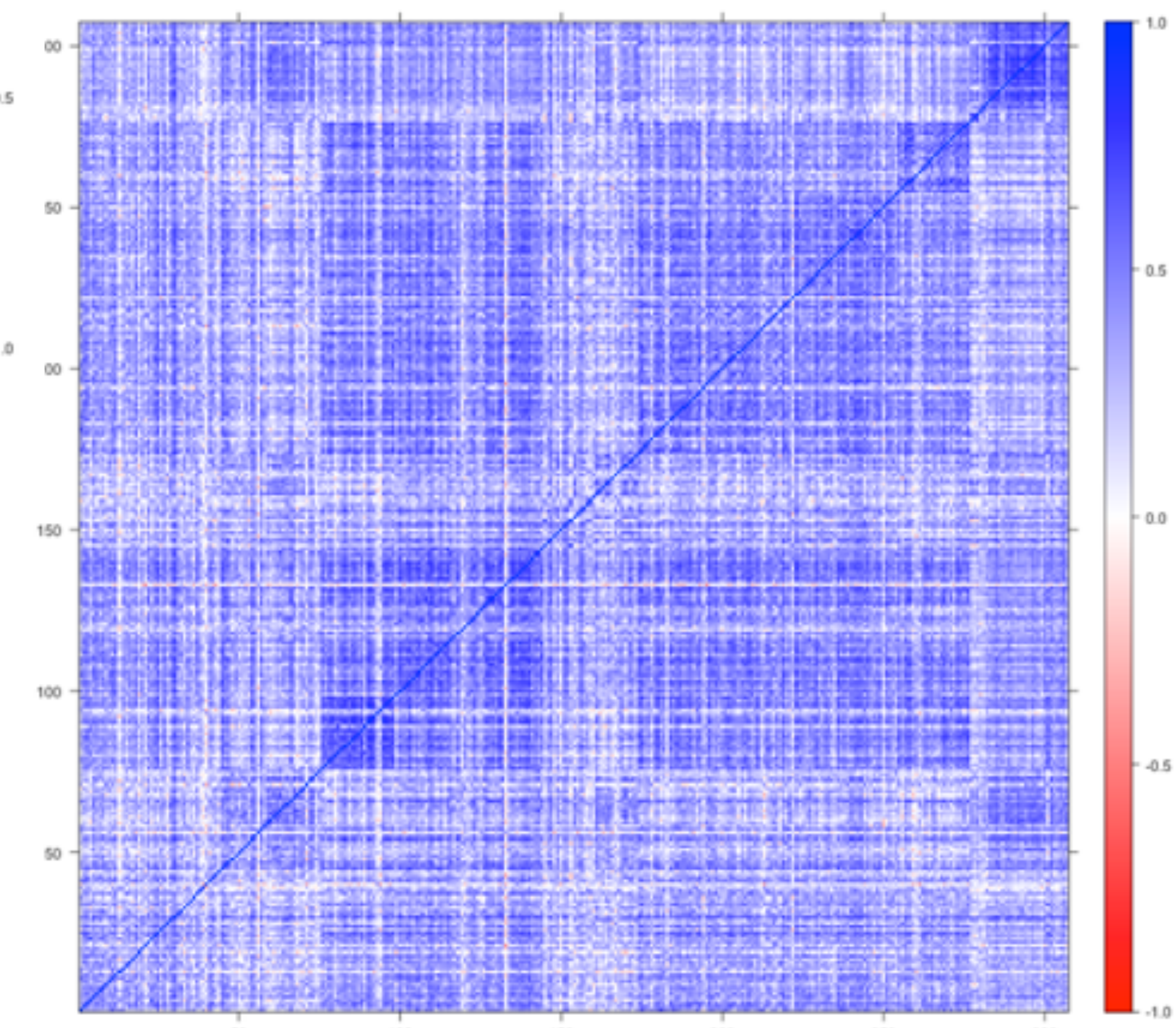
- All matrices are clustered by: a hierarchical, top-down clustering, based on the *k-means* splitting as introduced in ref. [2]

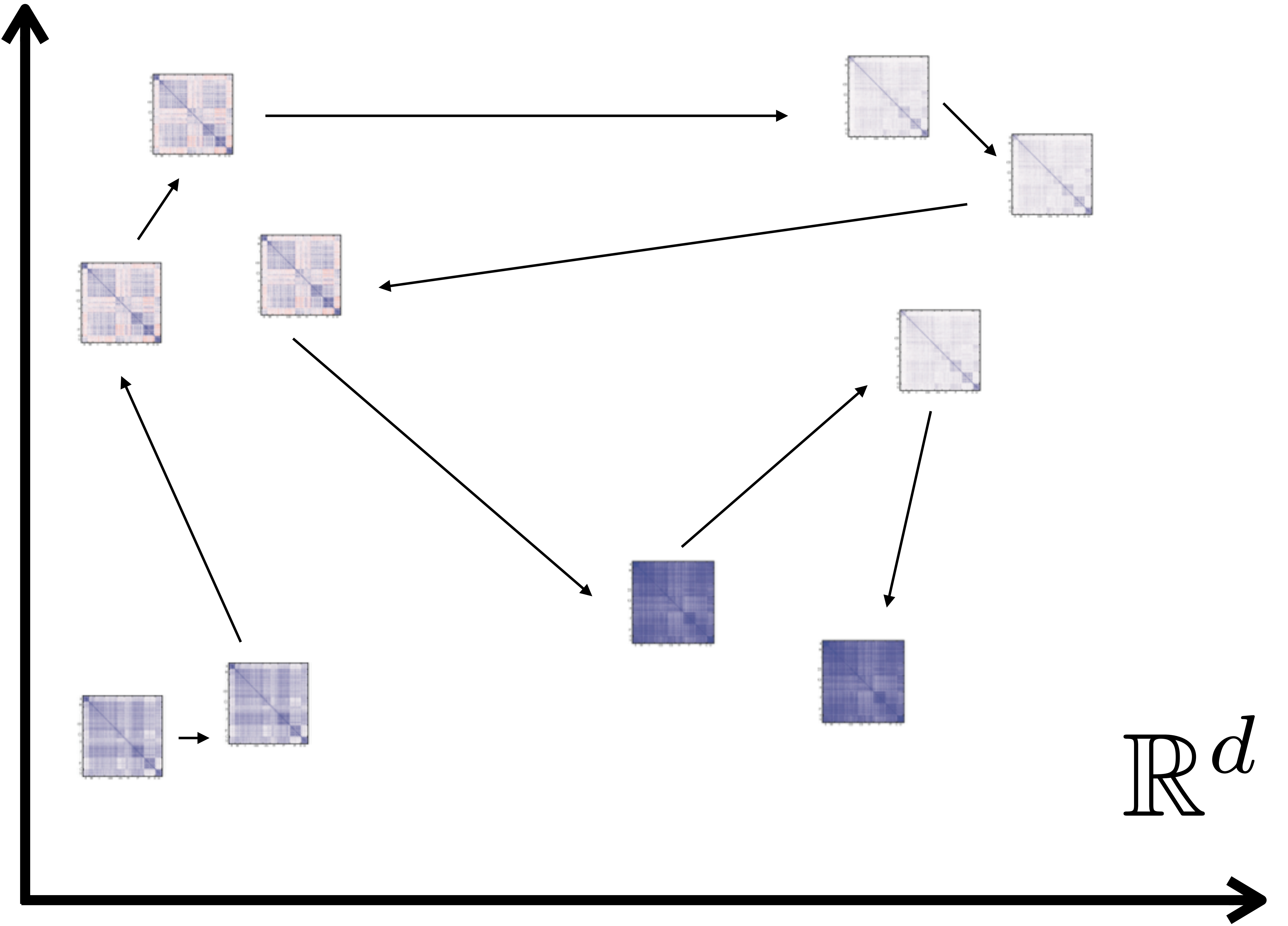
- The number of clusters is determined by a threshold

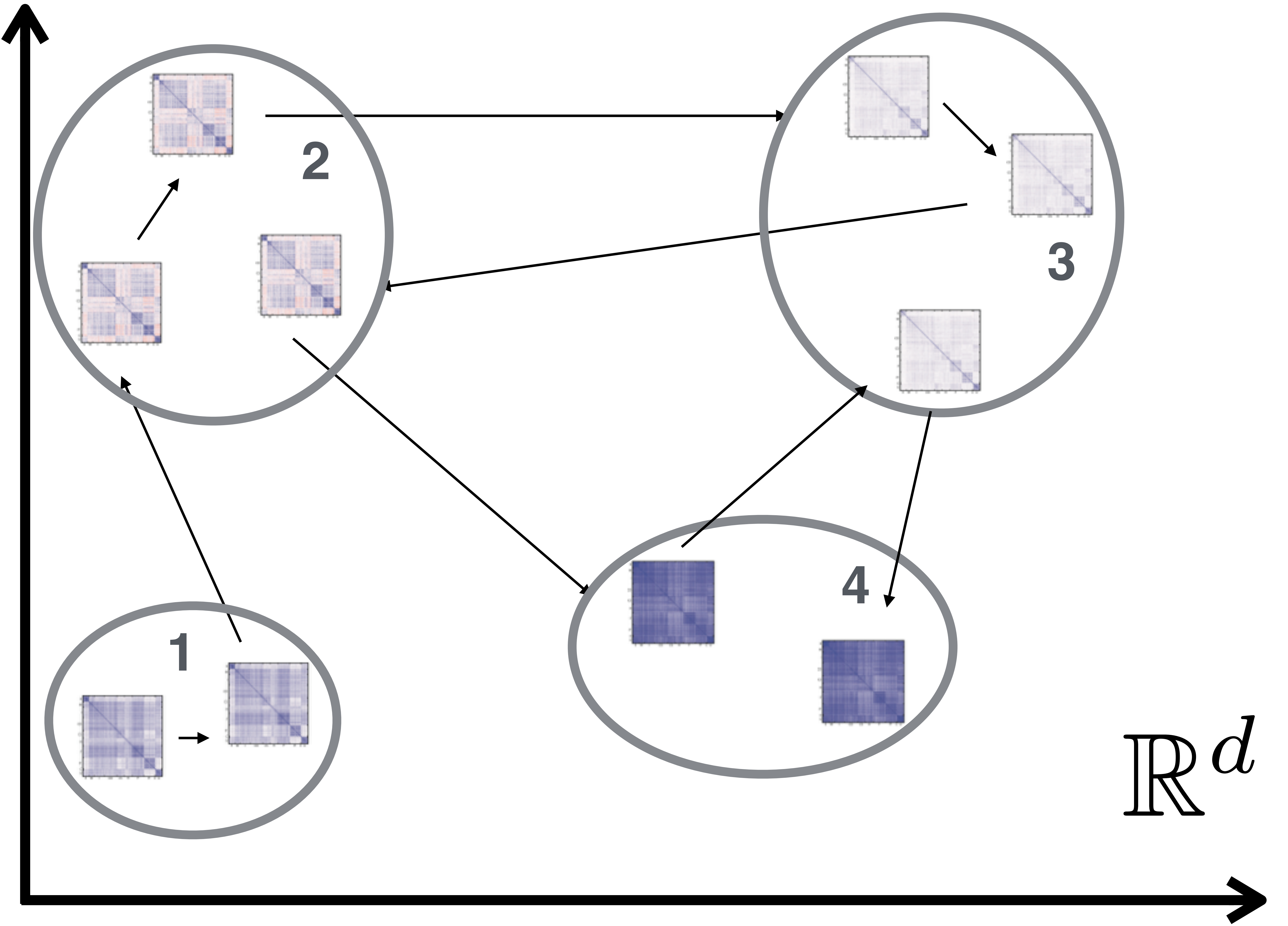
[2] **M.C. Münnix, T. Shimada, R. Schäfer, F. Leyvraz, T.H. Seligman, T. Guhr and H.E. Stanley**, *Identifying States of a Financial Market*, Scientific Reports **2** : 644 (2012)



Correlation Matrices

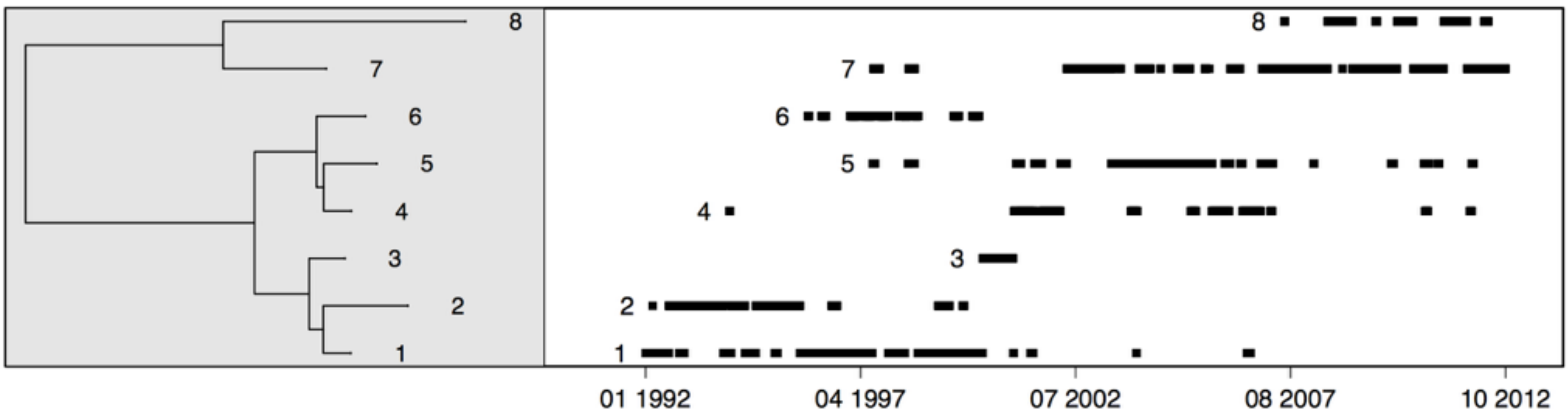



$\mathbb{R}^d$



2
3
1
4
$\mathbb{R}^d$

Market States



Mean Correlation

Analysed quantities: $c(t)$ and $\dot{c}(t)$ defined by:

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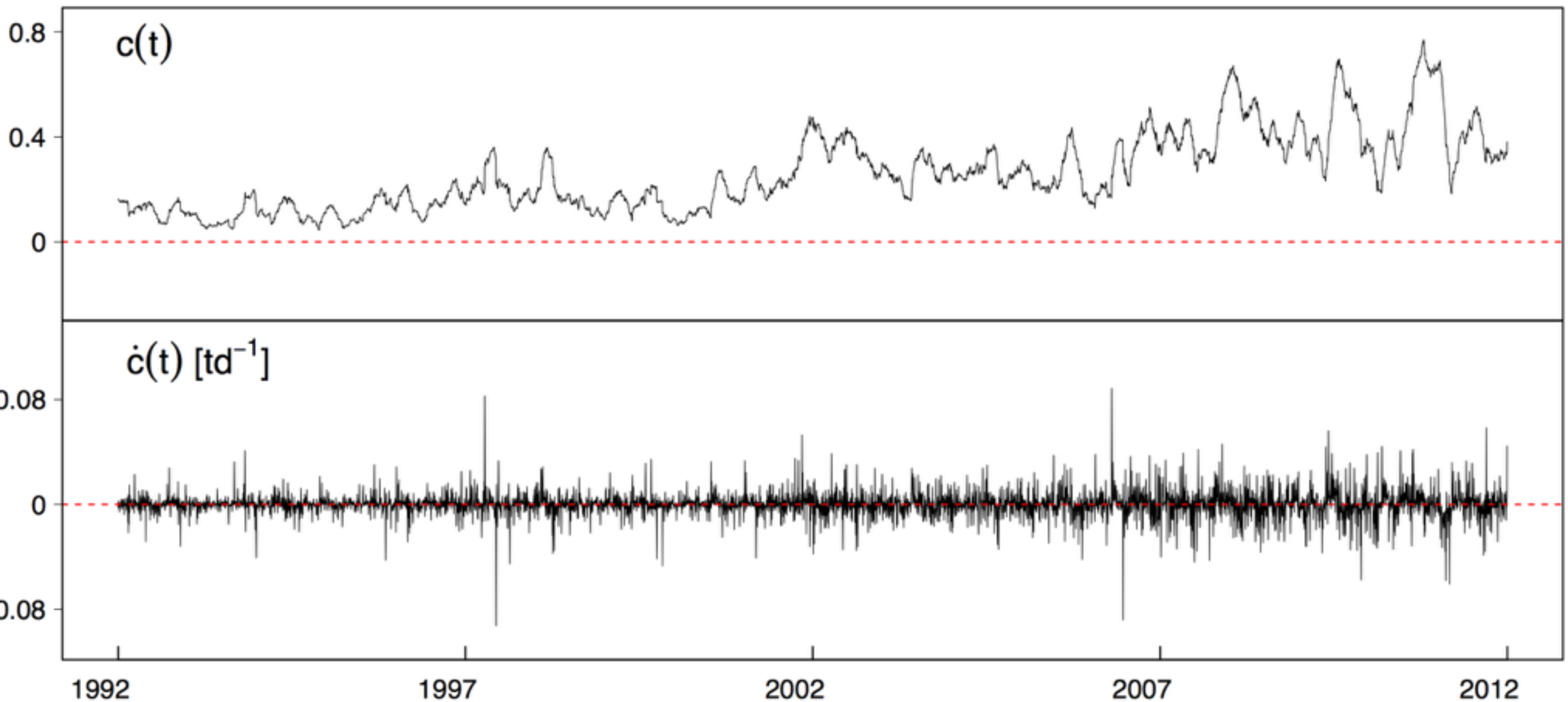
$$[\dot{c}(t)] = \frac{1}{\mathrm{Time}}$$

Mean Correlation



Stochastic Differential Equations
SDEs

Stochastic Differential Equation

- In *N* dimensions for $\vec{X} \in \mathbb{R}^N$

$$\frac{d}{dt}\vec{X}(t) = \vec{f}(\vec{X}) + \underline{g}(\vec{X}, t) \cdot \vec{\Gamma}(t)$$

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$$\vec{f}(\vec{X}, t)$$

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- With the **deterministic** part (**drift**)

$$\vec{f}(\vec{X}, t)$$

- And the **stochastic** part (**diffusion**)

$$\underline{g}(\vec{X}, t)$$

$$\frac{d}{dt}\vec{X}(t) = \vec{f}(\vec{X}) + \; ....$$

- With the **deterministic** part (**drift**)

$$\vec{f}(\vec{X}, t)$$

using Itō's interpretation

SDE: Drift Estimation

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For Itō SDEs $\frac{d}{dt}x(t) = f(x,t) + g(x,t) \cdot \Gamma(t)$ one has:

$$f(x) = \lim_{\tau \to 0} \frac{1}{\tau} \left\langle x(t+\tau) - x(t) \right\rangle \Big|_{x(t)=x}$$

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This can be estimated directly from the data [3, 4]

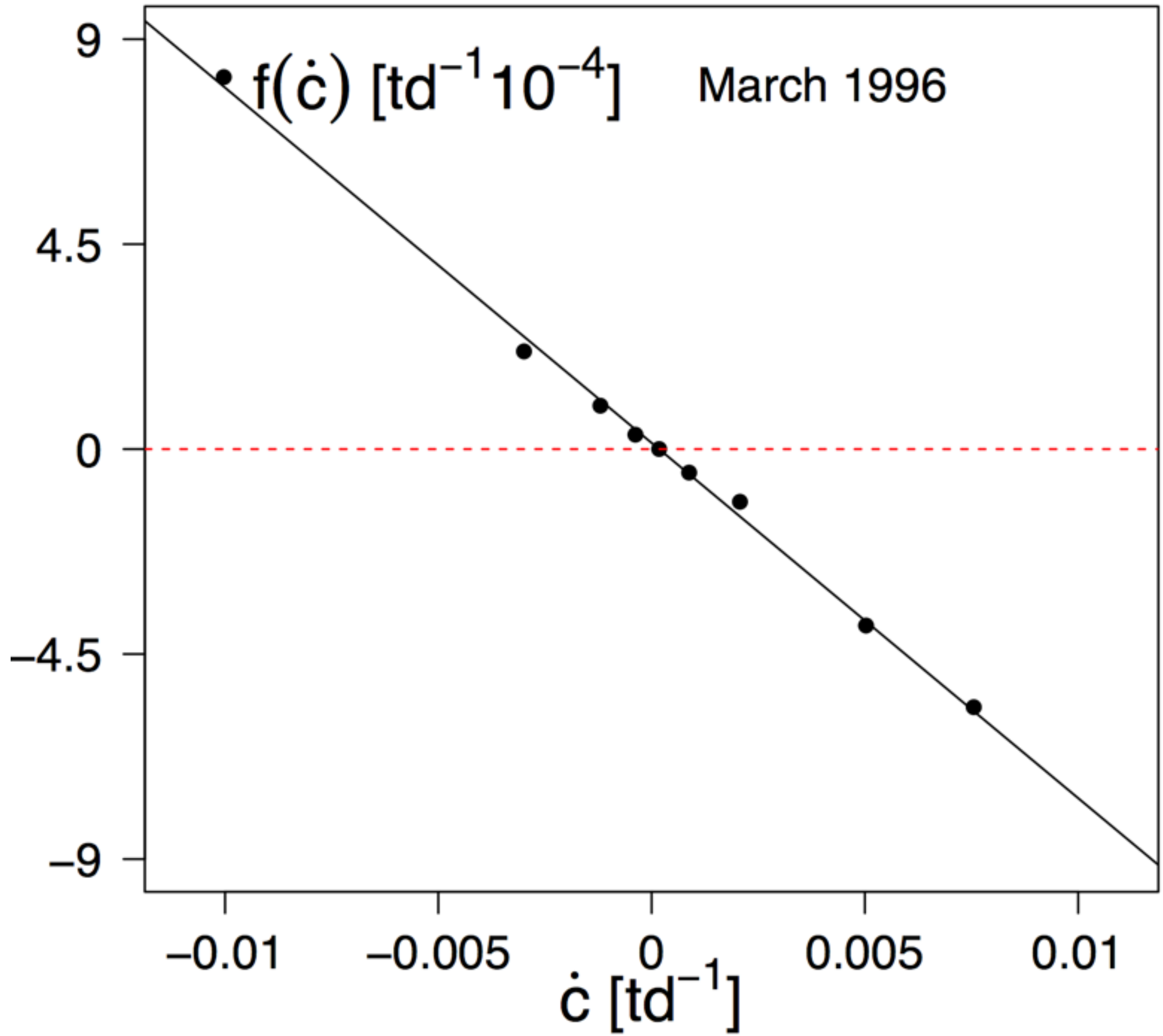
[3] **Siegert *et al.***, Physics Letters A 243, (1998)

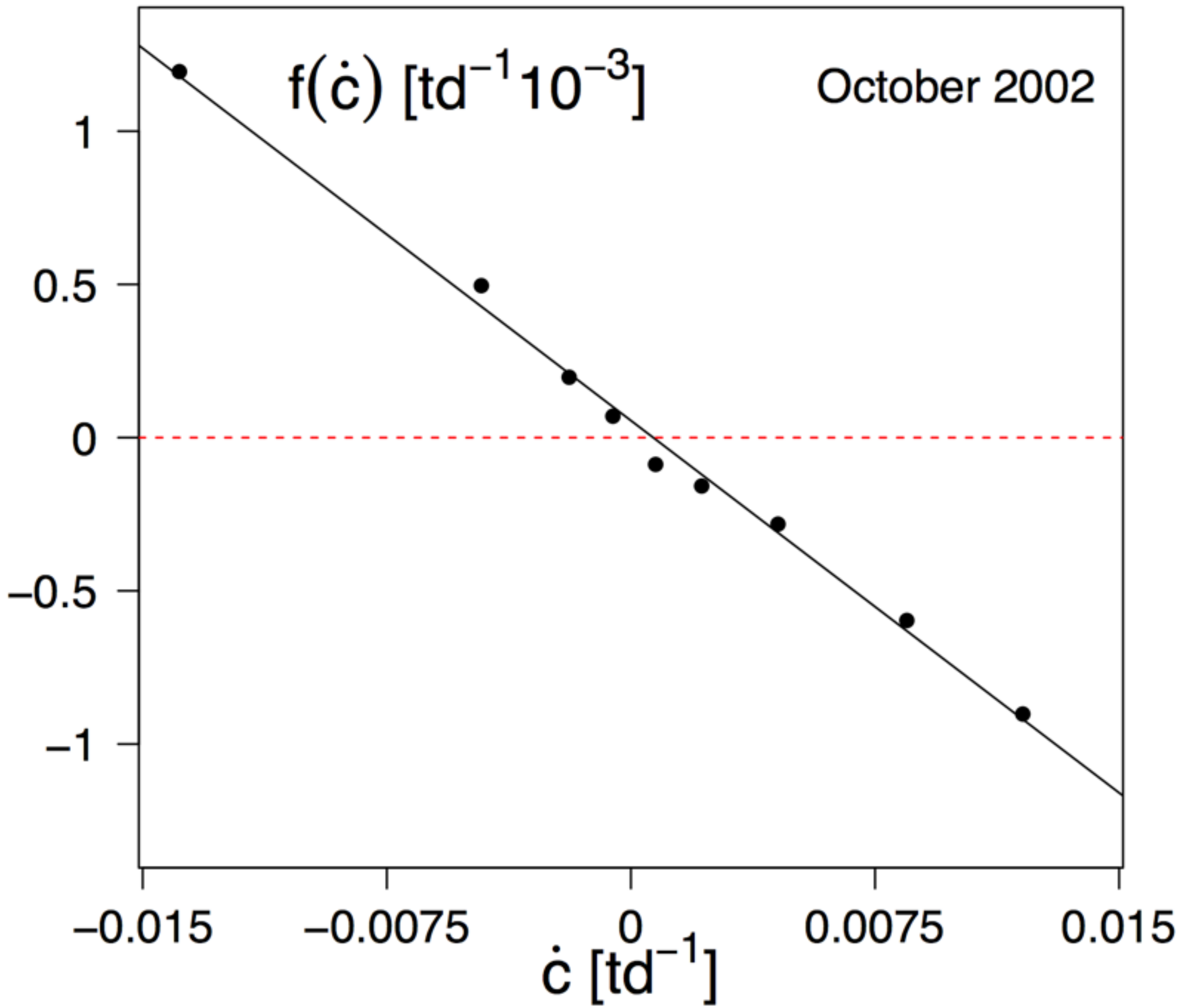
[4] **Friedrich *et al.***, Physics Letters A 271, (2000)

Linear Drag in Correlation Dynamics

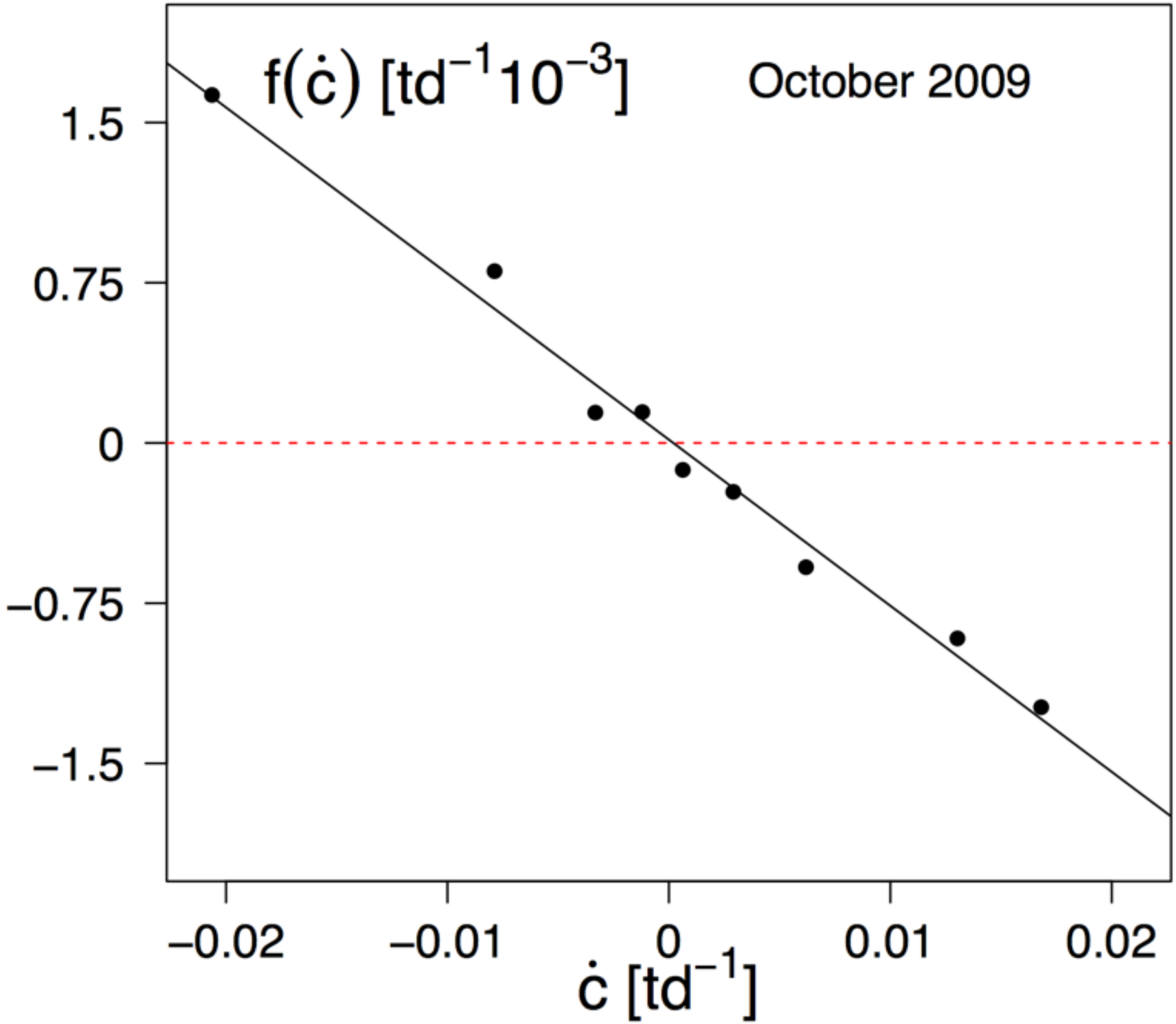
Linear Drag in Correlation Dynamics

- We estimated the drift $f(x)$ for $\frac{d}{dt}\dot{c}(t) = f(\dot{c}) + ...$
- Within windows of 4 tr. years sliding them by 0.5 tr. year


f(ċ) [td⁻¹10⁻⁴]
March 1996
9
4.5
0
–4.5
–9
–0.01
–0.005
0
0.005
0.01
ċ [td⁻¹]



$f(\dot{c})$ [$td^{-1}10^{-3}$]
October 2002
1
0.5
0
−0.5
−1
−0.015
−0.0075
0
0.0075
0.015
$\dot{c}$ [td^{-1}]



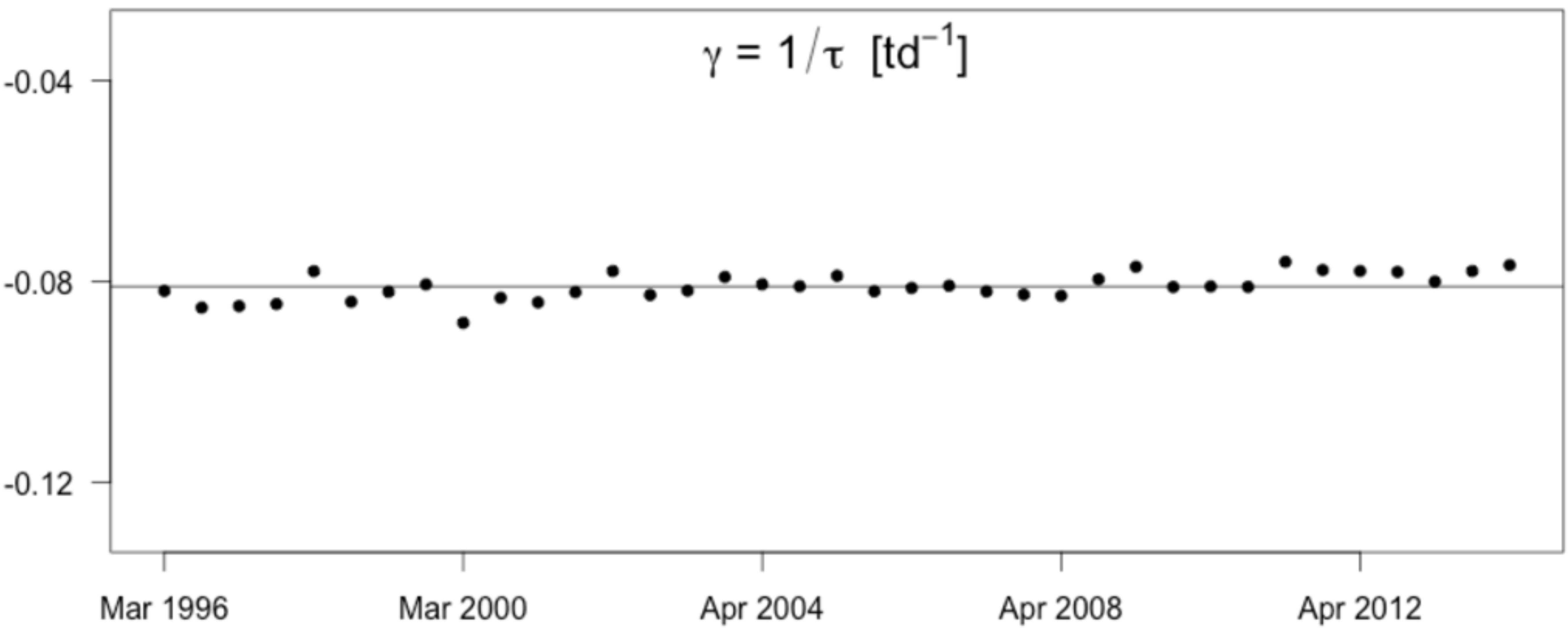
$f(\dot{c})$ [td^{-1}10^{-3}]
October 2009
1.5
0.75
0
−0.75
−1.5
−0.02
−0.01
0
0.01
0.02
$\dot{c}$ [td^{-1}]

Which yields linear drag

$$\frac{d}{dt}\dot{c}(t) = -\gamma(t)\dot{c}(t) + ...$$

$$\gamma(t) = \frac{1}{\tau(t)}$$

Universality and Characteristic Time



Meaning of Linear Drag

- Same as the **drift** ob a Brownian particle:

$$\frac{d}{dt}v = -\frac{1}{\tau}v + D \cdot W_t$$

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$$\frac{d}{dt}v = -\frac{1}{\tau}v + D \cdot W_t$$

- Or of an object moving through the air (e.g. skydiver)

$$\frac{d}{dt}v = (-\frac{1}{\tau}v + g) + 0$$

$$v(t) = v_{\mathrm{t}}\left[1 - \mathrm{e}^{-\frac{t}{\tau}}\right]$$

Before Jump: no Drag

Acceleration Phase

Reached Terminal Velocity

Drift Linear in $\dot{c}$

- Is the whole dynamics “determined” by the SDE?

$$\frac{d}{dt}\dot{c} = -\frac{1}{\tau}\dot{c}(t) + ...$$

- This can’t explain the fact $c(t) \in [-1, 1]$

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- Ansatz: a SDE of second order:

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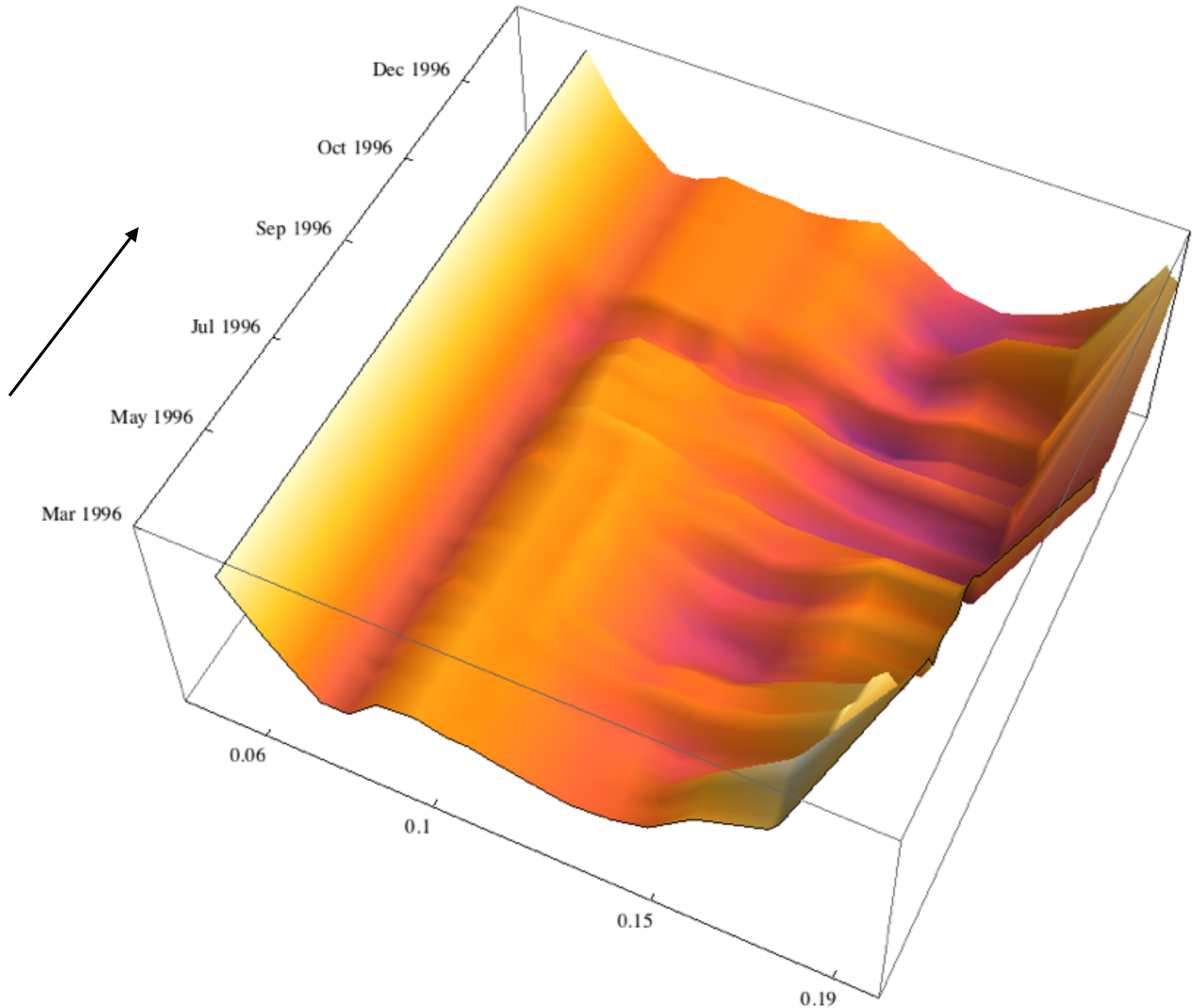
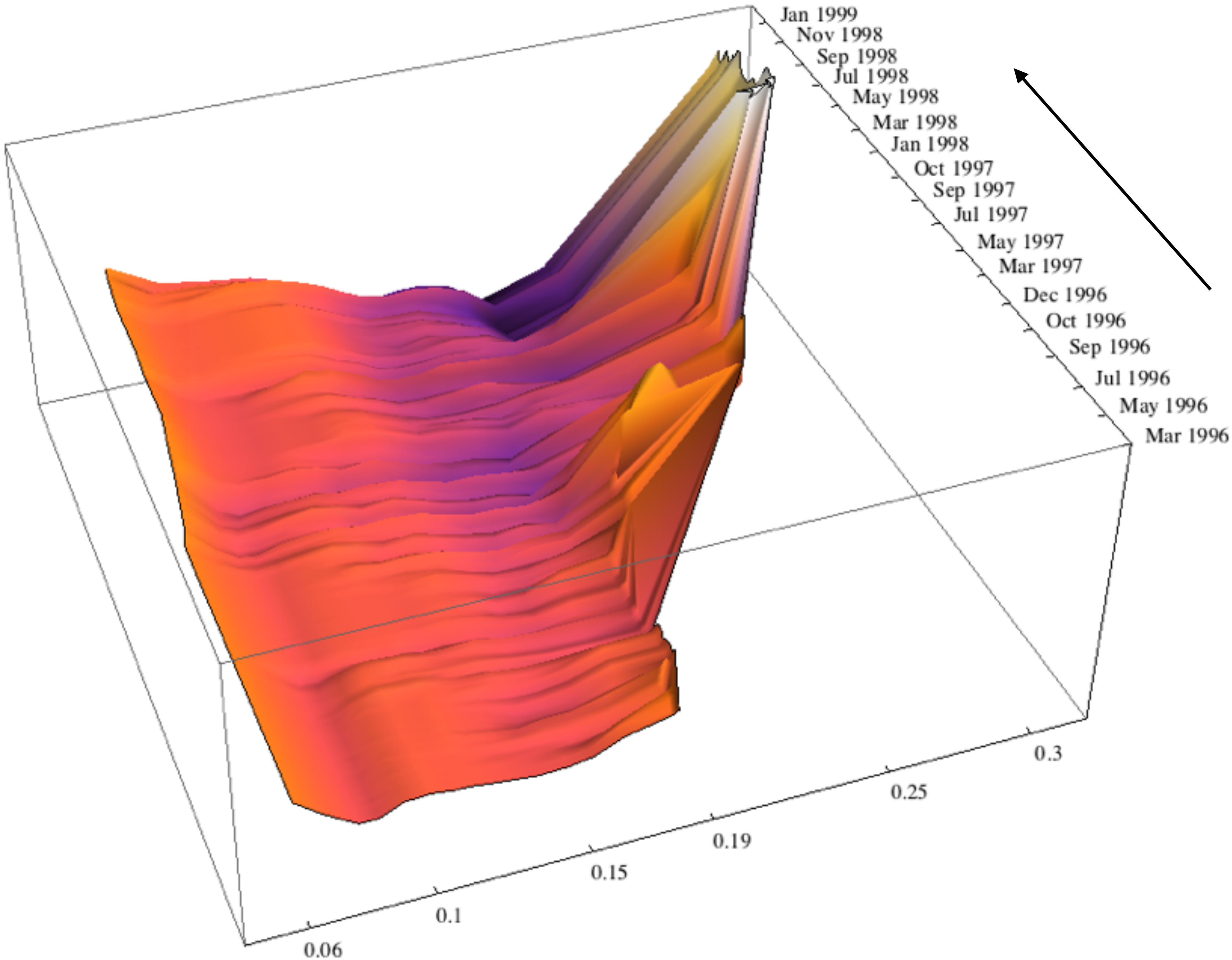
$$\frac{d}{dt}\dot{c} = -\frac{1}{\tau}\dot{c} + f(c, t) + ...$$

$$\frac{d}{dt}\left[\dot{c} + \frac{1}{\tau}c\right] = f(c, t) + ...$$

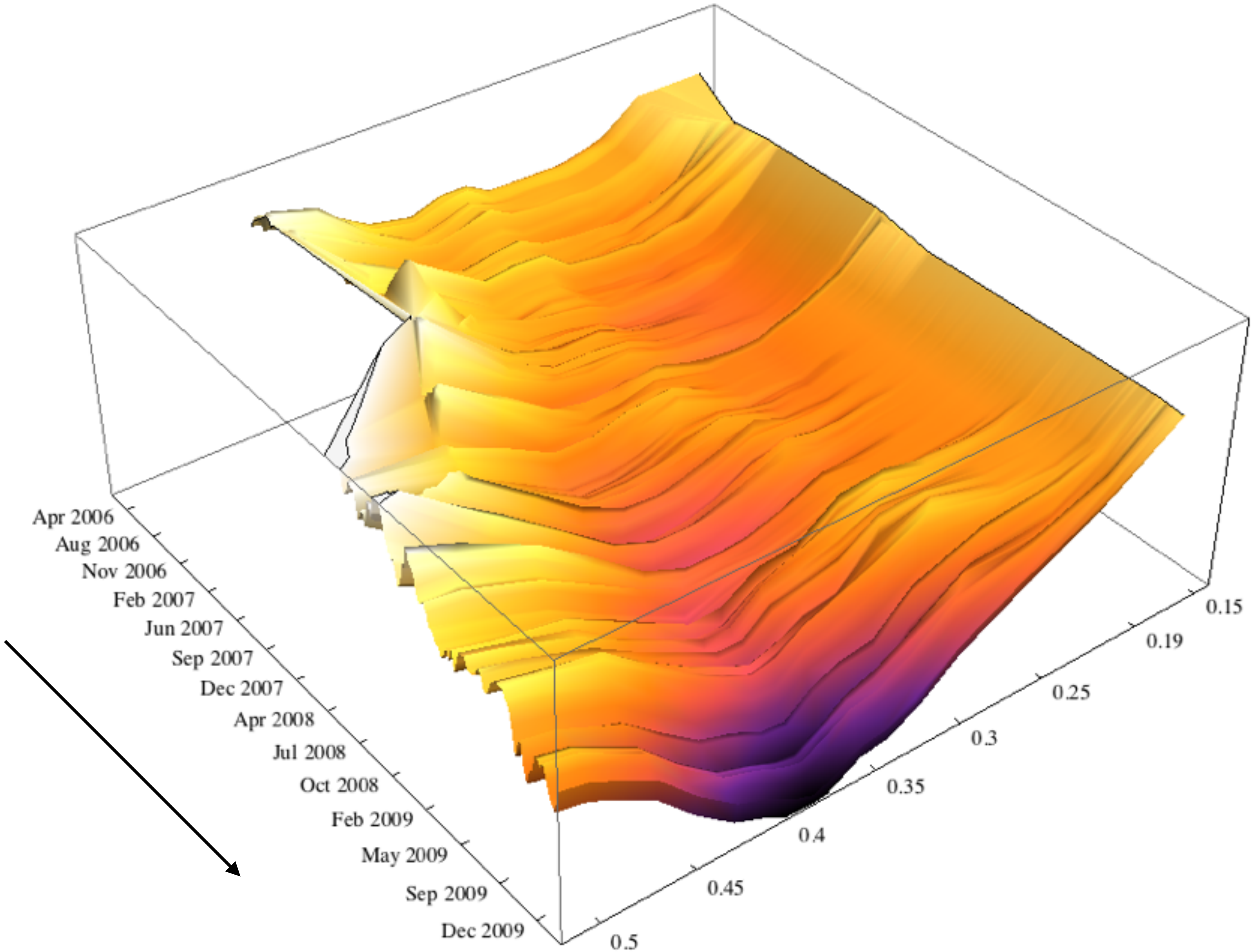
“External Potential”

$$V(c) := -\int f(c)dc$$

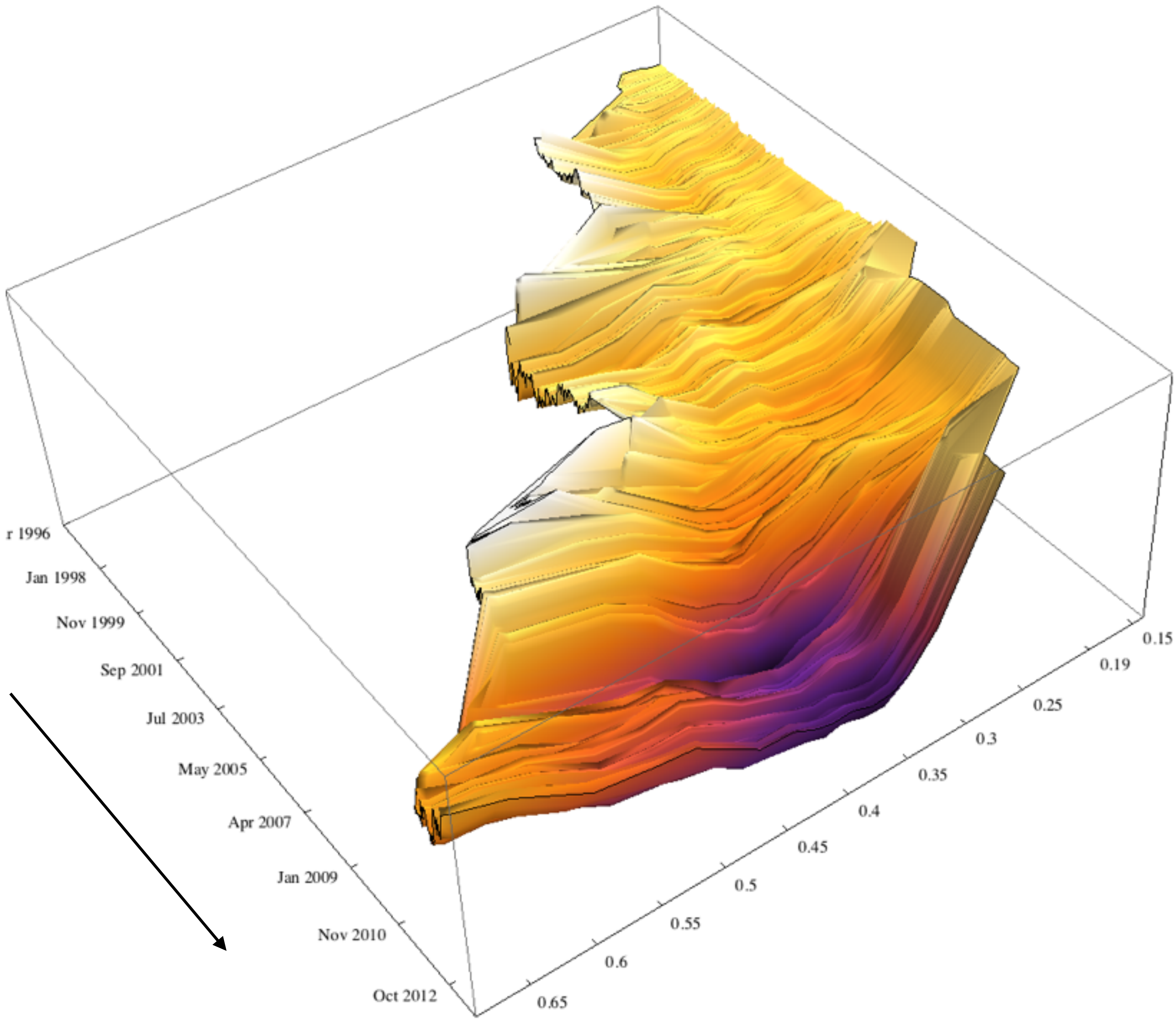

Dec 1996
Oct 1996
Sep 1996
Jul 1996
May 1996
Mar 1996
0.06
0.1
0.15
0.19



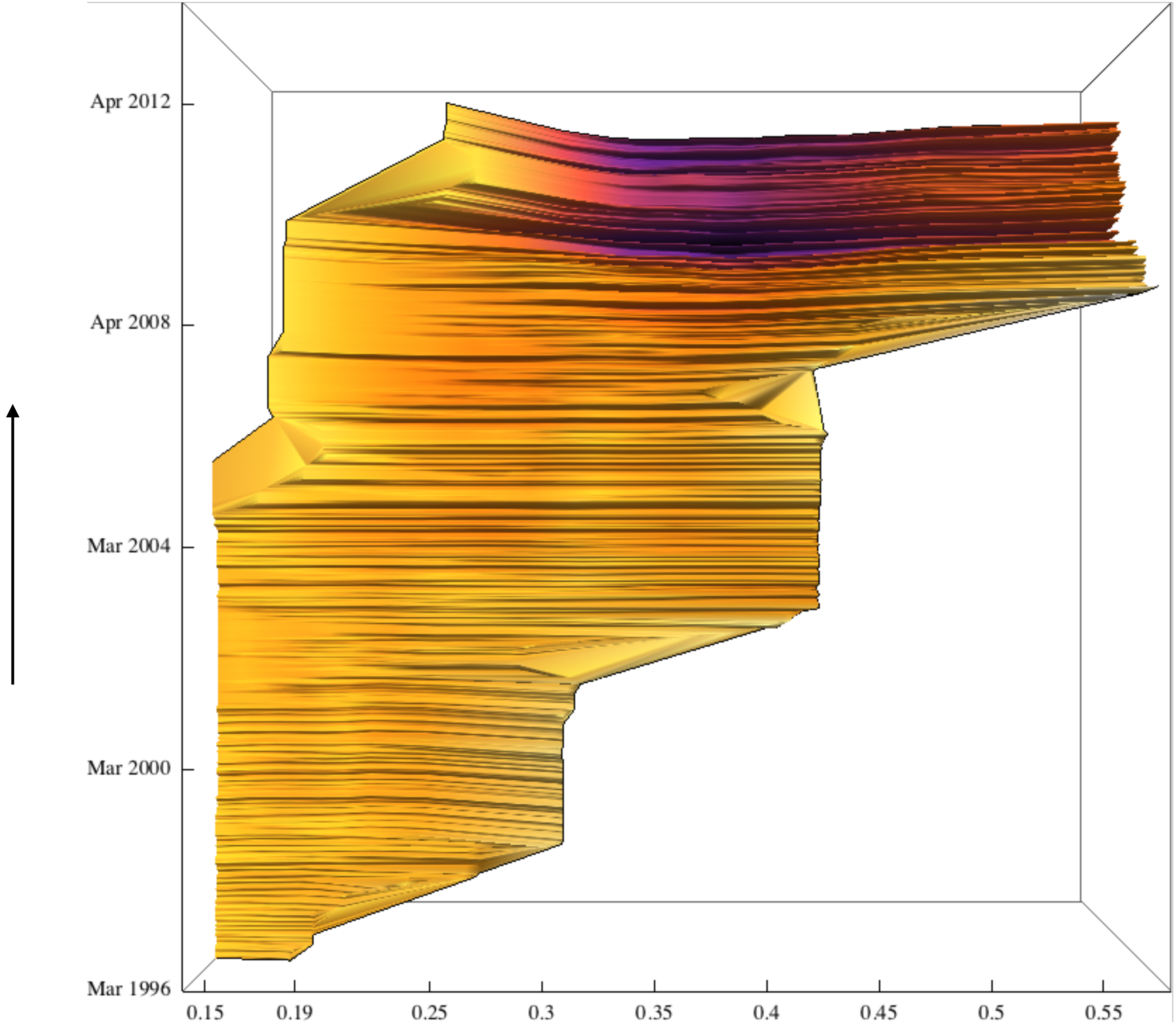
Jan 1999
Nov 1998
Sep 1998
Jul 1998
May 1998
Mar 1998
Jan 1998
Oct 1997
Sep 1997
Jul 1997
May 1997
Mar 1997
Dec 1996
Oct 1996
Sep 1996
Jul 1996
May 1996
Mar 1996
0.06
0.1
0.15
0.19
0.25
0.3



Apr 2006
Aug 2006
Nov 2006
Feb 2007
Jun 2007
Sep 2007
Dec 2007
Apr 2008
Jul 2008
Oct 2008
Feb 2009
May 2009
Sep 2009
Dec 2009
0.15
0.19
0.25
0.3
0.35
0.4
0.45
0.5



r 1996
Jan 1998
Nov 1999
Sep 2001
Jul 2003
May 2005
Apr 2007
Jan 2009
Nov 2010
Oct 2012
0.15
0.19
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65


Apr 2012
Apr 2008
Mar 2004
Mar 2000
Mar 1996
0.15
0.19
0.25
0.3
0.35
0.4
0.45
0.5
0.55

“External Potential” within Market States

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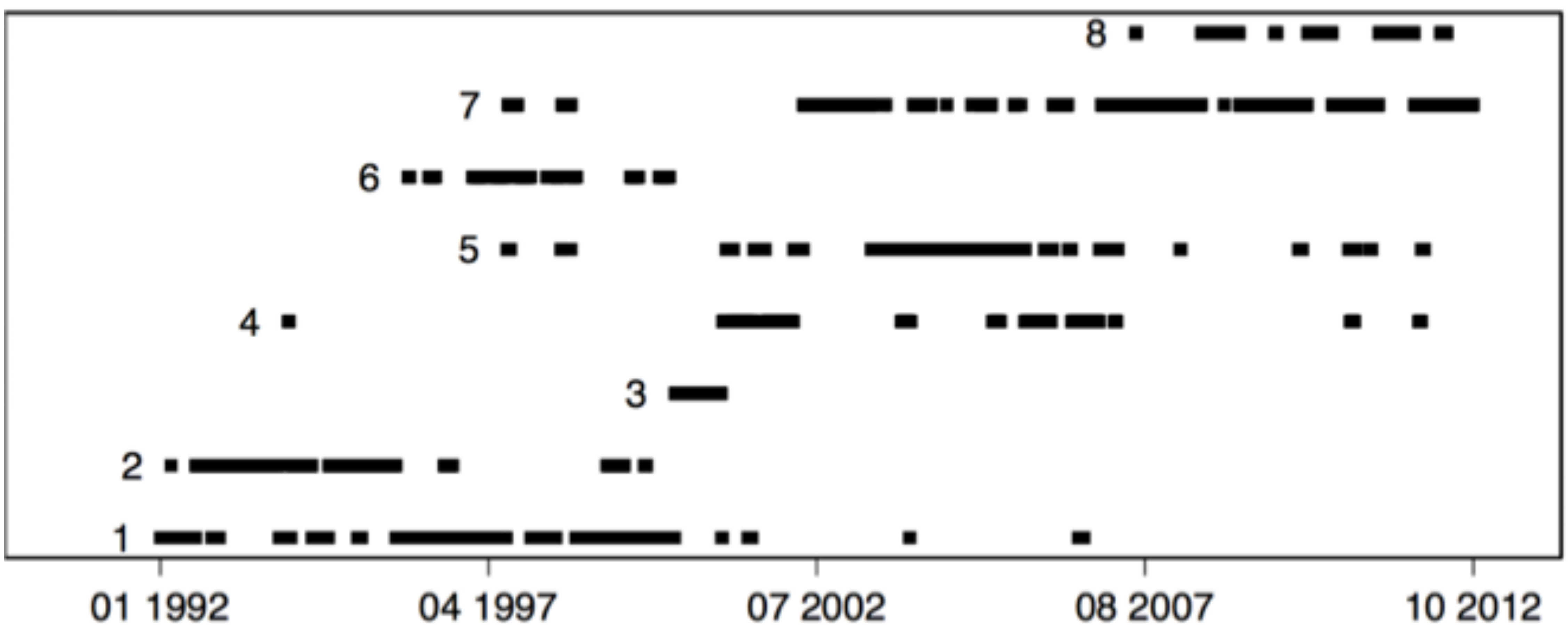

Potentials within MS

Recall: drift is a **conditional mean**:

$$f(x) = \lim_{\tau \to 0} \frac{1}{\tau} \left\langle x(t+\tau) - x(t) \right\rangle \Big|_{x(t)=x}$$

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For potentials within MS **refine** the condition:

$$f(x) = \lim_{\tau \to 0} \frac{1}{\tau} \left\langle x(t+\tau) - x(t) \right\rangle \Big|_{A}$$

$$A = \{x(t) = x\} \cup \{x(t), x(t+\tau) \in \text{state}\, \alpha\}$$

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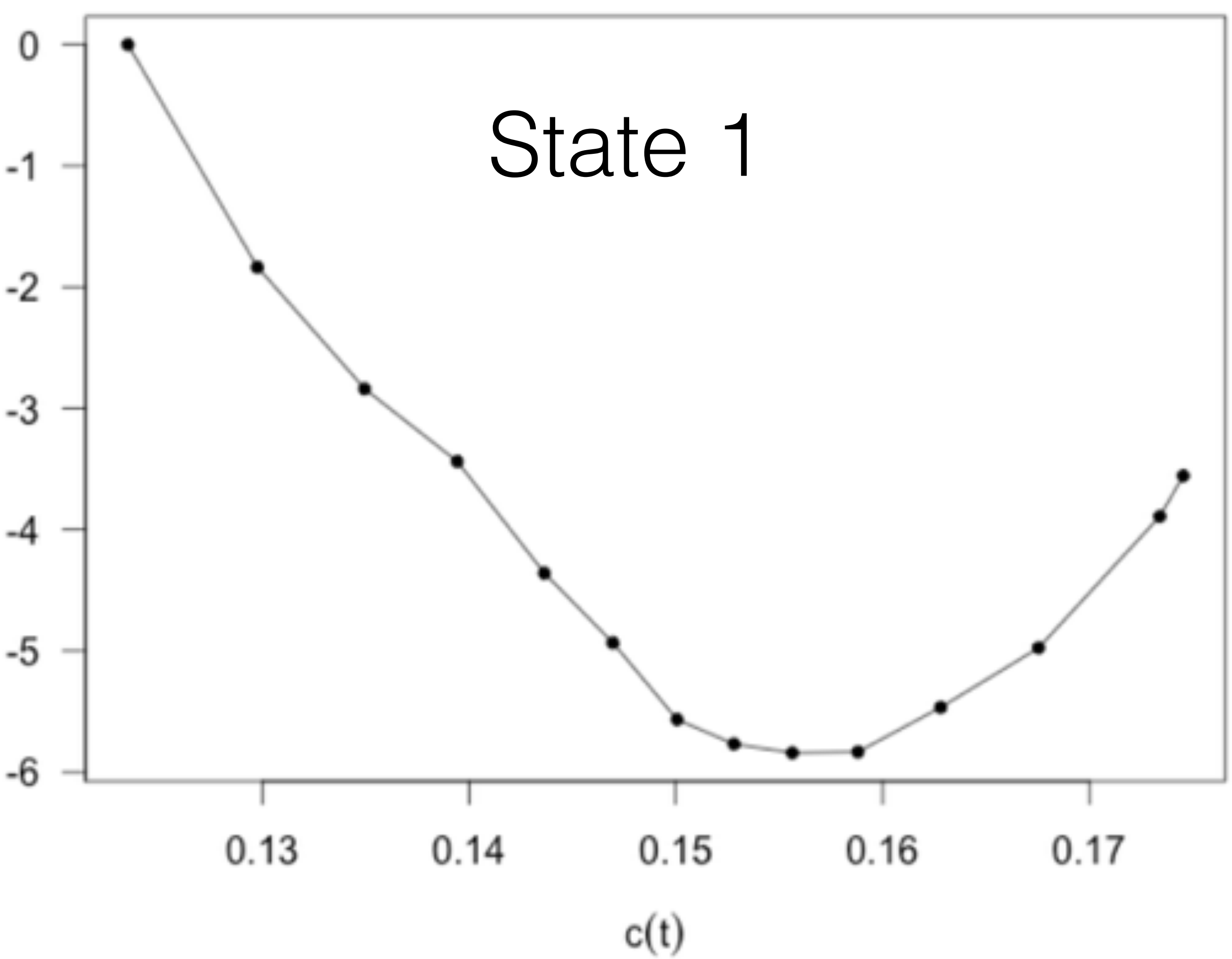
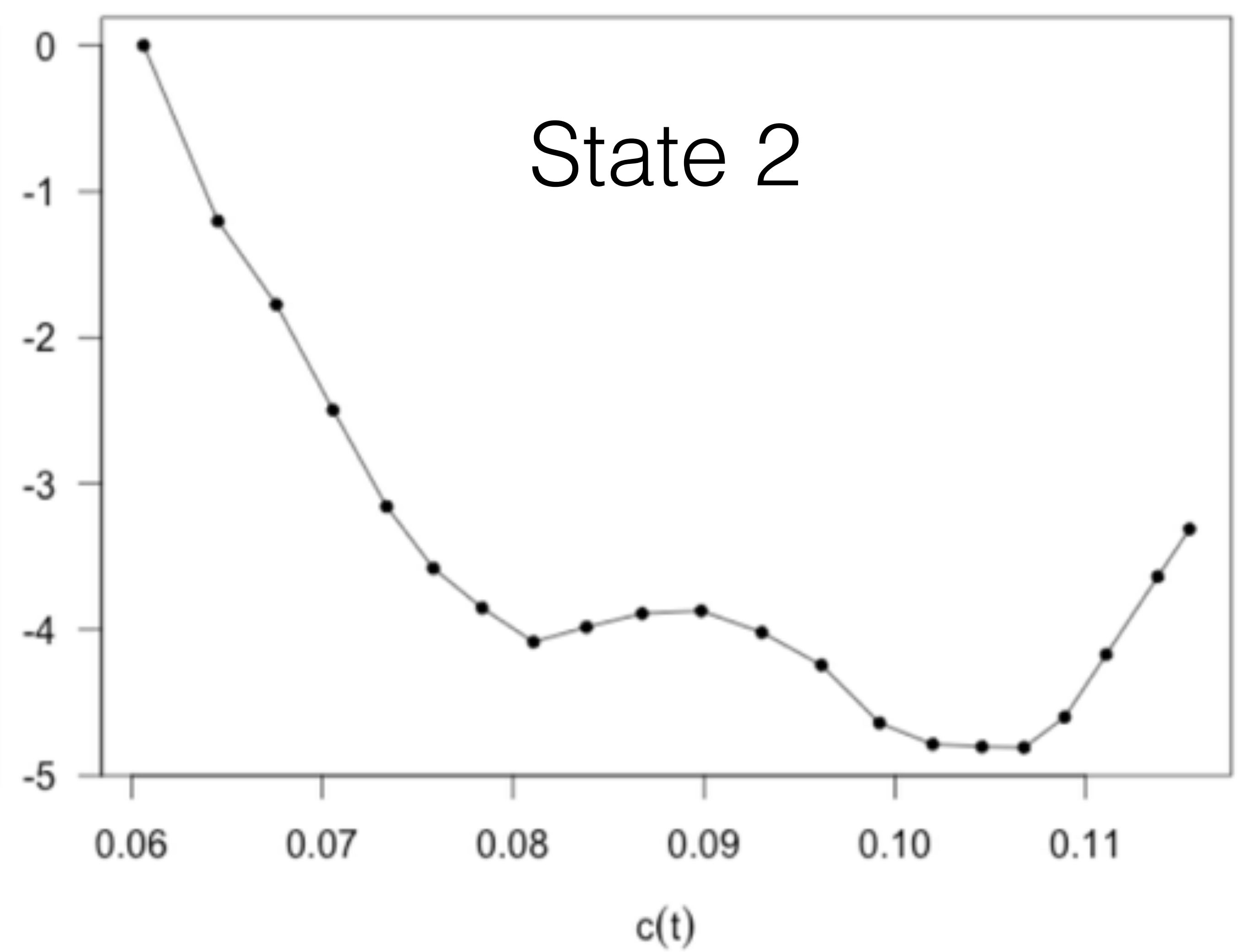
The "jumps" start and end in same market state

Potentials within MS





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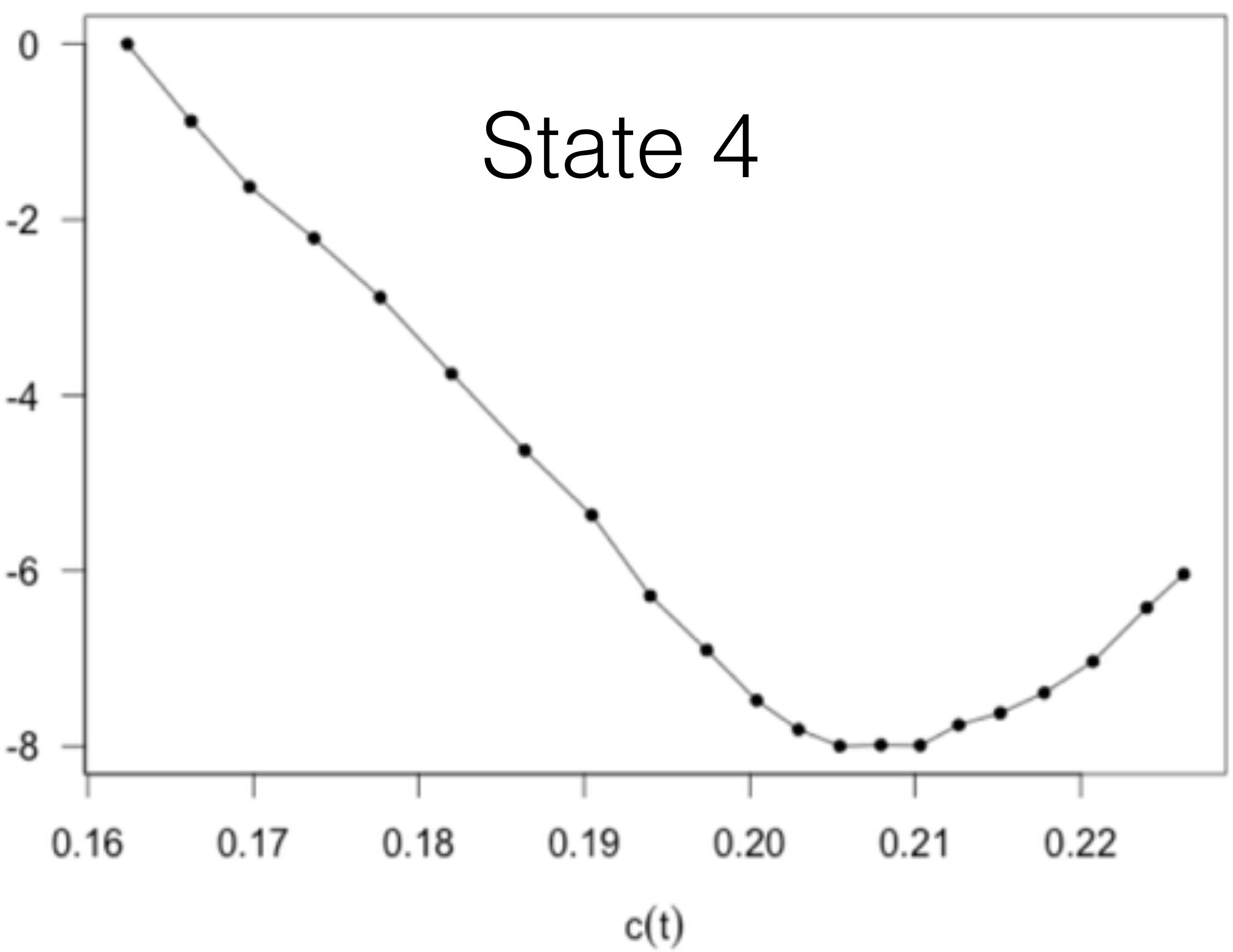

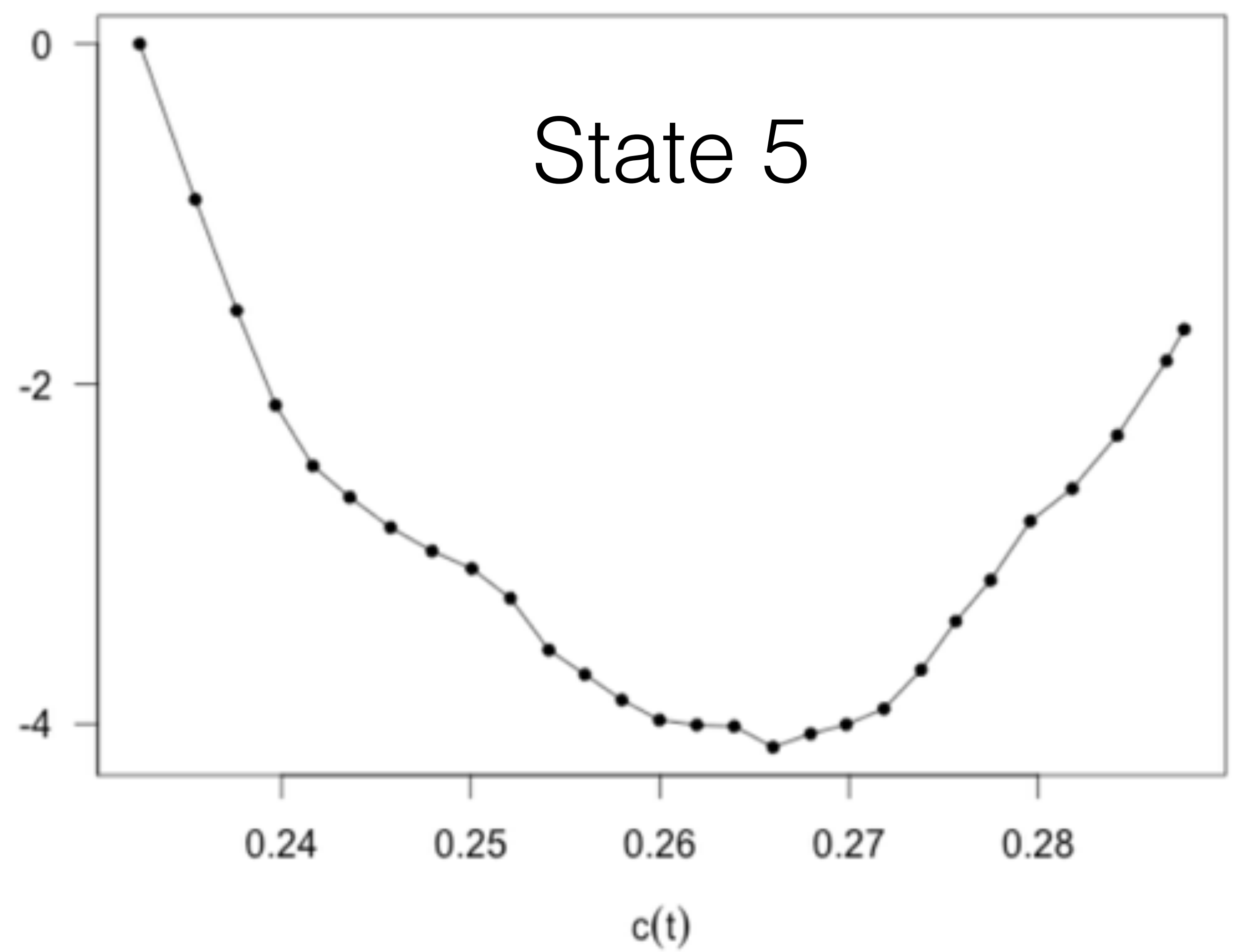

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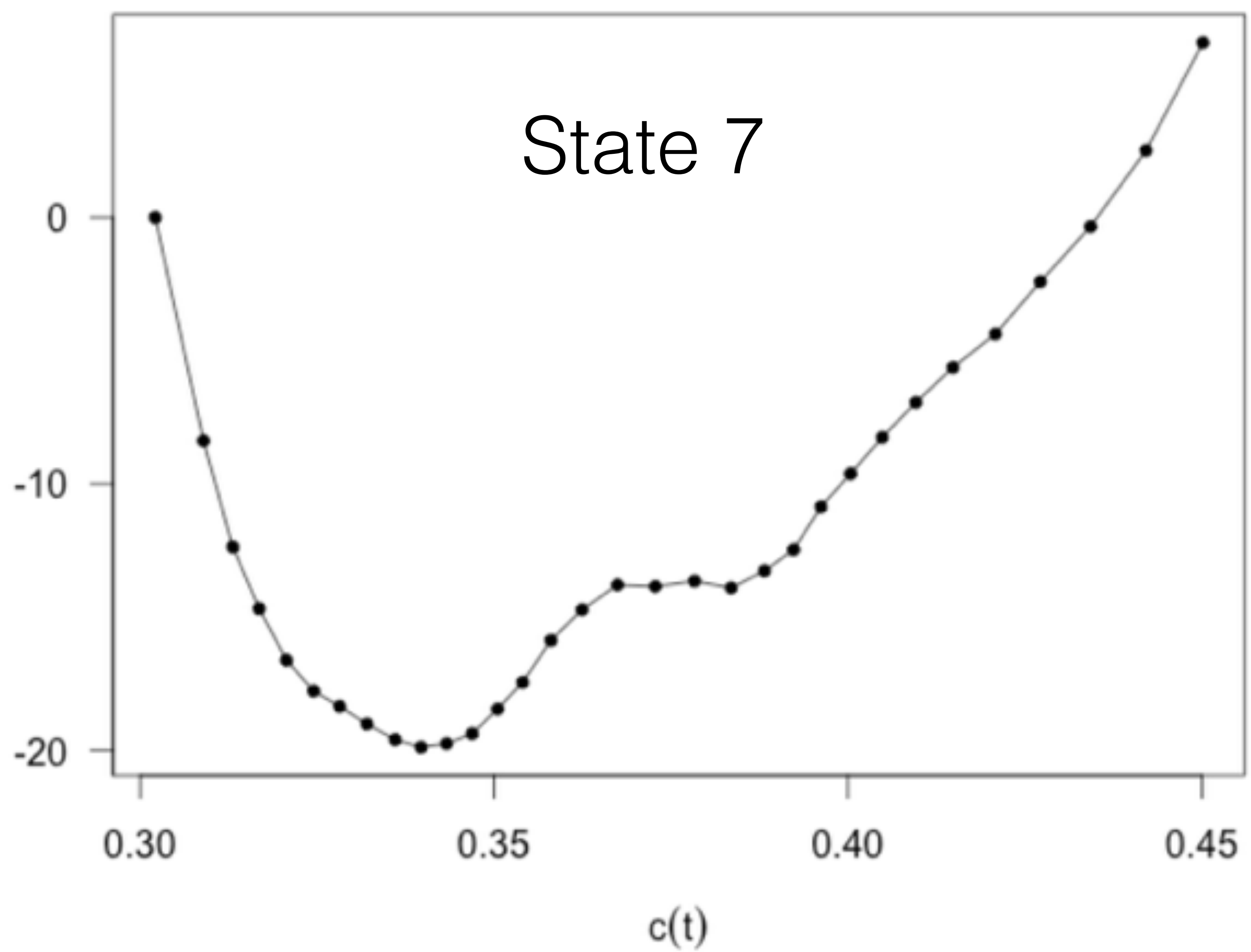

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- The dirt is linear in $\dot{c}$ - universality
- A characteristic time scale $\tau \approx 12 \text{ td}$
- Market states' can be clearly distinct by potentials

Thank you for your Attention

Noise dressing: **local normalisation**

$$\tilde{r} := (r - \mu)/\sigma$$