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Robust Control Approach for Input-Output Linearizable Nonlinear Systems Using High-Gain Disturbance Observer

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SUMMARY

This paper addresses a robust control approach for a class of input-output linearizable nonlinear systems with uncertainties and modeling errors considered as unknown inputs. As known, the exact feedback linearization method can be applied to control input-output linearizable nonlinear systems, if all the states are available and modeling errors are negligible. The mentioned two prerequisites denote important problems in the field of classical nonlinear control. The solution approach developed in this contribution is using disturbance rejection by applying feedback of the uncertainties and modeling errors estimated by a specific high-gain disturbance observer as unknown inputs. At the same time, the non-measured states can be calculated from the estimation of the transformed system states. The feasibility and conditions for the application of the approach on mechanical systems are discussed. A nonlinear MIMO mechanical system is taken as a simulation example to illustrate the application. The results show the robustness of the control design and plausible estimations of full rank disturbances.

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KEY WORDS: Disturbance observers; disturbance rejection; nonlinear control systems; robust control

1. INTRODUCTION

Due to the inevitable nonlinearities in real systems, many nonlinear control methods are developed. In the remarkable textbooks as [1, 2, 3] several classical nonlinear control methods (e.g., feedback

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linearization, sliding mode control, backstepping approach etc.) are described in detail. As well-known, classical methods have limitations in application. According to the situation in nonlinear control, researchers have struggled for decades to realize robust and practical solutions for nonlinear systems by proposing different approaches or mostly improving classical control methods. Considering the availability of the states and the most commonly used full state feedback, observer-based robust nonlinear control approaches have also been discussed in a large number of research works to achieve this goal.

In [4] an exact feedforward linearization approach based on differential flatness for nonlinear system control is proposed, which has the same robustness problem as the classical feedback linearization method. Hence, the authors discussed later in [5] its robustness with respect to uncertainties and disturbances. In [6], a robust nonlinear predictive control based on exact feedforward linearization is introduced. It demonstrates that the nonlinear flatness-based control methods are applicable and rational. Unfortunately, the assumptions with respect to disturbances and the availability of all the states is not discussed.

In [7, 8, 9, 10, 11] different methods are developed and applied to solve the robustness problem of the exact feedback linearization method. However, the modeling errors or the disturbances are considered with known bounds and/or known dynamical properties within most of the methods [7, 8, 9]. Robust feedback linearization methods are proposed by improving the design of the nonlinear feedback in [10, 11].

Besides the robustness, availability of the states has also been considered in nonlinear control design. Of course, observer-based control design, especially controllers based on state and disturbance observers or disturbance observers, have been studied to improve the robustness of classical nonlinear control and realize tasks like fault diagnosis, e.g., in [12, 13, 14, 15]. The discrete-time method in [13, 14] can solve the robust control problem of linear Multi-Input Multi-Output (MIMO) systems with mismatched disturbances that do not change significantly in two consecutive sampling instances. That means the rate of disturbances is limited. In [15] a robust control is proposed for a class of uncertain nonlinear systems without assumption of passifiability. The proposed control approach in [15] is based on an adaptive observer design and can partially linearize considered systems. Considering no disturbances, the adaptive control approach is suitable for a general class of nonlinear systems. However, it can only be applied to nonlinear systems with a basic linear time-invariant state space representation and additive nonlinearities, when external disturbances and especially measurement noise exist.

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On the other hand, the high-gain PI-Observer firstly presented in [16] is a specific type of disturbance observer, that estimates both the system states like a Luenberger observer and additionally the disturbances as unknown external inputs acting to the system. It takes the integration of the estimation errors as input to the observer and therefore generates extended states. The PI-Observer concept has been developed and applied by many authors [17, 18, 19, 20, 21, 22, 23, 24]. In [18, 20, 24] different design methods for the PI-Observer gains are introduced. In [17, 19, 22] the PI-Observer is applied mainly to improve the robustness of the state estimations. In [23] a robust PI-Observer is designed for stochastic linear systems under convex bounded uncertainty. In [21] the application of PI-Observer approach is to estimate both the original states and the bounded unknown inputs as extended states with high gains, without any information of the dynamics and bounds of the unknown inputs, which is also the function of the PI-Observer used here.

The high-gain PI-Observer as a state and disturbance observer can be applied to design an observer-based nonlinear robust control. Due to the simple linear structure of the PI-Observer, the best complementary method for the PI-Observer in nonlinear control design is the exact feedback linearization approach. The PI-Observer can offer the estimations of the states and unknown inputs for the exact feedback linearization and the linearized model generated by the feedback linearization is appropriate for the PI-Observer design. The combination of the two approaches may be a potential robust nonlinear control method for the class of nonlinear systems that is suitable for the classical exact feedback linearization method. Although the combined approach is also concentrated on a small part of nonlinear systems, the simple structure of the PI-Observer and the classical design of the exact feedback linearization ensure that the approach is easily understood by engineers and therefore a wide application in industry may be possible.

The key points of this contribution are the combination of advantages of the exact feedback linearization method and of the abilities of the PI-Observer. The application of the proposed approach to mechanical systems is revised based on the previous contributions of the authors [25, 26]. The stability and robustness are proven for the overall closed-loop.

The paper is organized as follows: in the second section, the considered group of nonlinear systems with disturbances/uncertainties and the robustness problem for these systems are stated. The proposed robust control approach is detailed in the third section. The application on mechanical systems is discussed in the fourth section with an example in the fifth section. The last section concludes the paper with a summary and conclusions.

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2. PROBLEM STATEMENT

A class of nonlinear systems with unknown inputs can be described by

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \underbrace{\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u}(t)}_{\text{Nominal model}} + \boldsymbol{E}\boldsymbol{d}(\boldsymbol{x}, t), \\ \boldsymbol{y}(t) &= \boldsymbol{h}(\boldsymbol{x}), \end{aligned} \tag{1}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$ denotes the state vector, $\boldsymbol{u}(t) \in \mathbb{R}^l$ the input vector, $\boldsymbol{y}(t) \in \mathbb{R}^m$ the output to be controlled. The vector $\boldsymbol{d}(\boldsymbol{x}, t) \in \mathbb{R}^s$ with $s \leq n$ together with the constant matrix $\boldsymbol{E} \in \mathbb{R}^{n \times s}$ represents the unknown inputs. Disturbances, modeling errors, parameter uncertainties or other uncertainties to the nominal model can be subsumed under $\boldsymbol{E}\boldsymbol{d}(\boldsymbol{x}, t)$, possibly in all the channels (when the rank of $\boldsymbol{E}$ equals n).

The problem to be solved is to design a controller, which can stabilize the system behavior or realize stable tracking and regulation control for the system in spite of the existence of disturbances $\boldsymbol{d}(\boldsymbol{x}, t)$.

Several assumptions are considered:

- The vector fields $\boldsymbol{f}(\cdot)$ on $\mathbb{R}^n$, $\boldsymbol{d}(\cdot)$ on $\mathbb{R}^s$, $\boldsymbol{g}(\cdot)$ on $\mathbb{R}^{n \times l}$, and $\boldsymbol{h}(\cdot)$ on $\mathbb{R}^m$ are smooth.
- The system has an equilibrium at $\boldsymbol{x} = 0$.
- The unknown input $\boldsymbol{d}(\boldsymbol{x}, t)$ and its derivatives are bounded, but the bounds and the related dynamical behavior is unknown.
- The nominal model of the system is available, input-output linearizable and the remaining zero dynamics is stable.
- In MIMO cases the number of the inputs is equal to the number of the outputs, namely $l = m$.

With the classical input-output linearization method as introduced in [2], the system model (1) can be transformed into the following form

$$\begin{bmatrix} y_1^{(r_1)}(t) \\ \vdots \\ y_m^{(r_m)}(t) \end{bmatrix} = \begin{bmatrix} v_1(t) \\ \vdots \\ v_m(t) \end{bmatrix} + \underbrace{\begin{bmatrix} \tilde{d}_1(\boldsymbol{x}, t) \\ \vdots \\ \tilde{d}_m(\boldsymbol{x}, t) \end{bmatrix}}_{\tilde{\boldsymbol{d}}(\boldsymbol{x}, t)} \tag{2}$$

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by feedback linearization with

$$\boldsymbol{u}(t) = -\boldsymbol{\Gamma}^{-1}(\boldsymbol{x}) \begin{bmatrix} \boldsymbol{L}_{\boldsymbol{f}}^{r_1} h_1(\boldsymbol{x}) \\ \vdots \\ \boldsymbol{L}_{\boldsymbol{f}}^{r_m} h_m(\boldsymbol{x}) \end{bmatrix} + \boldsymbol{\Gamma}^{-1}(\boldsymbol{x}) \begin{bmatrix} v_1(t) \\ \vdots \\ v_m(t) \end{bmatrix}, \tag{3}$$

with the decoupling matrix

$$\boldsymbol{\Gamma}(\boldsymbol{x}) = \begin{bmatrix} \boldsymbol{L}_{g_1} \boldsymbol{L}_{\boldsymbol{f}}^{r_1-1} h_1(\boldsymbol{x}) & \cdots & \boldsymbol{L}_{g_m} \boldsymbol{L}_{\boldsymbol{f}}^{r_1-1} h_1(\boldsymbol{x}) \\ \vdots & \ddots & \vdots \\ \boldsymbol{L}_{g_1} \boldsymbol{L}_{\boldsymbol{f}}^{r_m-1} h_m(\boldsymbol{x}) & \cdots & \boldsymbol{L}_{g_m} \boldsymbol{L}_{\boldsymbol{f}}^{r_m-1} h_m(\boldsymbol{x}) \end{bmatrix}, \tag{4}$$

and the transformed unknown effects

$$\tilde{\boldsymbol{d}}(\boldsymbol{x}, t) = \begin{bmatrix} \boldsymbol{L}_{\boldsymbol{Ed}}^{r_1-1} h_1 + \frac{d}{dt}(\boldsymbol{L}_{\boldsymbol{Ed}}^{r_1-2} h_1) + \frac{d^2}{dt}(\boldsymbol{L}_{\boldsymbol{Ed}}^{r_1-3} h_1) + \cdots + \frac{d^{(r_1-1)}}{dt}(\boldsymbol{L}_{\boldsymbol{Ed}} h_1) \\ \vdots \\ \boldsymbol{L}_{\boldsymbol{Ed}}^{r_m-1} h_m + \frac{d}{dt}(\boldsymbol{L}_{\boldsymbol{Ed}}^{r_m-2} h_m) + \frac{d^2}{dt}(\boldsymbol{L}_{\boldsymbol{Ed}}^{r_m-3} h_m) + \cdots + \frac{d^{(r_m-1)}}{dt}(\boldsymbol{L}_{\boldsymbol{Ed}} h_m) \end{bmatrix}, \tag{5}$$

with $r_i,\ i = 1, \cdots, m$, the smallest integer such that at least and firstly one of the inputs appear in $y_i^{(r_i)}(t)$. The total relative degree of the system is defined by $r = r_1 + r_2 + \cdots + r_m$. After the input-output linearization, the system model is transformed into a form with external dynamics (the input-output dynamics) and internal dynamics. Define new coordinates for the external dynamics as

$$\begin{array}{llll} \mathcal{X}_1^1(t) = y_1(t) = h_1(\boldsymbol{x}), & \mathcal{X}_2^1(t) = L_f h_1(\boldsymbol{x}), & \cdots & \mathcal{X}_{r_1}^1(t) = L_f^{r_1-1} h_1(\boldsymbol{x}) \\ & & \vdots & \\ \mathcal{X}_1^m(t) = y_m(t) = h_m(\boldsymbol{x}), & \mathcal{X}_2^m(t) = L_f h_m(\boldsymbol{x}), & \cdots & \mathcal{X}_{r_m}^m(t) = L_f^{r_m-1} h_m(\boldsymbol{x}), \end{array} \tag{6}$$

where the states are the m outputs y_i and their derivatives up to order r_i with $i = 1, \cdots, m$. Choosing $n - r$ variables $\eta_1(t), \cdots, \eta_{n-r}(t)$, which are independent with respect to each other and to the coordinates $\boldsymbol{\mathcal{X}}(t)$ and the new state vector $\begin{bmatrix} \boldsymbol{\mathcal{X}}(t) \\ \boldsymbol{\eta}(t) \end{bmatrix}$ for the system is completed.

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The external dynamics

$$\begin{aligned} \dot{\mathcal{X}}_1^i(t) &= \mathcal{X}_2^i(t) \\ &\vdots \\ \dot{\mathcal{X}}_{r_i}^i(t) &= v_i(t) + \tilde{d}_i(\boldsymbol{x}, t) \end{aligned} \tag{7}$$

for $i = 1, \cdots, m$ together with the internal dynamics (if $r < n$)

$$\dot{\boldsymbol{\eta}}(t) = \boldsymbol{w}(\boldsymbol{\mathcal{X}}, \boldsymbol{\eta}) + P(\boldsymbol{\mathcal{X}}, \boldsymbol{\eta})\boldsymbol{u}(t) + Q(\boldsymbol{\mathcal{X}}, \boldsymbol{\eta})\boldsymbol{d}(t) \tag{8}$$

describes the system dynamics. If $n = r$, no internal dynamics exists. The system is input-output linearized.

Based on the transformed form of the system with the external and internal dynamics in (7)-(8), linear control laws, e.g., state feedback with pole placement or LQR method, can be applied to control the transformed system (7). The performance of the control is apparently affected by the transformed unknown inputs, which makes classical nonlinear control ineffective and leads to the robustness problem of the classical nonlinear method. Additionally to realize the control design for the original system (1) with the classical nonlinear method, it has to be assumed that all the states $\boldsymbol{x}(t)$ are available (by measurements or reconstruction applying suitable observers assuming observability).

In the following section, a robust control method based on the combination of the exact feedback linearization and the PI-observer technique is proposed to solve the robustness problem and the availability problem of the states in the classical exact feedback linearization method.

3. ROBUST CONTROL METHOD BASED ON THE EXACT FEEDBACK LINEARIZATION AND THE PI-OBSERVER

The proposed approach shown in Fig. 1, takes the advantages of the exact feedback linearization method to get a transformed (input-output linearized) description of the system and then applies the PI-Observer to estimate the transformed states together with the transformed unknown inputs. Using a state feedback control and a disturbance rejection, a robust control for the transformed system can be designed. With the assumption of the stability of the remaining zero dynamics, which is a typical

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assumption controlling mechanical system and which can be easily realized, the whole control loop with the proposed control design is stable and robust.

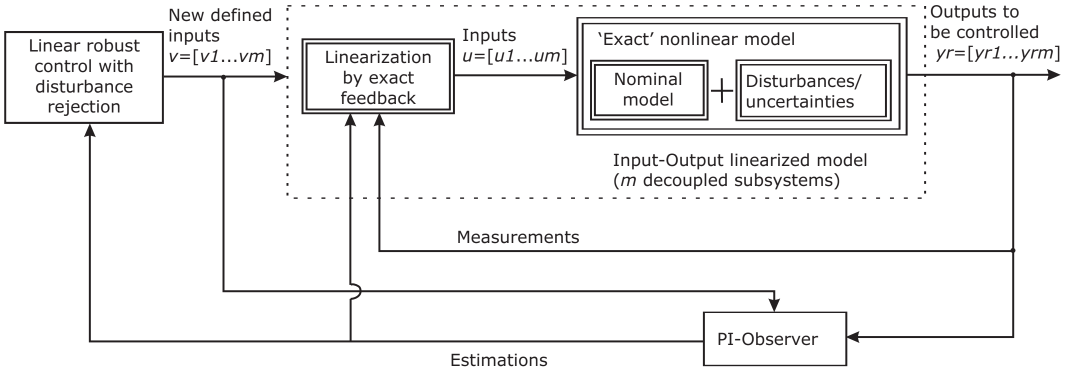

Figure 1. Sketch of the proposed approach

3.1. PI-Observer Design for the Estimations of Transformed System States and Unknown Inputs

From the transformed system model (2), the transformed system dynamics are decoupled in m subsystems which can be described uniformly by

$$y_i^{(r_i)}(t) = v_i(t) + \tilde{d}_i(\boldsymbol{x}, t), \quad i = 1, \cdots, m, \text{ or} \tag{9}$$

$$\begin{aligned} \dot{\mathcal{X}}_1^i(t) &= \mathcal{X}_2^i(t) \\ &\vdots \\ \dot{\mathcal{X}}_{r_i}^i(t) &= v_i(t) + \tilde{d}_i(\boldsymbol{x}, t). \end{aligned} \tag{10}$$

Every subsystem can be written in an individual state space form

$$\begin{aligned} \dot{\boldsymbol{\mathcal{X}}}^{\boldsymbol{i}}(t) &= \boldsymbol{\mathcal{A}_i}\boldsymbol{\mathcal{X}}^{\boldsymbol{i}}(t) + \boldsymbol{\mathcal{B}_i}v_i(t) + \boldsymbol{\mathcal{N}_i}\tilde{d}_i(\boldsymbol{x}, t), \\ y_i(t) &= \boldsymbol{\mathcal{C}_i}\boldsymbol{\mathcal{X}}^{\boldsymbol{i}}(t), \end{aligned} \tag{11}$$

with state vector $\boldsymbol{\mathcal{X}}^{\boldsymbol{i}}(t) = \begin{bmatrix} y_i(t) \\ \dot{y}_i(t) \\ \vdots \\ y_i^{(r_i)}(t) \end{bmatrix}_{r_i \times 1}$, system matrix $\boldsymbol{\mathcal{A}_i} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & 1 \\ 0 & \cdots & \cdots & 0 \end{bmatrix}_{r_i \times r_i}$,

input matrix $\boldsymbol{\mathcal{B}_i} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{r_i \times 1}$, matrix $\boldsymbol{\mathcal{N}_i} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{r_i \times 1}$, output matrix $\boldsymbol{\mathcal{C}_i} = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}_{1 \times r_i}$.

In this case, it is obvious that the system is fully controllable and fully observable according to the structure of $(\boldsymbol{\mathcal{A}_i}, \boldsymbol{\mathcal{B}_i})$ and $(\boldsymbol{\mathcal{A}_i}, \boldsymbol{\mathcal{C}_i})$.

The decoupled subsystems have an appropriate structure for the proposed PI-Observer design [18]. Therefore, PI-Observers can be constructed for each subsystem separately, for example

$$\begin{bmatrix} \dot{\hat{\boldsymbol{\mathcal{X}}}}^{\boldsymbol{i}}(t) \\ \dot{\hat{\tilde{d}}}_i(\boldsymbol{x}, t) \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{\mathcal{A}_i} & \boldsymbol{\mathcal{N}_i} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}}_{\boldsymbol{\mathcal{A}_{ei}}} \begin{bmatrix} \hat{\boldsymbol{\mathcal{X}}}^{\boldsymbol{i}}(t) \\ \hat{\tilde{d}}_i(\boldsymbol{x}, t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{\mathcal{B}_i} \\ \mathbf{0} \end{bmatrix} v_i(t) + \underbrace{\begin{bmatrix} \boldsymbol{\mathcal{L}_{1i}} \\ \boldsymbol{\mathcal{L}_{2i}} \end{bmatrix}}_{\boldsymbol{\mathcal{L}_i}} (y_i(t) - \hat{y}_i(t)),$$
$$\hat{y}_i(t) = \underbrace{\begin{bmatrix} \boldsymbol{\mathcal{C}_i} & \mathbf{0} \end{bmatrix}}_{\boldsymbol{\mathcal{C}_{ei}}} \begin{bmatrix} \hat{\boldsymbol{\mathcal{X}}}^{\boldsymbol{i}}(t) \\ \hat{\tilde{d}}_i(\boldsymbol{x}, t) \end{bmatrix}. \tag{12}$$

Furthermore, the requirement of a PI-Observer design [18, 27] is fulfilled: the extended system $(\boldsymbol{\mathcal{A}_{ei}}, \boldsymbol{\mathcal{C}_{ei}})$ is fully observable. With properly chosen observer gain matrices $\boldsymbol{\mathcal{L}_{1i}}$ and $\boldsymbol{\mathcal{L}_{2i}}$ with the corresponding dimension $r_i \times 1$ and 1×1, the transformed states $\boldsymbol{\mathcal{X}_i}(t)$ and the transformed disturbances $\tilde{d}_i(\boldsymbol{x}, t)$ can be estimated with the PI-Observer as $\hat{\boldsymbol{\mathcal{X}}}_{\boldsymbol{i}}(t)$ and $\hat{\tilde{d}}_i(\boldsymbol{x}, t)$.

Design of Observer Gain Matrices of a PI-Observer Based on the transformed system model (2) and the observer (12), the error dynamics of the extended system is described by

$$\begin{bmatrix} \dot{\boldsymbol{e}}^i(t) \\ \dot{\boldsymbol{f}_e}^i(t) \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{\mathcal{A}_i} - \boldsymbol{\mathcal{L}_{1i}}\boldsymbol{\mathcal{C}_i} & \boldsymbol{\mathcal{N}_i} \\ -\boldsymbol{\mathcal{L}_{2i}}\boldsymbol{\mathcal{C}_i} & \mathbf{0} \end{bmatrix}}_{\boldsymbol{\mathcal{A}_{ei,obs}}} \begin{bmatrix} \boldsymbol{e}^i(t) \\ \boldsymbol{f_e}^i(t) \end{bmatrix} - \begin{bmatrix} \mathbf{0} \\ \dot{\boldsymbol{d}}_{\boldsymbol{i}}(t) \end{bmatrix}, \tag{13}$$

with estimation errors defined as $\boldsymbol{e}^i(t) = \hat{\boldsymbol{\mathcal{X}}}^{\boldsymbol{i}}(t) - \boldsymbol{\mathcal{X}}^{\boldsymbol{i}}(t)$ and $\boldsymbol{f_e}^i(t) = \hat{\tilde{d}}_i(\boldsymbol{x}, t) - \tilde{d}_i(\boldsymbol{x}, t)$.

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It is proved in [18, 27, 28], with high gains in $\boldsymbol{\mathcal{L}_{2i}}$ and $\|\boldsymbol{\mathcal{L}_{2i}}\|_F \gg \|\boldsymbol{\mathcal{L}_{1i}}\|_F$[†]), the estimation errors $\left\|\boldsymbol{e}^i(s)\right\|_F$ and $\left\|\boldsymbol{f_e}^i(s)\right\|_F$ can be reduced to an arbitrary small value γ (but not to zero), if $\left\|s\,\tilde{\boldsymbol{d}}_{\boldsymbol{i}}(s)\right\|_F$ is bounded.

The Linear Quadratic Regulator (LQR) method is applied to design the high-gain PI-Observer feedback matrices by solving the algebraic matrix Riccati equation.

For a stable observer, suitable observer gains can be calculated, if for given positive definite matrices $\boldsymbol{Q}$, $\boldsymbol{R}$ the Riccati equation

$$\boldsymbol{\mathcal{A}_{ei}P} + \boldsymbol{P\mathcal{A}_{ei}}^T + \boldsymbol{Q} - \boldsymbol{P\mathcal{C}_{ei}}^T\boldsymbol{R}^{-1}\boldsymbol{\mathcal{C}_{ei}P} \;=\; \boldsymbol{0} \tag{14}$$

has a unique positive definite solution matrix $\boldsymbol{P}$. The observer feedback matrix is then calculated with $\boldsymbol{\mathcal{L}_i} = \boldsymbol{P\mathcal{C}_{ei}}^T\boldsymbol{R}^{-1}$, also other suitable approaches designing $\boldsymbol{\mathcal{L}_i}$ can be chosen.

3.2. Robust Control Design for the Transformed Systems

Realizing a robust control, a state feedback control with a disturbance rejection

$$v_i(t) = -\boldsymbol{\mathcal{K}^i}\hat{\boldsymbol{\mathcal{X}}}^{\boldsymbol{i}}(t) - \hat{\tilde{d}}_i(\boldsymbol{x}, t) \tag{15}$$

can be taken to stabilize the transformed system dynamics, because the estimations $\hat{\boldsymbol{\mathcal{X}}}^{\boldsymbol{i}}(t)$ and $\hat{\tilde{d}}_i(\boldsymbol{x}, t)$ are available from the PI-Observer and the transformed system is assumed as fully controllable. The state feedback control (gain matrix $\boldsymbol{\mathcal{K}^i}$) is designed by pole placement method with the characteristic polynomial to be a Hurwitz polynomial. Using the nonlinear feedback (3), the inputs to the original system dynamics can be constructed. At the same time, from the m PI-Observers the external unknown inputs $\tilde{\boldsymbol{d}}(\boldsymbol{x}, t)$ can be estimated in the transformed coordinations. Of course all the states $\boldsymbol{x}(t)$ and outputs $\boldsymbol{y}$(t) in the original coordinates should be available to realize the input-output linearization as usual.

3.3. Stability of the Closed-loop System

The following theorem is addressed in order to analyze the stability and robustness of the closed-loop system.

[†]The norm $\|\cdot\|_F$ denotes here the Frobenius norm, $\|A\|_F = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^2} = \sqrt{trace(A^*A)}$ for A in $R^{m\times n}$.

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Theorem 1

Assume the system (1) has relative degree r and its zero dynamics is locally asymptotically stable. With state feedback matrices $\boldsymbol{\mathcal{K}^i} = \begin{bmatrix} \mathcal{K}_1^i & \mathcal{K}_2^i & \cdots & \mathcal{K}_{r_i}^i \end{bmatrix}$ for every subsystem satisfying Hurwitz stability criterion and PI-Observers designed by LQR method with suitable choosing observer gains, the closed-loop system is locally asymptotically stable and robust against the unknown disturbances $\boldsymbol{d}$.

Proof

The external dynamics (7) is transformed with the control input designed by (15) into

$$\dot{\mathcal{X}}_{r_i}^i = -\mathcal{K}_1^i\hat{\mathcal{X}}_1^i - \mathcal{K}_2^i\hat{\mathcal{X}}_2^i - \cdots - \mathcal{K}_{r_i}^i\hat{\mathcal{X}}_{r_i}^i + \tilde{d}_i(\boldsymbol{x},t) - \hat{\tilde{d}}_i(\boldsymbol{x},t), \tag{16}$$

which can be analyzed after Laplace transformation in frequency domain

$$(s^{r_i} + \mathcal{K}_{r_i}^i s^{r_i-1} + \cdots + \mathcal{K}_2^i s + \mathcal{K}_1^i)\mathcal{X}_1^i(s) = \underbrace{\boldsymbol{\mathcal{K}^i}\left(\boldsymbol{\mathcal{X}^i}(s) - \mathcal{K}^i\hat{\mathcal{X}}^i(s)\right)}_{\boldsymbol{\mathcal{K}^i}\boldsymbol{e}^i(s)} + \underbrace{\tilde{d}_i(s) - \hat{\tilde{d}}_i(s)}_{\boldsymbol{f_e}^i(s)} \tag{17}$$

with $\boldsymbol{e}^i(s)$ and $\boldsymbol{f_e}^i(s)$ as the estimation errors from the PI-Observer design in (12). The output $y_i(s) = \mathcal{X}_1^i(s)$ accordingly the external dynamics is stable, because the characteristic polynomial in (17) is Hurwitz stable and the inputs to the transfer behavior, the estimation errors of the PI-Observer, converge to an arbitrarily small value γ.

Due to the fact that the zero dynamics is assumed to be stable, it can be concluded that the whole control loop is stable and robust to the disturbances $\boldsymbol{d}$. □

4. CONDITIONS FOR THE APPLICATION OF THE PROPOSED ROBUST CONTROL APPROACH ON MECHANICAL SYSTEMS

In the following subsections, the conditions for the application of the proposed approach on a typical class of mechanical systems will be discussed.

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4.1. Modeling of General Mechanical Systems

Without loss of generality, the discussed class of nonlinear mechanical systems is described by n second order differential equations

$$\begin{bmatrix} \ddot{q}_1(t) \\ \vdots \\ \ddot{q}_n(t) \end{bmatrix} = \begin{bmatrix} f_1(q_1, \dot{q}_1, \cdots, q_n, \dot{q}_n) \\ \vdots \\ f_n(q_1, \dot{q}_1, \cdots, q_n, \dot{q}_n) \end{bmatrix} + \begin{bmatrix} g_{11}(q_1, \dot{q}_1, \cdots, q_n, \dot{q}_n) & \cdots & g_{1l}(q_1, \dot{q}_1, \cdots, q_n, \dot{q}_n) \\ \vdots & \ddots & \vdots \\ g_{n1}(q_1, \dot{q}_1, \cdots, q_n, \dot{q}_n) & \cdots & g_{nl}(q_1, \dot{q}_1, \cdots, q_n, \dot{q}_n) \end{bmatrix} \boldsymbol{u}(t) + \begin{bmatrix} d_1(q_1, \dot{q}_1, \cdots, q_n, \dot{q}_n, t) \\ \vdots \\ d_n(q_1, \dot{q}_1, \cdots, q_n, \dot{q}_n, t) \end{bmatrix}, \tag{18}$$

namely

$$\ddot{\boldsymbol{q}}(t) = \underbrace{\boldsymbol{f}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}, \dot{\boldsymbol{q}})\boldsymbol{u}(t)}_{\text{Nominal model}} + \boldsymbol{d}(\boldsymbol{q}, \dot{\boldsymbol{q}}, t), \tag{19}$$

$$\boldsymbol{y}_m(t) = \boldsymbol{q}(t), \tag{20}$$

$$\boldsymbol{y}_c(t) = C_c\boldsymbol{q}(t), \tag{21}$$

with $\boldsymbol{q}(t) = \begin{bmatrix} q_1(t) \\ q_2(t) \\ \vdots \\ q_n(t) \end{bmatrix} \in \mathbb{R}^n$, input vector $\boldsymbol{u}(t) \in \mathbb{R}^l$, outputs to be controlled $\boldsymbol{y}_c(t) \in \mathbb{R}^m$ and measurements $\boldsymbol{y}_m(t) \in \mathbb{R}^n$. The vector fields $\boldsymbol{f}(\cdot)$ on $\mathbb{R}^n$ and $\boldsymbol{g}(\cdot)$ on $\mathbb{R}^{n\times l}$ are assumed as smooth. The assumed system structure (19) is typical for systems like mass-spring systems, multibody systems or systems modeled by finite element method.

4.2. Conditions for the Application of the Proposed Robust Control Approach

Several assumptions are made here for mechanical systems to adopt the proposed robust control approach:

- all the displacements are measurable, see (20),

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- part of displacements is to be controlled, see (21),
- the number of inputs is equal to the number of outputs to be controlled, namely $l = m$ in subsection 4.1 and
- the nominal system model in (19) is input-output linearizable.

With the assumptions given above, the system (19) can be written in a general form with $2n$ first order differential equations

$$\dot{\boldsymbol{\phi}}(t) = \begin{bmatrix} \phi_{n+1} \\ \vdots \\ \phi_{2n} \\ \boldsymbol{f}(\boldsymbol{\phi}) \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{n\times 1} \\ \boldsymbol{g}(\boldsymbol{\phi}) \end{bmatrix} \boldsymbol{u}(t) + \begin{bmatrix} \mathbf{0}_{n\times 1} \\ \boldsymbol{d}(\boldsymbol{\phi}, t) \end{bmatrix},$$
$$\boldsymbol{y}_{control}(t) = \boldsymbol{C}_{contr}\boldsymbol{\phi}(t),$$
$$\boldsymbol{y}_{measure}(t) = \boldsymbol{C}_{measure}\boldsymbol{\phi}(t), \tag{22}$$

with the state vector $\boldsymbol{\phi}(t) = \begin{bmatrix} \boldsymbol{q}(t) \\ \dot{\boldsymbol{q}}(t) \end{bmatrix}$ and output matrices $\boldsymbol{C}_{contr}$ and $\boldsymbol{C}_{measure}$.

For such systems, the application of the proposed approach is feasible, because the requirements of the approach are fufilled.

- Availability of all the states

  For the nonlinear feedback in the input-output linearization process in (3)-(5), usually all the system states, the outputs, and the derivatives of the outputs in the original coordinates are required. This includes for the application on mechanical systems:

  – All the displacements as well as the outputs to be controlled are assumed as measurable.
  – The velocities, which are also states in the original coordinates, can be estimated

    i) either by PI-Observers as the states in the transformed coordinates when the corresponding displacements are to be controlled

    ii) or by additional PI-Observers designed for estimation of the internal dynamics if the corresponding displacements do not appear in the outputs.
  – The derivatives of the outputs can be obtained from the PI-Observers as the states in the transformed coordinates.

- Possibility of full rank disturbance estimation

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Another important aspect is that the estimations of modeling errors/disturbances as unknown inputs possibly with full rank in the original coordinates in (18) are available based on the estimations of transformed unknown inputs by the PI-Observers.

5. APPLICATION EXAMPLE FOR THE ROBUST NONLINEAR CONTROL METHOD ON A MECHANICAL SYSTEM

5.1. Modeling of a Nonlinear MIMO Mass-spring System

An example of nonlinear MIMO mechanical systems, based on the benchmark system [29] shown in Fig. 2, is given to illustrate the proposed method. The system is modelled by

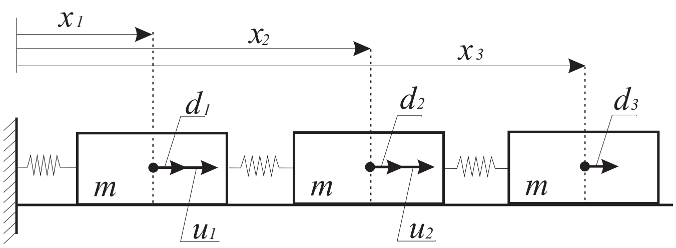

Figure 2. Nonlinear MIMO mechanical system example

$$
\begin{aligned}
m\ddot{x}_1 &= k(-2x_1 + x_2) + k_p[-x_1^3 + (x_2 - x_1)^3] + u_1 + d_1, \\
m\ddot{x}_2 &= k(x_1 - 2x_2 + x_3) + k_p[(x_3 - x_2)^3 - (x_2 - x_1)^3] + u_2 + d_2, \\
m\ddot{x}_3 &= k(x_2 - x_3) + k_p(x_2 - x_3)^3 + d_3, \\
y_{meas} &= \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T, \text{ and} \\
y_{contr} &= \begin{bmatrix} y_1 & y_2 \end{bmatrix}^T = \begin{bmatrix} x_1 & x_3 \end{bmatrix}^T.
\end{aligned} \tag{23}
$$

The parameters used in simulation are $m = 0.5\ kg$, $k = 217.0\ N/m$, and $kp = 63.5\ N/m^3$. The dynamics of the disturbances in the inputs d_1, d_2, d_3 are assumed as unknown to the control design but present in the simulation as $d_1 = 5$, $d_2 = 10sin(5t)$, and $d_3 = 20sin(10t)$ additionally to the nominal model presented in [29], which can be considered as uncertainties in the parameters or external disturbances.

The input-output linearized form of the system can be written in

$$
\ddot{y}_1 \quad = \quad v_1 + \frac{d_1}{m} = v_1 + \bar{\eta}_1, \tag{24}
$$

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$$y_2^{(4)} = v_2 + [\frac{k}{m} + \frac{3k_p}{m}(x_2 - x_3)^2](\frac{d_2}{m} - \frac{d_3}{m}) = v_2 + \bar{\eta}_2, \tag{25}$$

if the inputs are chosen as

$$u_1 = m\left[v_1 - \frac{k}{m}(-2x_1 + x_2) - \frac{k_p}{m}\left[-x_1^3 + (x_2 - x_1)^3\right]\right] \text{ and} \tag{26}$$

$$\begin{aligned} u_2 = & \frac{m}{\frac{k}{m} + \frac{3k_p}{m}(x_2 - x_3)^2} \\ & \{v_2 - \left[\frac{k}{m} + \frac{3k_p}{m}(x_2 - x_3)^2\right]\left[\frac{k}{m}(x_1 - 3x_2 + 2x_3) + \frac{k_p}{m}\left[2(x_3 - x_2)^3 - (x_2 - x_1)^3\right]\right] \\ & -6\frac{k_p}{m}(x_2 - x_3)(\dot{x}_2 - \dot{x}_3)^2\}. \end{aligned} \tag{27}$$

The remaining zero dynamics/internal dynamics

$$\ddot{x}_2 = \frac{1}{m}\left[k(x_1 - 2x_2 + x_3) + k_p[(x_3 - x_2)^3 - (x_2 - x_1)^3] + u_2 + d_2\right]$$

is stable, if the disturbance d_2 is bounded.

Two PI-Observers are designed for the transformed decoupled dynamics (24) and (25)

$$\begin{aligned} \dot{z}_a &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} z_a + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \hat{\bar{\eta}}_1 + L_{1a}(y_1 - \hat{y}_1), \\ \dot{\hat{\bar{\eta}}}_1 &= L_{2a}(y_1 - \hat{y}_1), \\ \hat{y}_1 &= \begin{bmatrix} 1 & 0 \end{bmatrix} z_a, \end{aligned} \tag{28}$$

and

$$\begin{aligned} \dot{z}_b &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} z_b + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} v_2 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \hat{\bar{\eta}}_2 + L_{1b}(y_2 - \hat{y}_2), \\ \dot{\hat{\bar{\eta}}}_2 &= L_{2b}(y_2 - \hat{y}_2), \\ \hat{y}_2 &= \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} z_b, \end{aligned} \tag{29}$$

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with the state vectors $z_a = \begin{bmatrix} \hat{y}_1 \\ \hat{\dot{y}}_1 \end{bmatrix}$ and $z_b = \begin{bmatrix} \hat{y}_2 \\ \hat{\dot{y}}_2 \\ \hat{\ddot{y}}_2 \\ \hat{y}_2^{(3)} \end{bmatrix}$, to estimate the transformed states and disturbances, namely $\hat{x}_1$, $\hat{\dot{x}}_1$, $\hat{\bar{\eta}}_1$, $\hat{x}_3$, $\hat{\dot{x}}_3$, $\hat{\ddot{x}}_3$, $\hat{x}_3^{(3)}$, and $\hat{\bar{\eta}}_2$. To construct the inputs in (26) and (27), besides the displacements x_1, x_2, and x_3 the velocities $\dot{x}_2$ and $\dot{x}_3$ are also required. As a transformed coordinate, the velocity $\dot{x}_3$ can be estimated by the PI-Observer (29). To estimate the velocity $\dot{x}_2$, an additional PI-Observer is designed by

$$\begin{aligned} \dot{z}_c &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} z_c + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_3 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \hat{\bar{\eta}}_3 + L_{1c}(x_2 - \hat{x}_2), \\ \hat{\dot{\bar{\eta}}}_3 &= L_{2c}(x_2 - \hat{x}_2), \end{aligned} \tag{30}$$

with the state vector $z_c = \begin{bmatrix} \hat{x}_2 \\ \hat{\dot{x}}_2 \end{bmatrix}$, $v_3 = \frac{k}{m}\left[(x_1 - 2x_2 + x_3) + k_p[(x_3 - x_2)^3 - (x_2 - x_1)^3] + u_2\right]$, and $\bar{\eta}_3 = \frac{d_2}{m}$.

With the estimations from the three PI-Observers mentioned above, the system (18) can be transformed into an input-output linearized form with nonlinear feedback (26) and (27). To realize the robust control, linear control methods can be applied to the linearized model (24) and (25), for example as linear state feedback control

$$v_1 = -20\hat{\dot{x}}_1 - 100(x_1 - x_{1ref}) - \hat{\bar{\eta}}_1, \tag{31}$$

$$v_2 = -200\hat{x}_3^{(3)} - 15000\hat{\ddot{x}}_3 - 500000\hat{\dot{x}}_3 - 6250000(x_3 - x_{3ref}) - \hat{\bar{\eta}}_2. \tag{32}$$

The desired values taken in the simulation are $x_{1ref} = 0.25$ and $x_{3ref} = 0.3$. The dynamics of the disturbances d_1, d_2, and d_3 are calculated from the estimations $\hat{\bar{\eta}}_1$, $\hat{\bar{\eta}}_2$, and $\hat{\bar{\eta}}_3$.

5.2. Simulation Results

The simulation results in comparison with classical input-output linearization method are given in Fig. 3. It should be noted that classical nonlinear control methods cannot be directly applied, because not all the states are assumed as measurable.

According to the illustrated results, the proposed approach shows firstly robustness against the external disturbances and the reconstruction of the unmeasured states. In comparison, the classical

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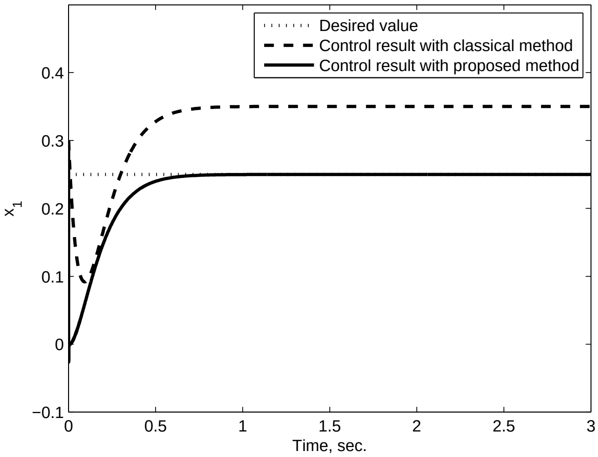
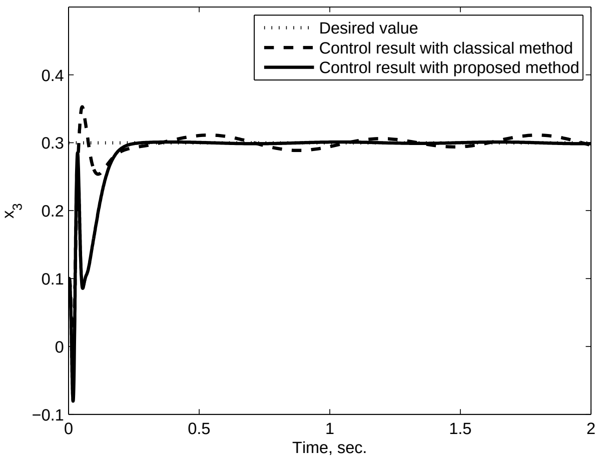


(a)



(b)

Figure 3. Comparison of the control results

input-output linearization method with estimated states from Luenberger observer and the same state feedback control as in the proposed approach is strongly influenced by the disturbances and leads to large control error, especially in controlling x_1.

Secondly, the proposed approach estimates the velocities based on the measurements of the displacements and therefore avoids numerical differentiation of the measured signals to get the information of all the states.

At last, the proposed approach not only realizes a robust control for the considered class of nonlinear MIMO systems, but also generates plausible estimations of the unknown disturbances/modeling error. This may be in addition suitable for extended Fault Detection

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and Isolation (FDI) approaches and/or Structural Health Monitoring techniques supervising the condition and states of the system by for example evaluating residuals. That is a breakthrough in both application and nonlinear robust control. The estimated disturbances for the example are shown in Fig. 4.

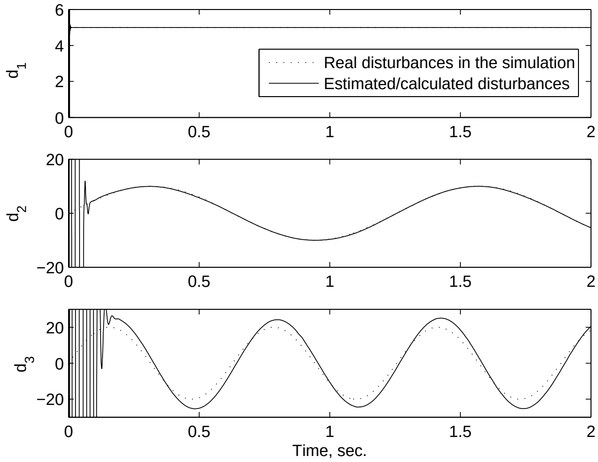

Figure 4. Estimation of the disturbances

6. SUMMARY AND CONCLUSIONS

In this contribution, a robust control design approach is proposed for input-output linearizable nonlinear systems with modeling errors or uncertainties. To realize the input-output linearization, the information of the states is derived by measurements together with the estimations generated by PI-Observers. A robust disturbance rejection control is realized by using state feedback of the linearized model and estimations of the unknown inputs, which include possible modeling errors and uncertainties with unknown dynamics. The stability and robustness of the presented approach are proven. Additionally the estimations of unknown disturbances/modeling errors are given by the PI-Observer design. The conditions are listed for the application on mechanical systems. The simulation results of a mechanical MIMO system illustrate the effects and advantages of the proposed robust control method with use of PI-Observer technique.

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