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# AUTOMATIC GENERATION OF THE EQUATIONS OF MOTION OF THE MOVING NONLINEAR ELASTIC BEAM

DIRK SÖFFKER

*Safety Control Engineering, University of Wuppertal, Gaußstraße 20, D-42097 Wuppertal, Federal Republic of Germany*

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The contribution contains the description of the procedure for automatic generation of the equations of motion of the geometric nonlinear beam. The corresponding matrix representation of a model of 3rd order is given. Here, due to the geometric nonlinear beam kinematic, coupling effects between the elastic variables can be represented. This effects are relevant for lightweight structures for space applications undergoing large reference motion. 'Stiffning effects' and parametrically excited effects between longitudinal and bending vibrations *e.g.*, are important. The introduced modeling procedure allows the representation of such kind of effects known as important from the literature. Here these effects are not considered as additional linearized terms. Additionally also the length-variability of beam-like structures is considered. The clearness and effectiveness of matrix methods are combined with beam theory of 3rd order.

*Keywords:* Nonlinear modeling; elastic structure; stiffning effect; weakning effect; elastic robot; beam structure

## 1. INTRODUCTION

In the last years modeling, simulation, and control of flexible structures have made an essential progress, especially stimulated by the requirements of space operations. For this application field, flexible lightweight robots can enhance the range of the work space of space robots very well, if the length of the robot arm is variable. A literature review to related works can be found in [1]. Furthermore in [1] a discussion about conceptional aspects of choosing modeling methods



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are given. Here only a brief introduction of the literature will be given. Contributions presented in the corresponding literature [2, 3, 4, 5, 6] mostly consider special configuration models or experimental setups and perform simple approximations or even assume linear or linearized models for the robot arm. Actual developments concentrate on the development of the automatic generation of equations of motion for a given structure using symbolic description [7] or to the problem of considering higher order couplings added to a linear description [8, 9, 10, 11, 12]. The question about the choice of the coordinate system describing the elastic coordinates is also important [9, 13]. Finally all the equations are linearized with respect to the elastic variables, so the equations have to be developed up to second order to get the 'correct' linearization. These aspects are widely discussed in the last decade.

It is known that for a beam with constant length the longitudinal vibration may excite bending vibrations [9]. In the case of a robot arm with variable length like a telescopic arm, it is expected that even stronger vibrations occur. In this way it seems to be important to consider also corresponding couplings, which represents the energy transport between different vibration planes.

## 2. GEOMETRIC NONLINEAR BEAM KINEMATICS

Defining the cartesian coordinate system $(E)$ to describe the elastic variables $u_x$, $u_y$, $u_z$ in the movable joint coordinate system $(G)$, (Fig. 1), the vector $r(t)$ of an finite mass point dm in the finite (and stiff) disc $(P)$ in the inertial coordinate system $(I)$ is given by

$$r(t) = r_{OP}^{(I)}(t) = r_{OG}^{(I)}(t) + T\left( l_{GE'}^{(G)}(t) + s_{E'E}^{(G)}(t) + u_{EP'}^{(G)}(t) + \hat{T} t_{P'P}^{(E)}(t) \right), \tag{1}$$

where $r_{OG}^{(I)}(t)$, $r_{GP}^{(I)}(t)$ are the vectors of the joint and of the mass point from the joint resp., in the inertial coordinate system, where the vector of the elastic displacements $u_{EP'}^{(G)}(t)$ describes only the deformation of

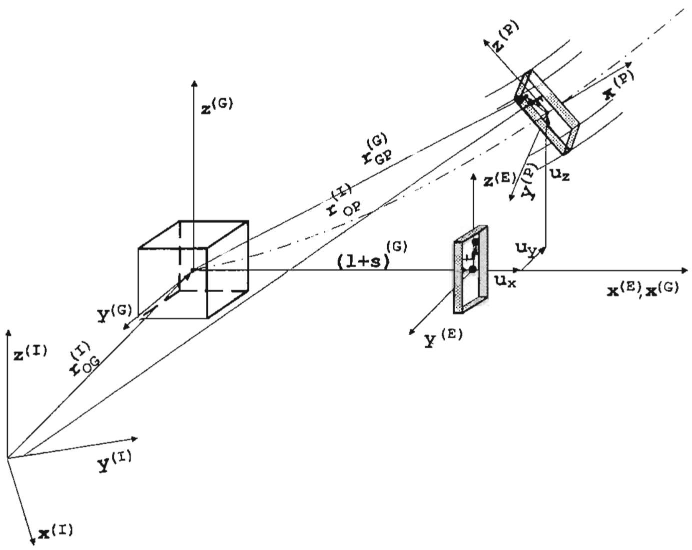


FIGURE 1 Coordinate systems.

the neutral axis of the considered cross-section. The whole vector $r^{(I)}_{GP}(t)$ (described in the joint coordinate system) contains also the vectors $l^{(G)}_{GE'}(t)$, $s^{(G)}_{E'E}(t)$ describing the joint oriented boundaries of the considered finite element and the position of the focussed undeformed disc in the element coordinate system, and the vector $\hat{T}t^{(E)}_{P'P}$ to describe the finite mass point of the stiff disc in the deformed position. The rotation matrices $T$ and $\hat{T}$ are needed to define the element- and joint coordinate based variables of $r(t)$ (Eq. (1)), which are given in the inertial coordinate system. Using the angles $1-\psi, 2-\Theta, 3-\phi$ and the corresponding order of rotations, $T$ is given as the function

$$T(\phi, \Theta, \psi) = \begin{pmatrix} \cos\Theta\cos\psi & \cos\Theta\sin\psi & -\sin\Theta \\ \sin\phi\sin\Theta\cos\psi - \cos\phi\sin\psi & \sin\phi\sin\Theta\sin\psi & \sin\phi\cos\Theta \\ \cos\phi\sin\Theta\cos\psi & \cos\phi\sin\Theta\sin\psi & \\ +\sin\phi\sin\psi & -\sin\phi\cos\psi & \cos\phi\cos\Theta \end{pmatrix}$$

and can be expressed defining the operation point $\phi = \Theta = \psi = 0$, by the linearized expression

$$\hat{T}(\alpha, \beta, \gamma) = \begin{pmatrix} 1 - \frac{\beta^2}{2} - \frac{\gamma^2}{2} & \gamma & -\beta \\ \alpha\beta - \gamma & 1 - \frac{\alpha^2}{2} - \frac{\gamma^2}{2} & \alpha \\ \beta + \alpha\gamma & \beta\gamma - \alpha & 1 - \frac{\alpha^2}{2} - \frac{\beta^2}{2} \end{pmatrix}. \quad (2)$$

In [1] the development of $\beta, \gamma, \varepsilon$ is given in detail. Using the results

$$\beta = u'_y - u'_x u'_y, \ \gamma = u'_z - u'_x u'_z, \ \text{and} \quad \varepsilon = \sqrt{(1 + u'_x)^2 + u'^2_y + u'^2_z} - 1 \quad (3)$$

the elastic strain $\varepsilon$ and the curvatures $\kappa_1, \kappa_2, \kappa_3$

$$\varepsilon = u'_x + \frac{1}{2}(u'^2_y + u'^2_z) \quad \kappa_1 = -\alpha' + u''_y u'_z \quad (4)$$

$$\kappa_2 = -u''_y + u'_y u''_x + u''_y u'_x - \alpha' u'_z \quad \kappa_3 = -u''_z + u''_z u'_x + u'_z u''_x + \alpha' u'_y \quad (5)$$

are determined as functions of the elastic variables. This costly procedure and the determinations of the virtual values are given in [1] in detail.

## 3. PRINCIPLE OF VIRTUAL WORK

Applying the principle of virtual work $\delta V_e = \delta W_a + \delta W_m$ with $\delta V_e$ as virtual potential energy, the virtual work of the external forces $\delta W_a$, and the virtual work of the inertia forces and moments $\delta W_m$, neglecting the influence of shear forces for the long beam, assuming, without loss of generality, constant material behaviour for Youngs modulus and the shear coefficient with E, G and the geometric parameters $A, I_x, I_y, I_z$, the calculation of the virtual potential energy of the deformed beam gives

$$\begin{aligned}\delta V_e = \int_0^1 \Bigg[ \Bigg( \Big(1 - \frac{1}{2}u_y'^2 + u_z'^2\Big)\delta u_x' + (u_y' - u_x'u_y')\delta u_y' \\ + (u_z' - u_x'u_z')\delta u_z' \Bigg) EA \Big(u_x' + \frac{1}{2}(u_y'^2 + u_y'^2)\Big)\Bigg] \\ + [(-\delta\alpha' + u_y''\delta u_z' + u_z'\delta u_y'')GI_t(-\alpha' + u_y''u_z')] \\ [(u_y''\delta u_x' + u_y'\delta u_x'' - u_z'\delta\alpha' - \alpha'\delta u_z' + u_x''\delta u_y' \\ + (u_x' - 1)\delta u_y'')EI_y(u_y'u_x'' - u_y'' + u_x'u_y'' - u_z'\alpha')] \\ +[(+u_z''\delta u_x' - u_z'\delta u_x'' - u_y'\delta\alpha' + \alpha'\delta u_y' + u_x''\delta u_z' \\ + (u_x' - 1)\delta u_z'')EI_z(-u_z'' + u_z'u_x'' + u_x'u_z'' + \alpha'u_y')]dx. \end{aligned} \quad (6)$$

Considering all products up to 2nd order to develop a beam model of 3rd order, using the usual separation principle (here given for the bending displacement $u_y(x, t)$) and therefore applying a vector of shape functions $f_y(t)$

$$\begin{aligned} u_y(x,t) &= f_y^T(x)u_y(t), \\ u_y'(x,t) &= f_y'^T(x)u_y(t), \\ u_y''(x,t) &= f_y''^T(x)u_y(t), \end{aligned} \quad (7)$$

$$\begin{aligned} \delta u_y(x,t) &= \delta u_y^T(t)f_y, \\ \delta u_y'(x,t) &= \delta u_y^T(t)f_y', \\ \delta u_y''(x,t) &= \delta u_y^T(t)f_y''. \end{aligned} \quad (8)$$

all appearing products can be discretized, like-as an example

$$\delta u_y'' EI_y y'' = \delta u_y^T EI_y f_y'' f_y''^T u_y, \quad (9)$$

and also in quadratic form – as an example -

$$\begin{aligned} u_y''\delta u_x' EI_y u_y'' &= \delta u_x^T EI_y f_x' f_y''^T u_y f_y''^T u_y \\ &= \delta u_x^T EI_y f_y'' u_y^T f_y'' f_x'^T u_y \end{aligned} \quad (10)$$

Using a systematic description for all the elastic variables and combinations of states and shape functions, the proposed procedure leads to the complete expression for the potential energy

$$
\delta \mathrm{V}_{\mathrm{c}_i} = \left(\delta u_x^T \delta\beta_\alpha^T \delta u_y^T \delta u_z^T\right) \bullet \int_0^l
\left(\begin{array}{c|c|c|c}
 & & (1/2)\mathrm{EA} f'_x f'^T_y u_y f'^T_y & (1/2)\mathrm{EA} f'_x f'^T_z u_z f'^T_z \\
\mathrm{EA} f'_x f'^T_x & 0 & -\mathrm{EI}_y f'_x f''^T_y u_y f''^T_y & -\mathrm{EI}_z f'_x f''^T_z u_z f''^T_z \\
 & & -\mathrm{EI}_y f''_x f'^T_y u_y f''^T_y & -\mathrm{EI}_z f''_x f'^T_z u_z f''^T_z \\
\hline
 & & +\mathrm{EI}_y f'_\alpha f'^T_z u_z f''^T_y & -\mathrm{GI}_t f'_\alpha f''^T_y u_y f'^T_z \\
0 & \mathrm{GI}_t f'_\alpha f'^T_\alpha & -\mathrm{EI}_z f'_\alpha f''^T_z u_z f'^T_y & +\mathrm{EI}_y f'_\alpha f'^T_y u_y f''^T_z \\
\hline
\mathrm{EA} f'_y f'^T_y u_y f'^T_x & -\mathrm{GI}_t f''_y f'^T_z u_z f'^T_\alpha & \mathrm{EI}_y f''_y f''^T_y & \\
-\mathrm{EI}_y f''_y f''^T_y u_y f'^T_x & +\mathrm{EI}_z f''_y f'^T_z u_z f'^T_\alpha & -\mathrm{EI}_y f'_y f''^T_x u_x f''^T_y & 0 \\
-\mathrm{EI}_y f'_y f''^T_y u_y f''^T_x & -\mathrm{EI}_z f'_y f''^T_z u_z f'^T_\alpha & -\mathrm{EI}_y f''_y f'^T_x u_x f''^T_y & \\
\hline
\mathrm{EA} f'_z f'^T_z u_z f'^T_x & -\mathrm{GI}_t f'_z f''^T_y u_y f'^T_\alpha & & \mathrm{EI}_z f''_z f''^T_z \\
-\mathrm{EI}_z f''_z f''^T_z u_z f'^T_x & +\mathrm{EI}_z f''_z f'^T_y u_y f'^T_\alpha & 0 & -\mathrm{EI}_z f''_z f'^T_x u_x f''^T_z \\
-\mathrm{EI}_z f'_z f''^T_z u_z f''^T_x & & & -\mathrm{EI}_z f''_z f''^T_x u_x f'^T_z
\end{array}\right)
\begin{pmatrix} u_x \\ \beta_\alpha \\ u_y \\ u_z \end{pmatrix} \mathrm{dx}
\tag{11}
$$

which contains linear elements (like $\mathrm{EI}_y f''_y f''^T_y$) and nonlinear elements (like $\mathrm{EI}_y f'_x f''^T_y u_y f''^T_y u_y$), which have to be integrated over the actual length of the beam.

The complete procedure resulting to concrete matrix coefficients can be done automatically.

To consider the nonlinear length -variant beam the following procedure is applied.

For a general nonlinear matrix -element

$$
\tilde{F} = \int_0^l f_a(\mathrm{x}) f_b^T(\mathrm{x}) u(\mathrm{t}) f_c^T(\mathrm{x}) \mathrm{dx} \tag{12}
$$

the procedure/the length integration $\int_0^l \cdots \mathrm{dx}$ can be substituted to the form

$$\tilde{F} = u_1(t)\int_0^1 f_{b1} f_a f_c^T dx + u_2(t)\int_0^1 f_{b2} f_a f_c^T dx + u_3(t)\int_0^1 f_{b3} f_a f_c^T dx + u_4(t)\int_0^1 f_{b4} f_a f_c^T dx \tag{13}$$

$$= \sum_i u_i(t) \underbrace{\int (f_{bi}(x) f_a(x) f_c^T(x)) dx}_{D_{f_i f_a f_b^T}} = \sum_i (u_i(t) D_{f_i f_a f_b^T}(x)). \tag{14}$$

Here $f_a$, $f_b$, $f_c$ denotes the vectors of shape functions. The vectors $f_i$ and $u$ consist of the scalar expressions

$$f_i = \begin{pmatrix} f_{i1} \\ f_{i2} \\ f_{i3} \\ f_{i4} \end{pmatrix} \quad \text{resp.} \quad u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix}. \tag{15}$$

As an example the length dependent matrix $D_{f_i f_a f_b^T}(x)$ is scalar multiplied by time – dependent states of the beam. Here, because of the length dependency, a time – and state dependent stiffness matrix element is obtained, which avoids the usual 'length-integration' in every time step. The integration procedure is replaced by the updating procedure of the matrices $D$. The generation procedure of the actual element – and global matrices is realized by the steps

- of **defining all the matrix – products** $f_{bi}(f_a f_c^T)$, which are necessary. For this procedure only the set of the polynomial coefficients of the used shape functions are necessary. Knowing all arising vector function combinations, this procedures work automatically. Here Hermite polynomials are used and the resulting coefficients are calculated one time automatically by using a MATLAB – program [1]. In general all shape functions can be used. Therefore only the polynomial coefficient have to be changed.
- of **calculating all** $D(x)$dx – **terms**, using the previously calculated matrix coefficients multiplied by the actual length and later also with the actual state. This step has to be done for all states of the

modeled knots. For every matrix element position the related elements have to be collected, so that the output of this subprogram gives actual matrix coefficients of the related beam element energy expression.

- of **summarizing all beam elements** by using the knowledge about the structure. This step gives the global matrices of the beam, which represent the actual coefficients of the equations of motion.

All developed matrix-elements are developed and given in [1].

The virtual work of a mass element described by the vector $r_{OP}^{(I)^T}$ (Eq. 1) is

$$\delta \mathrm{W_m} = \int\int\int \rho\left(\delta r_{OP}^{(I)^T} \ddot{r}_{OP}^{(I)}\right)\mathrm{dm} \tag{16}$$

applying the notation given in (1). Using the above mentioned variations and assumptions, and assuming addionally that the material is homogenius with $\rho$ = const., Eq. (16) results to

$$\begin{aligned}\delta \mathrm{W_m} = -\varrho \int\int\int \Big(&\delta r_{OG}^{(I)^T} + (\delta l^{(G)^T} + \delta s^{(G)^T} \\ &+ \delta u^{(G)^T} + \delta t^{(E)^T} \hat{T}^T) T^T\Big) \bullet \\ \Big(&\ddot{r}_{OG}^{(T)} + \ddot{T}(l^{(G)} + s^{(G)} + u^{(G)} + \hat{T}t^{(E)}) \\ &+ 2\dot{T}(\dot{l}^{(G)} + \dot{s}^{(G)} + \dot{u}^{(G)} + \dot{\hat{T}}t^{(E)}) \\ &+ T(\ddot{l}^{(G)} + \ddot{s}^{(G)} + \ddot{u}^{(G)} + \ddot{\hat{T}}t^{(G)}\Big)\mathrm{dV}.\end{aligned} \tag{17}$$

To make the previous equation integrable and applicable for the equations generation procedure with respect to the variable length and to apply the introduced separation procedure Eq. (7, 8), terms of the $\hat{T}t^{(E)}$, $\dot{\hat{T}}t^{(E)}$, $\ddot{\hat{T}}t^{(E)}$ with the vector $t^{(E)} = [0\ \mathrm{y}\ \mathrm{z}]^T$, describing the mass points of the finite stiff disc (Fig. 1), can be reformulated using only the geometry dependent matrices

$$N_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad N_{1a} = \begin{pmatrix} 0 & -\mathrm{z} & \mathrm{y} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad N_{1b} = \begin{pmatrix} 0 & 0 & 0 \\ \mathrm{z} & 0 & 0 \\ -\mathrm{y} & 0 & 0 \end{pmatrix},$$

$$N_{2a} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{y}{2} \\ 0 & -\frac{y}{2} & 0 \end{pmatrix}, \quad N_{2b} = \begin{pmatrix} 0 & 0 & 0 \\ -\frac{y}{2} & 0 & 0 \\ -\frac{y}{2} & 0 & 0 \end{pmatrix}, \quad N_3 = \begin{pmatrix} z & 0 & -y \\ 0 & 0 & 0 \\ 0 & y & 0 \end{pmatrix} \text{ as} \tag{18}$$

$$\hat{T}t^{(E)} = N_0 u' + N_{1a} u' + N_{1b}\beta_\alpha + N_{2a} u'^2 + N_{2b}\beta_\alpha^2 + N_3 u'_m fg. \tag{19}$$

Here y, z denote the coordinates of the considered beam element of the ($P$)-coordinate system (Fig. 1). The expression (19) contains beside the geometry matrices the linear vector of the elastic variables $u$, a quadratic expression $u^2$, and also a mixed expression $u_m$ of the elements. The virtual expression of the kinematic part of Eq. (19) follows using the calculation rules as

$$\begin{aligned} \delta t^{(E)T} \hat{T}^T = {} & \delta t^T N_0^T + \delta u'^T N_{1a}^T + \delta\beta_\alpha^T N_{1b}^T \\ & + \delta u'^{2^T} N_{2a}^T + \delta\beta_\alpha^{2^T} N_{2b}^T + \delta u_m'^T N_3^T. \end{aligned} \tag{20}$$

With the knowledge of the geometry of the beam, 2 of the 3 integrations of (17) can be done in advance using the introduced 'geometric'-matrices (18) as

$$\iint N_i \mathrm{dA} \quad \text{or} \quad \iint N_i \omega^* N_j \mathrm{dA}. \tag{21}$$

Using in general a fully matrix $\omega^*$, assuming

- axes-symmetric cross sections,
- that the neutral axis coincides with the center-point of the cross section area, and
- the choice of a principal axes coordinates system for the (P)-coordinate system of the disc

the square-integration $I_{y1} = \int_y\int_z z\,dzdy$, $I_{z1} = \int_y\int_z y\,dzdy$ $I_y = I_{y2} = \int_y\int_z z^2\,dzdy$ $I_z = I_{z2} = \int_y\int_z y^2\,dzdy$ $I_{yz} = \int_y\int_z yz\,dzdy$ $I_r = I_{r2} = \int_y\int_z z^2 + y^2\,dzdy = I_{z2} + I_{y2}$ are defined as usual, which allows solving the integrations in the way, that only the rotation-dependent parts of $\omega^*$, have to be multiplied in the actualization step of the modeling (or simulation) procedure with precalculated matrix elements. For the length variable beam the length integration can be solved by the same procedure using subprograms mentioned before. For the virtual work of the inertia terms it follows formally a very complex series of terms.

The virtual work of the external forces and moments $\delta W_a$ consider beside the gravitation influence, which is given in detail in [1] and not considered here, also the reaction forces of the changing mass of the element, which appear, due to the length variable distance of the beam knots, only in axial direction. The complete idea behind the strategy of handling variables mass systems is given in [14].

In this way for all coupling terms of the elastic coordinates and combinations of shape function and node variables specific products arise, which are implemented by the introduced systematic scheme. Using this length-dependent matrices as a library; stiffness, damping and mass matrices for a beam element can be generated in every time step of numerical calculations. Organized in modules, the whole procedure is done automatically, *cf.* Figure 2. The user of the modelling module defines only the nature of the model (linear, nonlinear with some coupling terms, modeling with all coupling terms, *cf.* [1], the configuration of the length variable beams, and the detailed

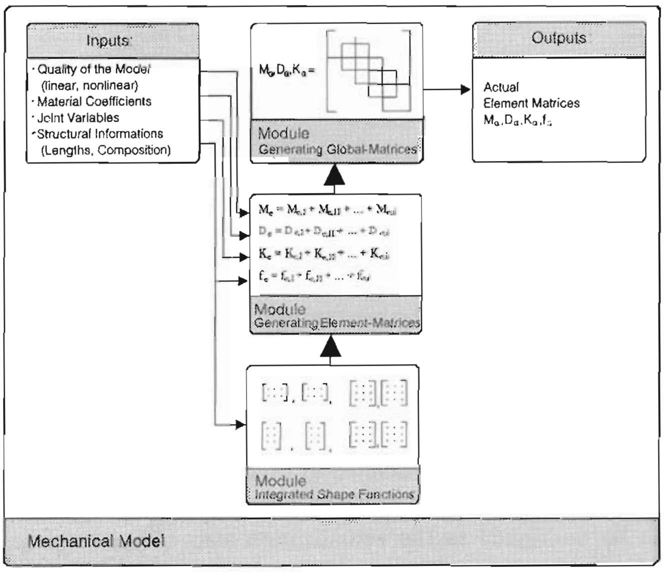


FIGURE 2 Modeling procedure.

geometry of each beam element. The lowest level of the modeling procedure generates length depending matrices by scalar multiplying of given matrices. This matrices depend on the used shape functions implemented as a set of parameters. The second level contains the construction of beam-wise element matrices of second order depending on the actual states and on the given reference motion. The third level generates the global matrices from the element matrices, which leads, in the case of a beam-like structure, to partial adding of matrices in the usual linear manner illustrated in Figure 2.

In the case of length-constant beam the lowest level procedures are done only ones, then, due to the reference motion dependency, the matrices of the 2nd and 3rd level has to be updated to get the higher order model representation.

In the case of a linear model no multiplication with actual system states are needed: the generated matrices are constant, state independent and well-known.

The global system matrices are found by adding the specified element matrices in the usual way, resulting to the describtion for the global system

$$M_g\ddot{u}_g + D_g\dot{u}_g + (K_g + K_{gg})u_g = f_{g1} + f_{g2} + f_{gg}, \tag{22}$$

with the global mass matrix $M_g(u, l_{ei})$, the global damping matrix $D_g(u, \omega, l_{ei})$, the global stiffness matrix $K_g(u, \omega, \dot{\omega}, l_{ei})$, and vectors $f_{g1}, f_{g2}, (u, l_{lei})$ resulting from translational and rotational motion of the joint, and the matrices of gravitational effects $f_{gg}\,(K_{gg})$ [1]. Here $l_{lei}$ denotes the dependency of the actual length and position of each beam element.

## 4. DYNAMIC STUDIES

The quality of the model will be shown by simulation of a very flexible telescopic robot arm of three elements for space applications. The geometric parameters of the system are given in [1]. The whole length of the beam is 11.8 m, the weight is 48 kg. The telescopic robot is modeled using 5 beam elements. Results of using two different models are shown: a) the linear model by using the state independent matrices and b) the nonlinear model of 3rd order.

**Example – Planar Rotatory Maneuver**

Using a rotational planar maneuver for the robot described in [1] with the maneuver time $T_0 = 15$ sec. and the stationary rotatory speed $\Omega = 2$ rad/sec, starting with zero initial conditions, the dynamic excitation leads to elastic deformations (Fig. 3). Here the length constant case is considered in detail.

In correspondence with results known from the literature [7, 8, 9, 12] a negative axial displacement appears in the nonlinear case. Furthermore, it is known that the stationary vibrations using the nonlinear modeling are larger than in the linear case. In contrast to the known literatur it should be mentioned that the bending vibration for the nonlinear model is larger than in the linear case. In the literature [10, 12] it is mentioned that so-called stiffning terms are neglected in some program codes, which stiffs the system, so for the rotational maneuver smaller vibration should be expected. The analysis of this effects in [1], leads to the conclusion that these kind of modeling is useful for very elastic structures, where terms of 3rd order play an important rule not only for stiffening but also for weakening effects, as shown in the example.

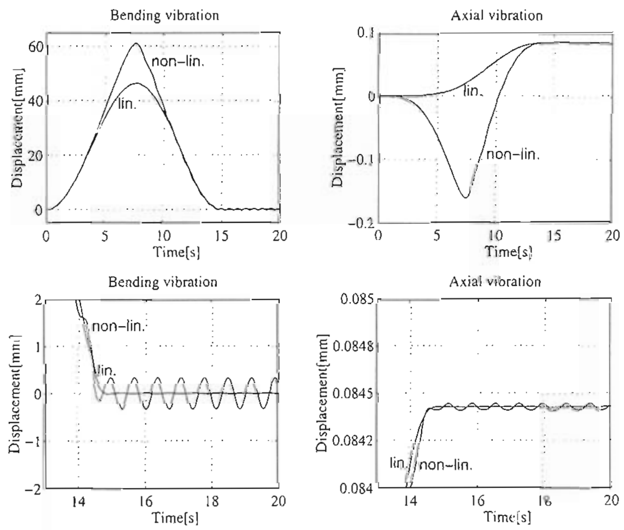


FIGURE 3 Axial-and bending vibrations for a rotational maneuver.

Other examples are given in [1]. They defines the relevant terms and show that this terms are responsible for the weakning effects for weak structures. In [15] a simulation for the well known NASA minimast is given using the introduced modeling technique. The results show that for this type of structure (this configuration of material and geometry parameters) the mentioned terms are not relevant.

As a conclusion it can be stated, that for weak structures weakning effects can play an important rule. This effects only can be described by coupling terms of higher order. This terms can not be integrated by known procedures. Here a suggestion is given for consideration of this type and order of coupling terms combined with a suggestion for handling of the terms.

## 5. CONCLUSIONS

This contribution investigates the automatic generation of the equations of motion of the nonlinear beam to describe the related dynamical behavior. Using the nonlinear beam kinematics up to terms of 2nd order, developing a new technique handling the quadratic expressions, which includes model-updating of the state- and time-dependent element matrices, time consuming integrations for the length variation of the beam can be avoided and are replaced by matrix operations which works automatically and time efficiently. The scheme of (linear) mass, damping and stiffnes matrices can also be used to handle nonlinear effects of higher order. Simulations show the importance of these stiffning and weakning effects.

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