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Citation: *Journal of Applied Physics* **117**, 054305 (2015); doi: 10.1063/1.4907217

View online: <http://dx.doi.org/10.1063/1.4907217>

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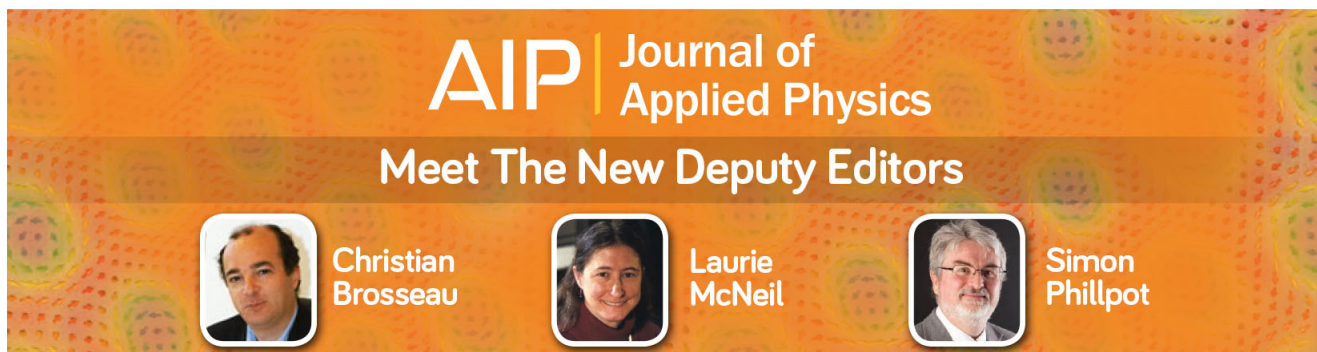
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The effect of charged quantum dots on the mobility of a two-dimensional electron gas: How important is the Coulomb scattering?

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(Received 19 November 2014; accepted 21 January 2015; published online 3 February 2015)

We have investigated the influence of a layer of charged self-assembled quantum dots (QDs) on the mobility of a nearby two-dimensional electron gas (2DEG). Time-resolved transconductance spectroscopy was used to separate the two contributions of the change in mobility, which are: (i) The electrons in the QDs act as Coulomb scatterers for the electrons in the 2DEG. (ii) The screening ability and, hence, the mobility of the 2DEG decreases when the charge carrier density is reduced by the charged QDs, i.e., the mobility itself depends on the charge carrier concentration. Surprisingly, we find a negligible influence of the Coulomb scattering on the mobility for a 2DEG, separated by a 30 nm tunneling barrier to the layer of QDs. This means that the mobility change is completely caused by depletion, i.e., reduction of the charge carrier density in the 2DEG, which indirectly influences the mobility. © 2015 AIP Publishing LLC.

[<http://dx.doi.org/10.1063/1.4907217>]

I. INTRODUCTION

Self-assembled quantum dots (QDs)¹ with their three-dimensional confinement for electrons and/or holes act as artificial atoms² in a semiconductor crystal. With their discrete energy levels, QDs can be used in optical devices such as QD lasers³ and amplifiers⁴ as well as building blocks for quantum light sources,⁵ e.g., in single photon sources^{6,7} or as a solid-state device to generate indistinguishable photons^{8,9} for future quantum networks.¹⁰ In such optical quantum devices, the QDs are coupled to the light field for initialisation, manipulation and read-out of the excitonic (electron-hole) states. In electrical devices, the preparation and read-out can be done by a nearby two-dimensional electron gas (2DEG), which is coupled to the layer of QDs.^{11,22} This is also the read-out principle of a QD flash memory¹² and can be used to prepare and detect excited many particle spin states.¹³ The coupling of the QDs to the 2DEG in the electrical read-out is mediated by Coulomb interaction between the electrons inside the dots and the electrons flowing through the 2DEG. The charged QDs change the electron density n in the 2DEG and its mobility μ . This influence of the electrons stored inside the dots on the transport properties in the 2DEG is, hence, of importance for future electrical QD-based devices.

A number of groups have previously investigated the influence of charged QDs on the mobility and carrier concentration in the 2DEG.^{14,15} Sakaki *et al.*¹⁶ investigated the change in mobility and carrier concentration for different distances of the charged QDs to the two-dimensional system, while Ribeiro *et al.*¹⁷ changed the QD density to study the influence on the mobility. However, on one hand, the charged QDs are mainly treated as Coulomb potentials that

decrease the mobility by electron scattering in the 2DEG,^{17,18} while the influence of the change in charge carrier concentration¹⁹ is often neglected.^{18,20} On the other hand, Zhukov *et al.*¹⁵ surprisingly measured a increasing mobility in the 2DEG for charged QDs. All this studies lead us to the question “How important is Coulomb scattering for the observed change in mobility of a nearby 2DEG?.”

We have also studied in the past the coupling between the charged QDs and the 2DEG in Capacitance-voltage (C-V) and conductance measurements with single electron resolution and modelled the influence on the mobility of the 2DEG.²¹ Using the time-resolved transconductance spectroscopy,²² we were able to measure directly the change in mobility $\Delta\mu$ and charge carrier concentration Δn .²³ However, up to now, we were also not able to separate the two contributions that change the mobility in the 2DEG: (i) The electrons in the QDs act as Coulomb scatterers (with fixed position but tunable charge) for electrons in the 2DEG.^{17,24} (ii) The screening ability of the 2DEG decreases when the carrier density n is reduced by charge transfer into the QDs. Thus, the mobility itself depends on n .²⁵ Using time-resolved transconductance spectroscopy, we can now measure the influence of the charged QDs as tunable scatterers with single electron resolution, corresponding to a filling of the s- and p-shell with individual electrons. Surprisingly, we find an almost negligible contribution of the charged QDs as Coulomb scatterers for QDs separated by a 30 nm tunnelling barrier from the 2DEG. This is in good agreement with simulations using the Stern-Howard model.

II. EXPERIMENTS

The investigated sample has been grown by molecular beam epitaxy (MBE) as an inverted high electron mobility transistor (HEMT) structure with an layer of self-assembled InAs QDs. The active region consists of a 300 nm

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$\text{Al}_{0.34}\text{Ga}_{0.66}\text{As}$ layer followed by a silicon δ -doping and a 16 nm AlGaAs spacer layer. A 2DEG is formed between the AlGaAs spacer and the 30 nm tunneling barrier consisting of a 15 nm GaAs, a 10 nm AlGaAs, and a 5 nm GaAs layer, see Fig. 1(a). The 2DEG serves as back contact and sensitive charge sensor in this sample.^{11,22} The InAs QDs were grown on top of the tunneling barrier and covered by 30 nm GaAs and a Superlattice consisting of GaAs and AlAs. The device is patterned into a Hall-bar structure (Fig. 1(b)) using standard lithography methods. A source and drain contact and four side contacts are formed by evaporation of Ni, AuGe, and Au. The Hall-bar is covered by a 350 μm long, 200 μm wide, and 50 nm thick gold layer, used as gate electrode that allows to control the occupation level of the dots and simultaneously the carrier density of the 2DEG electrostatically. The electron channel area is $5 \times 10^3 \mu\text{m}^2$ which corresponds to approximately 4×10^5 QDs below the gated area.

The results of two different measurement techniques are shown in the following to analyse the transport properties of the 2DEG: (1) the *equilibrium* transconductance spectroscopy¹¹ and (2) the *non-equilibrium* transconductance spectroscopy.²² We first start with the equilibrium transconductance spectroscopy to characterize the relative contributions of $\Delta n/n$ and $\Delta\mu/\mu$ to the overall change in

conductance $\Delta\sigma/\sigma$ for charged and uncharged QDs (see also Marquardt *et al.*²³). The measurements are performed at 4 K with an applied magnetic field of $B = 0.5$ T and a constant source-drain-current $I_{SD} = 3 \mu\text{A}$. The transverse and longitudinal voltages V_{xy} and V_{xx} , respectively, are measured, while voltage pulses with an amplitude of $\Delta V_G = 50$ mV are applied to the gate. We observe an exponential decrease in the voltages V_{xx} and V_{xy} , when electron tunneling from the 2DEG into the QD takes place.²³ The tunneling of electrons is controlled by the gate voltage, which shifts the levels of the QDs. Tunneling from the 2DEG into the QDs is possible, if the levels of the QDs are in resonance with the Fermi energy of the 2DEG. We shift the offset gate voltage by about 10 mV from one pulse to the next and obtain the mobility and charge carrier density in the 2DEG, from the transverse and longitudinal voltages V_{xx} and V_{xy} , respectively, using the Drude model:

$$n = \frac{I_{SD}B}{V_{xy}e}, \quad (1)$$

$$\mu = \frac{V_{xy}L}{BWV_{xx}}, \quad (2)$$

with the width and length of the Hall-bar W and L , the source-drain-current I_{SD} , and the magnetic field B .

III. RESULTS AND DISCUSSION

The transport properties are calculated using Eqs. (1) and (2), respectively. From the difference in V_{xx} and V_{xy} at $t = 0$ ms (no tunneling has taken place and all additional carriers induced by ΔV_G are in the 2DEG) and $t = 50$ ms (equilibrium, carriers have tunneled into the QD) we get the change in mobility $\Delta\mu$, conductivity $\Delta\sigma$ and charge carrier density Δn (Fig. 1c) for the different gate voltages V_G . In the measurement the s_1 - and s_2 - states can be distinguished and a broad shoulder for the four p-states is measured. We find that the relative change in mobility $\Delta\mu/\mu$ exceeds the change in charge carrier density $\Delta n/n$ by a factor of 1.8, in good agreement with Ref. 23.

In the measurement shown in Fig. 1(c), we are only able to measure the total change in mobility $\Delta\mu$. However, the change in mobility of the 2DEG by the charged QDs consists of two parts which are: The mobility in the 2DEG is reduced as the charged QDs (i) act as Coulomb scatterers for the electrons flowing through the 2DEG (referred to as $\Delta\mu_{QD}$) and (ii) reduce the screening ability of the 2DEG as the charged QDs decrease the charge carrier density in the 2DEG by depletion²⁵ (referred to as $\Delta\mu_n$). The total change in mobility $\Delta\mu$ is, hence, given by: $\Delta\mu = \Delta\mu_n + \Delta\mu_{QD}$.

To determine the strength of the different parts of the change in mobility, the relation between mobility and carrier concentration *without an influence by the charged QDs* is needed. The dependence between the mobility and the carrier density in a 2DEG can be described by different scattering mechanisms. The most important scattering mechanisms for electrons in the 2DEG at low temperatures are scattering with ionized doping atoms in the δ -doping and 3D background impurities. Taking into account these two scattering

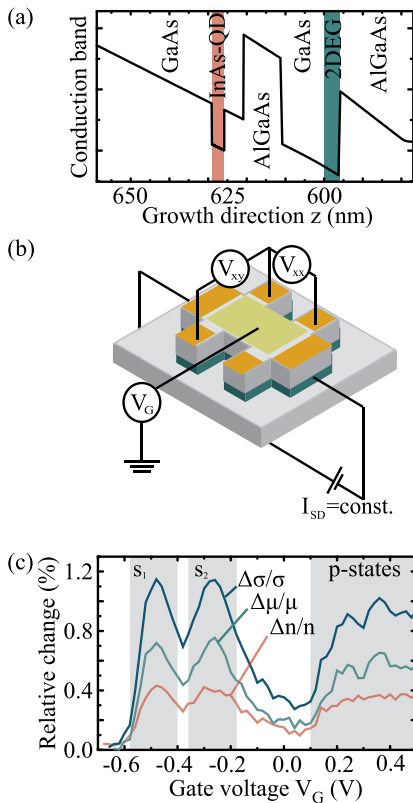


FIG. 1. (a) Conduction band structure of the active part of the sample with InAs QDs, tunneling barrier and a 2DEG. (b) Hall-bar structure with 2DEG, QD-layer, gate and ohmic contacts. The transverse (V_{xx}) and longitudinal voltage (V_{xy}) are measured while voltage pulses are applied to the gate contact. (c) Relative change of the conductivity $\Delta\sigma/\sigma$, the mobility $\Delta\mu/\mu$, and the charge carrier density $\Delta n/n$ in the 2DEG as function of the gate voltage V_G . The relative change relates to the difference in conductivity, charge carrier density, and mobility for charged and uncharged QDs. The measured change in mobility exceeds the change in carrier density by a factor of 1.8.

mechanisms, the mobility μ depends on the carrier concentration by $\mu(n) \propto n^{\frac{2}{3}}$.¹⁹

The carrier density in the 2DEG is determined by the gate voltage V_G . To obtain the relation between mobility, carrier density, and the gate voltage, we use the time-resolved transconductance spectroscopy in the *non-equilibrium* measurement scheme.¹³ We use a pulse scheme where the QDs are always discharged at position (1) in Fig. 2(a) and, hence, we are able to measure the transport properties of the 2DEG for empty QDs like in a reference sample without any charged QDs (schematically sketched in the left inset in Fig. 2(a)). The electrons tunnel into the dot states on an average time scale of milliseconds, seen as transients in Fig. 2(a). After approximately 10 ms, the equilibrium situation is reached, and at $t=10$ ms the QDs are charged with 1 to 6 electrons depending on the gate voltage at position (2), see Fig. 2(a) and corresponding inset below. Fig. 2(b) shows the mobility μ and carrier concentration n in the 2DEG for charged QDs (red line), corresponding to position (2) in Fig. 2(a), and uncharged QDs (blue line) corresponding to position (1) in Fig. 2(a). A reduced mobility and charge carrier density in the 2DEG are found for charged QDs (red lines).

To separate the two influences of the change in mobility ($\Delta\mu_{QD}$ and $\Delta\mu_n$), we take the $n(V_G)$ and $\mu(V_G)$ dependence in Fig. 2(b) to plot in Fig. 3(a) the $\mu(n)$ relation for charged (red dashed line) and uncharged QDs (blue solid line). On the

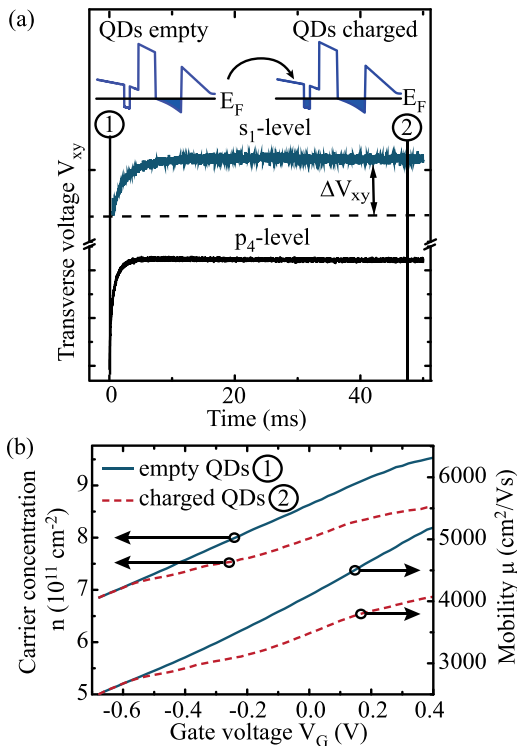


FIG. 2. (a) Time evolution of the transverse voltage for two different gate voltages. The s_1 - and p_4 -level are in resonance with the 2DEG for the upper and lower transient, respectively. At $t=0$ ms, the QDs are empty and at the time $t=50$ ms charged with one or six electrons. (b) Charge carrier density n and mobility μ for charged (red) and uncharged (blue) QDs. A reduced charge carrier density and mobility of the 2DEG are found if the QDs are charged with 1.6 electrons.

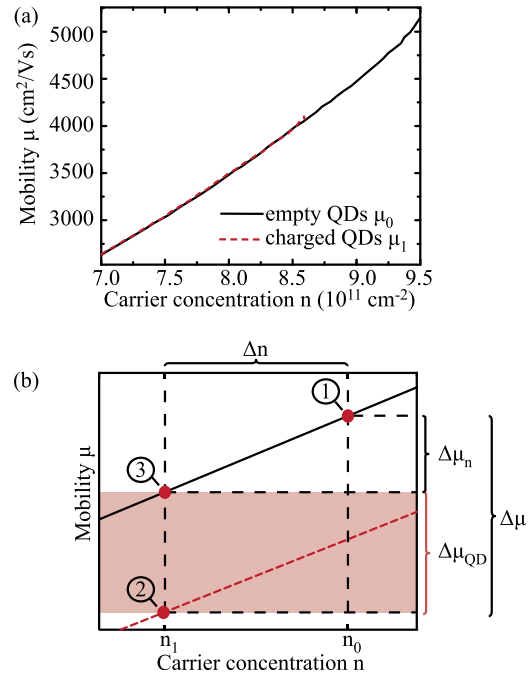


FIG. 3. (a) Mobility in the 2DEG for charged (red line) and uncharged QDs (black line) versus carrier concentration n . The very small difference between the two mobilities cannot be distinguished in this presentation. (b) Schematic picture for the evaluation procedure to derive the mobility changes $\Delta\mu$, $\Delta\mu_n$, and $\Delta\mu_{QD}$ from the $\mu(n)$ -diagram above; the difference in mobility for charged and uncharged QDs is highly enhanced.

scale used in Fig. 3(a), both lines are indistinguishable. Therefore, Fig. 3(b) sketches the situation schematically again to explain our evaluation procedure in detail using the $\mu(n)$ -diagram.

A specific gate voltage corresponds to two specific points in the $\mu(n)$ -diagram, one for empty QDs, labelled (1), and another one for charged QDs, labelled (2) in Figs. 2 and 3(b). Note that, when moving from (1) to (2) for a given gate voltage V_G , both n (x-axis) and μ (y-axis) change as a specific number of electrons Δn are transferred from the 2DEG to the QDs. The $\mu(n)$ -diagram allows us to evaluate the change in mobility *at constant carrier density* (vertical dashed lines in Fig. 3(b)) by comparing point (2) with point (3) at a different gate voltage; this corresponds to a mobility change in the 2DEG due to Coulomb scattering with the charges in the QDs only, labelled $\Delta\mu_{QD}$. We can also separate the contributions due to depletion $\Delta\mu_n$ on the y-axis by moving from (1) to (3) and the sum $\Delta\mu = \Delta\mu_{QD} + \Delta\mu_n$ by moving vertically from (1) to (2) in Fig. 3(b).

The change in mobility due to Coulomb scattering $\Delta\mu_{QD}$ is plotted as green dotted line in Fig. 4(a), showing values that are within the accuracy of the measurement close to $0 \text{ cm}^2/\text{Vs}$. The change in mobility due to the depletion of the 2DEG $\Delta\mu_n$ is plotted as red solid line in Fig. 4(a). It shows a step like behavior, with steps at gate voltages where the QDs get charged with an additional electron. This part on the change in mobility $\Delta\mu_n$ is almost on top of the total change in mobility $\Delta\mu$ (dashed blue line in Fig. 4(a)), i.e., the depletion of the 2DEG is the only reason for the mobility change for a sample structure with QDs separated by a 30 nm tunneling barrier from the 2DEG.

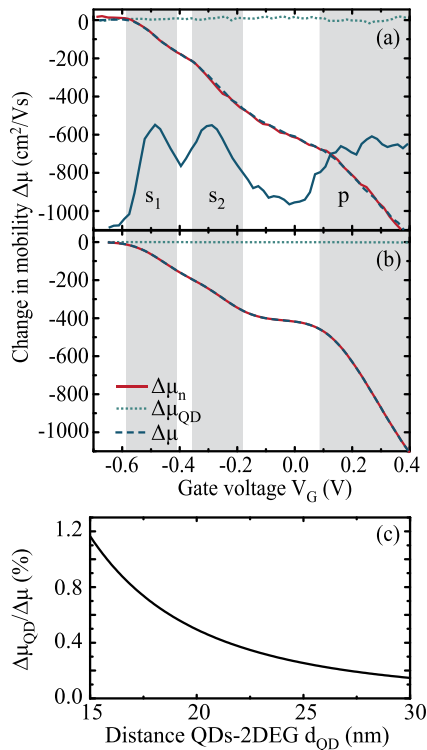


FIG. 4. (a) The green dashed line displays the measured change in mobility $\Delta\mu_{QD}$ due to Coulomb scattering caused by the charged QDs. The red solid line represents the change in mobility $\Delta\mu_n$, caused by the depletion of the 2DEG, which leads to a reduced screening of random background Coulomb scatterers, while the blue dashed line shows the overall change in mobility $\Delta\mu = \Delta\mu_n + \Delta\mu_{QD}$. (b) Calculation of the different parts of the change in mobility using the Stern-Howard-model (see text for details). (c) Mobility change due to Coulomb scattering with the electrons stored inside the dots; calculated for different distances between QDs and 2DEG d_{QD} .

To compare these experimental findings with theoretical expectations, we use the Stern-Howard-model.^{17,21,26} Within this model, we calculate the mobility, taking into account three different scattering processes: (i) The Coulomb scattering caused by the charged QDs μ_{QD} ; (ii) Coulomb scattering caused by the ionised atoms in the layer of the δ -doping μ_{2D} ; (iii) Coulomb scattering by 3D background impurities μ_{3D} .

The average scattering time of electrons passing the 2DEG with the charged QDs by Coulomb interaction can be calculated within this model by:

$$\frac{1}{\tau_{QD}} = \frac{\pi\hbar n_{QD} q_{QD}}{8m(k_F d_{QD})}, \quad (3)$$

with the density of the QDs n_{QD} , the Fermi-wavenumber $k_F = \sqrt{2\pi n_{0/1}}$, and the distance between QDs and 2DEG d_{QD} . As the QDs get charged, the number of carriers in the 2DEG is reduced by depletion, however, we assume for simplicity that every electron stored in a QD will deplete one electron in the 2DEG (see Marquardt *et al.*²³ for more details). A step function $q_{QD}(V_G) = 0 \dots 6$ describes the average number of electrons in the QDs and we derive the charge carrier density in the 2DEG for charged QDs $n_1 = n_0 - \Delta n$ with $\Delta n = n_{QD} q_{QD}$. A QD density of $1.2 \cdot 10^{10}$ cm⁻² and a distance between the QDs and the 2DEG of 30 nm has been used.

The average time for scattering with the ionized doping atoms in the layer of the δ -doping is also calculated using

Eq. (3); taking into account an impurity density of $3 \cdot 10^{12}$ cm⁻² and a distance between 2DEG and δ -doping of 16 nm. The average time for scattering with 3D-impurities is the only free parameter and fitted to the data, using the well established scattering formula for background ionized impurities.¹⁹ From this fit to the data, we get a density of 3D background impurities of about $2.6 \cdot 10^{17}$ cm⁻³. This unexpected high value in comparison to the normal background doping level of $10^{13} - 10^{14}$ cm⁻³ in MBE-grown samples could be explained by the QD-strain-induced potential modulations in the 2DEG.¹⁶

The overall mobility is now calculated by using Matthiessens' rule $\frac{1}{\mu_1(n_1)} = \frac{1}{\mu_{QD}(n_1)} + \frac{1}{\mu_{2D}(n_1)} + \frac{1}{\mu_{3D}(n_1)}$. This includes Coulomb scattering μ_{QD} and a reduced carrier density $n_1 = n_0 - \Delta n$ by depletion from the charged QDs, where n_0 is the charge carrier density in the 2DEG without depletion. The mobility $\mu_0(n_0)$ for empty QDs is calculated using $\frac{1}{\mu_0(n_0)} = \frac{1}{\mu_{2D}(n_0)} + \frac{1}{\mu_{3D}(n_0)}$. Having calculated both mobilities for charged and empty dots in analogy to the lines in Fig. 3(b), we can obtain (i) the mobility change due to Coulomb scattering with the electrons in the QDs $\Delta\mu_{QD} = \mu_1(n_1) - \mu_0(n_1)$ [position (2) minus position (3) in Fig. 3(b)] and (ii) due to depletion $\Delta\mu_n = \mu_0(n_1) - \mu_0(n_0)$ [position (3) minus position (1)]. Both contributions $\Delta\mu_n$ and $\Delta\mu_{QD}$, respectively, are shown in Fig. 4(b) together with the overall change in mobility $\Delta\mu$, which is the summation of both parts.

The calculated change in mobility due to Coulomb scattering from the charged QDs, $\Delta\mu_{QD}$, shows a negligible contribution (green, dotted line). The overall change in mobility $\Delta\mu$ is completely caused by depletion of the 2DEG $\Delta\mu_n$, in very good agreement with the observations made in the measurements. We also calculated the influence of the Coulomb scattering in Fig. 4(c) as a function of the distance between the QDs and the 2DEG where the QDs are charged with one electron. For a distance of only 15 nm, the influence on the mobility increases up to 1.2%, however, still a small part of the total change in mobility.

IV. SUMMARY

In conclusion, we have investigated the influence of charged QDs on the mobility of a 2DEG using time-resolved transconductance spectroscopy. This method allows us to unambiguously separate the influence of the electrons in the QDs as Coulomb scatterers from the indirect effect, that electrons in the QDs also change the 2DEGs charge carrier density. Surprisingly, a negligible influence of Coulomb scatterers on the mobility is found for a distance of 30 nm between the QD layer and the 2DEG. The measurements are confirmed by calculations using the Stern-Howard model. Reducing the distance to only 15 nm, the model proposes still a surprisingly small influence on the mobility in the range of only 1.2%.

ACKNOWLEDGMENTS

The authors gratefully acknowledge financial support by the DFG (Contract No. GE 2141/1-1) in the framework of

the NanoSci-E+ project QD2D of the European Commission, the Mercator Research Center Ruhr (MERCUR), Project No. PR-2010-0008 of Stiftung Mercator, as well as the project Hochfunktionale Speicher (HOFUS) within the VIP program of the BMBF. A. Ludwig and A. D. Wieck acknowledge gratefully support of Mercur Pr-2013-0001, BMBF-Q.com-H 16KIS0109, and the DFH/UFA CDFS-05-06.

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