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State estimation of dynamical systems with nonlinearities by using proportional-integral observer

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In this paper the application of a PI observer technique to dynamical systems with nonlinearities is proposed. The PI observer has two feedback loops, a proportional loop and an integral loop of the estimation error. In this way the PI observer combines the structures of the practical orientated nonlinearity observer developed by the third author and the classical Luenberger observer. The structure and the estimation performance of the PI observer are discussed and analysed. The results show that the PI observer can estimate the states not only of linear systems, but also, more significantly, of systems with any arbitrary external input which appear as unknown input, nonlinearity or unmodelled dynamics. It is shown that the PI observer works with weak assumptions, which can be fulfilled by many classes of systems to be observed. Owing to the weak assumptions it can improve many observer-based technical solutions as diagnosis or control based on observers. In the paper the conditions are given and proved. The design method is declared and carried out with illustrative examples of a linear system and of a nonlinear system of a link manipulator with flexible joints. The results are good and they show the efficiency of the PI observer. In the case of nonlinear systems the advantages of 'robustness' and the model independency of the proposed observer scheme can be shown clearly.

1. Introduction

For more than 20 years Luenberger observers have been known and used very intensively (Luenberger 1971) as classical observers. Based on a linear, time-invariant and deterministic description of the plant, the observer can reconstruct unmeasurable states using measurements of outputs. This permits the employment of the observer scheme to dynamic systems of the form

$$\dot{x} = \mathbf{A}x + \mathbf{B}u, \quad y = \mathbf{C}x \tag{1}$$

with the state vector x of order n, the vector of measurements y of order l, and the known input vector u of order m. The system matrix $\mathbf{A}$, the input matrix $\mathbf{B}$ and the output matrix $\mathbf{C}$ are of appropriate dimensions. The Luenberger observer has many successful applications in the state reconstruction of linear time-invariant systems. However, it is not directly applicable to nonlinear systems or systems with unknown inputs. In the last decade, therefore, several observer techniques have been developed for such systems like unknown-input observers (UIOs) (Hou and Müller 1992, Yang and Wilde 1988) or nonlinear observers (NLOs) (Misawa and Hedrick 1989, Nijmeijer and van der Schaft 1991).

The UIO requires that the outputs of the system contain the complete information about the unknown inputs. In most practical applications this condition is not fulfilled. Nonlinear observer techniques are often very restricted in the application to special classes of nonlinearities. In this way knowledge of the nonlinearity is assumed. At the same time the design and the implementation of NLOs is very

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complex and difficult, e.g. the Lie-algebraic method (Nijmeijer and van der Schaft 1991) requires the integration of some coupled partial differential equations. Then, with respect to practical uses it would be helpful if there were methods which could be used more generally and possibly without knowledge of the exact structure and parameters of the system.

In control theory it is well known that for controlling unknown systems or for achieving steady-state accuracy, integral terms are useful. Because of the duality observability and controllability it will be an extension of the Luenberger observer, which uses only the current information of the estimation error, to extend the observer using also information about the past to obtain the same advantages. Since this type of observer uses proportional and integral information, it is called a PI observer. It is known from literature that the PI observer design is useful for linear systems with constant disturbances (Anderson and Moore 1989). This kind of observer scheme was first proposed by Wojciechowski (1978) for SISO-linear time-invariant systems. Further development was made by Kaczorek (1979) and Shafai (1985) for multivariable systems, with the aim of improving their robustness against parameter variations and step disturbances. Beale and Shafai (1989) presented a special methodology in which the authors use the additional degrees of freedom provided by the integration path to design robust control systems against uncertainties. In this paper the PI observer is developed from another point of view. Continuing the ideas of Johnson (1976), who introduced linear models for disturbances acting upon linear systems, and Müller and Lückel (1977) who gave the conditions and proofs for modelling disturbances as linear models also acting upon linear systems, this paper deals with the idea of constructing a disturbance model for more general use, especially for the practical case in which no information about the disturbance is available. The following aspects are the points of consideration: the usual Luenberger observer fails if the system (1) is only roughly known or/and there are additional unknown inputs caused by nonlinearities. Using PI observer techniques this disadvantage can be compensated for, but only for piecewise constant disturbances. If the unknown input is caused by modelling errors or nonlinearities this assumption is not fulfilled. Because of the structure of the proposed PI observer, the estimations of both the state and nonlinearities are obtained. This will be an advantage for several other cases of nonlinearities (Söffker *et al.* 1993 b).

The paper is organized as follows. Section 2 introduces the idea and structure of the developed PI observer. Furthermore, an interpretation on the means of a generalization of the so-called nonlinearity observer of Müller (1990) is given. In § 3 the estimation behaviour is analysed and the design procedure is declared. Examples are given in § 4, where the developed scheme is used to observe linear and nonlinear systems.

2. Proportional integral observer

The linear model (1) can be used to describe a class of dynamical systems with an acceptable accuracy. However, there are many nonlinear systems that cannot be modelled as (1) owing to inherent nonlinear effects. Therefore, a more general description of such systems is

$$\left.\begin{aligned} \dot{x} &= \mathbf{A}x + \mathbf{B}u + \mathbf{N}f(x, u, t) \\ y &= \mathbf{C}x \end{aligned}\right\} \qquad (2)$$

In (2) the vector function $f(x, u, t)$ of order r describes the nonlinearities, unknown inputs and unmodelled dynamics of the plant and may be a nonlinear function of states, control inputs and time. The matrix $\mathbf{N}$ is the corresponding distribution matrix. Without loss of generality it is assumed that the matrix $\mathbf{N}$ has full column rank, and the matrix $\mathbf{C}$ has full row rank.

Based on the well-known method of disturbance rejection control (DRC) (Johnson 1976, Davison 1972, Müller and Lückel 1977), several successful practical and theoretical applications concerning machine diagnosis (Söffker *et al.* 1993 a) and also observer-based control have been studied by Ackermann (1989) and Neumann and Moritz (1990). In all of these cases an approximation

$$f \approx \mathbf{H}v \tag{3}$$

of the vector of nonlinearities f (friction torques, forces caused by the crack) was used. In these works the nonlinearities have been considered as general disturbances and referred to as unknown inputs. In the theory of DRC the linear time-invariant system with the unknown inputs $\mathbf{N}f$ caused by nonlinearities, unknown inputs or unmodelled dynamics is extended by a linear exosystem, i.e. by a linear dynamical model of these inputs:

$$\begin{bmatrix} \dot{x} \\ \dot{\boldsymbol{v}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{NH} \\ 0 & \mathbf{F} \end{bmatrix} \begin{bmatrix} x \\ \boldsymbol{v} \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix} u \tag{4}$$

$$y = [\mathbf{C} \quad 0] \begin{bmatrix} x \\ \boldsymbol{v} \end{bmatrix} \tag{5}$$

Here the matrix $\mathbf{N}$ relates the fictitious approximations $\mathbf{H}\boldsymbol{v}$ of the unknown inputs f to the states where they appear. The signal characteristic of these inputs will be approximated by a linear dynamical system with the system matrix $\mathbf{F}$. Using (4) and (5) an extended observer can be designed, so the estimate $\hat{\boldsymbol{v}}$ of $\boldsymbol{v}$ represents the approximation of the disturbances.

In the applications of Söffker *et al.* (1993 a) and Ackermann (1989) it is noticed that using

$$F = 0 \tag{6}$$

leads to a very good reconstruction of the diagnosted nonlinearity (Söffker *et al.* 1993 a) and, in combination with control, to very good compensation results. This means that without exact knowledge about the dynamical behaviour of the unknown inputs f, a very general approach is possible by assuming the disturbance as piecewise constant.

Although this approach has been successfully applied to many problems, as mentioned above, the interpretation of the interaction among the fictitious model $\mathbf{F}$ of the exosystem of (4), the designed extended system as a base for constructing observers and the observer itself it not yet exactly clear. Söffker and Müller (1993) presented some new justifying arguments which point at a new interpretation.

In this way the synthetic procedure of modelling disturbances by fictitious models, shown before, will be seen as a natural comprehensible extension of the well-known Luenberger observer as a PI observer. Figure 1 depicts the structure of an observer using the information proportional and integral to the estimation error.

Here a second loop with two gain matrices $\mathbf{L}_2$ and $\mathbf{L}_3$ and integrator is used additionally. It is obvious that the proportional and integral feedback loops of the developed observer structure correspond to the well known PI control structure. Now the problem is how to determine the matrices $\mathbf{L}_1$, $\mathbf{L}_2$ and $\mathbf{L}_3$ such that the corresponding PI observer has the performance defined by Luenberger.

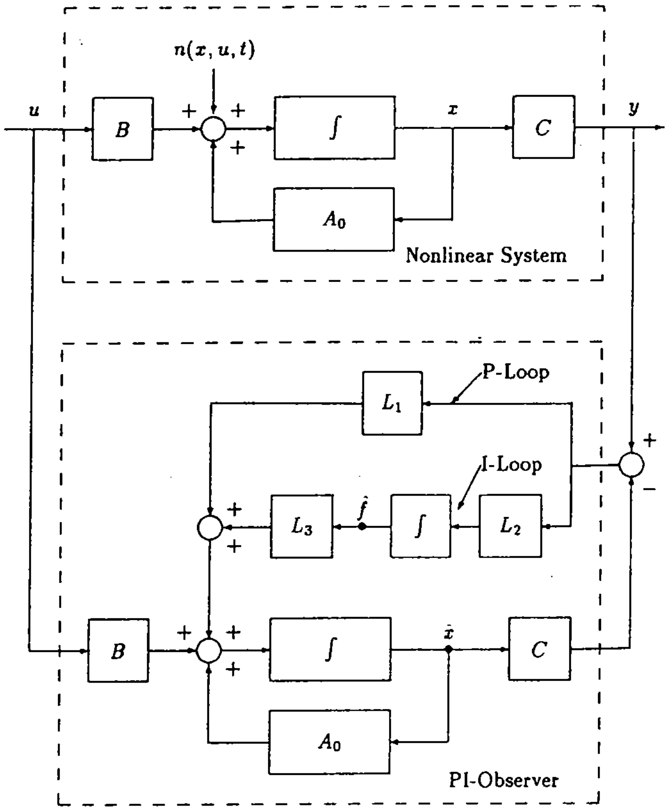

Figure 1. Structure of PI observer.

3. Estimation performance analysis

It was shown in the preceding section that the PI observer is an extension of the Luenberger observer. The PI observer uses not only the information proportional to the estimation error, but also an integral of the estimation error. From this it could be predicted that the PI observer will be of a better estimation performance and of some special properties which will be discussed below.

From the structure of the PI observer depicted in Fig. 1, it follows that the dynamics of a PI observer are described by

$$\left.\begin{aligned} \dot{\hat{x}} &= \mathbf{A}\hat{x} + \mathbf{L}_3\hat{f} + \mathbf{B}u + \mathbf{L}_1(y - \hat{y}) \\ \dot{\hat{f}} &= \mathbf{L}_2(y - \hat{y}) \end{aligned}\right\} \tag{7}$$

where $\hat{y} = \mathbf{C}\hat{x}$. Writing (7) in a matrix form gives

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{f}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{L}_3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{f} \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix} u + \begin{bmatrix} \mathbf{L}_1 \\ \mathbf{L}_2 \end{bmatrix} (y - \hat{y}) \tag{8}$$

or

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{f}} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{A} - \mathbf{L}_1\mathbf{C} & \mathbf{L}_3 \\ -\mathbf{L}_2\mathbf{C} & \mathbf{0} \end{bmatrix}}_{A_e} \begin{bmatrix} \hat{x} \\ \hat{f} \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix} u + \begin{bmatrix} \mathbf{L}_1 \\ \mathbf{L}_2 \end{bmatrix} y \tag{9}$$

Now the problem is how to design the gain matrices $\mathbf{L}_1$, $\mathbf{L}_2$ and $\mathbf{L}_3$, such that the observer can approximately estimate the states x of the plant.

Define the estimation error as $e(t) = \hat{x}(t) - x(t)$. Then from (1), (2) and (3) we have that

$$\begin{bmatrix} \dot{e} \\ \dot{\hat{f}} \end{bmatrix} = \mathbf{A}_e \begin{bmatrix} e \\ \hat{f} \end{bmatrix} \tag{10}$$

in the case of system (1), or

$$\begin{bmatrix} \dot{e} \\ \dot{\hat{f}} \end{bmatrix} = \mathbf{A}_e \begin{bmatrix} e \\ \hat{f} \end{bmatrix} - \begin{bmatrix} \mathbf{N} \\ 0 \end{bmatrix} f \tag{11}$$

in the case of system (2) with unknown inputs or nonlinearities. From (10) the following result can be obtained.

Theorem 1: *If the pair* (**A**, **C**) *is observable, then there exists a PI observer with any dynamics for the system* (1), *such that*

$$\lim_{t \to \infty} [\hat{x}(t) - x(t)] = 0$$

for any initial states $x(0)$, $\hat{x}(0)$ *and* $\hat{f}(0)$.

Proof: From the dynamics (9) of the PI observer it can be seen that the dynamics or poles of the PI observer (9) can be arbitrarily assigned if and only if the matrix pair

$$\left(\begin{bmatrix} \mathbf{A} & \mathbf{L}_3 \\ \mathbf{0} & 0 \end{bmatrix}, [\mathbf{C} \quad 0] \right)$$

is observable, i.e.

$$\operatorname{rank} \left\{ \begin{bmatrix} s\mathbf{I} - \mathbf{A} & -\mathbf{L}_3 \\ 0 & s\mathbf{I} \\ \mathbf{C} & \mathbf{0} \end{bmatrix} \right\} = n + \dim(\hat{f}) \tag{12}$$

holds for all $s \in \mathbb{C}$. Furthermore, the condition (12) is equivalent to

$$\operatorname{rank} \left\{ \begin{bmatrix} \mathbf{A} & \mathbf{L}_3 \\ 0 & 0 \end{bmatrix} \right\} = n + \dim(\hat{f}) \tag{13}$$

when $s = 0$ and

$$\operatorname{rank} \left\{ \begin{bmatrix} s\mathbf{I} - \mathbf{A} \\ \mathbf{C} \end{bmatrix} \right\} = n \tag{14}$$

when $s \neq 0$. The condition (13) implies that the dimension of integrator must be less than or equal to that of the outputs. Since the matrix $\mathbf{L}_3$ may be arbitrarily selected, rank condition (13) holds if and only if

$$\operatorname{rank} \left\{ \begin{bmatrix} \mathbf{A} \\ \mathbf{C} \end{bmatrix} \right\} = n \tag{15}$$

Combining the conditions (14) and (15) leads to

$$\operatorname{rank} \left\{ \begin{bmatrix} s\mathbf{I} - \mathbf{A} \\ \mathbf{C} \end{bmatrix} \right\} = n \tag{15}$$

for all $s \in \mathbb{C}$, i.e. (**A**, **C**) is observable.

Additionally, when the condition (12) is satisfied the eigenvalues of the matrix $\mathbf{A}_e$ can be arbitrarily placed by means of the matrices $\mathbf{L}_1$ and $\mathbf{L}_2$, i.e. the eigenvalues of $\mathbf{A}_e$ can be placed at any location in the left-half complex plane. This guarantees that

$$\lim_{t \to \infty} e(t) = 0 \quad \text{and} \quad \lim_{t \to \infty} \hat{f}(t) = 0 \tag{17}$$

for any $e(0)$ and $\hat{f}(0)$. This completes the proof of Theorem 1. □

A main motivation to study the PI observer is to reconstruct the states of the system (2) with nonlinearities. The following two theorems give the results in case of the system (2).

Theorem 2: *Assume that*

$$\lim_{t \to \infty} f(x, u, t)$$

exists. Then there exists a PI observer with any dynamics for the system (2), *such that*

$$\lim_{t \to \infty} [\hat{x}(t) - x(t)] = 0$$

for any initial states $x(0)$, $\hat{x}(0)$ *and* $\hat{f}(0)$ *if* $(\mathbf{A}, \mathbf{C})$ *is observable and*

$$\operatorname{rank}\left\{\begin{bmatrix} \mathbf{A} & \mathbf{N} \\ \mathbf{C} & 0 \end{bmatrix}\right\} = n + r \tag{18}$$

Proof: We prove Theorem 2 following the construction method. Let $L_3 = N$. Then the dynamics (11) of the estimation error of PI observer (9) become

$$\begin{bmatrix} \dot{e} \\ \dot{\hat{f}} \end{bmatrix} = \mathbf{A}_e \begin{bmatrix} e \\ \hat{f} \end{bmatrix} - \begin{bmatrix} 0 \\ \mathbf{I} \end{bmatrix} f \tag{19}$$

where

$$\mathbf{A}_e = \begin{bmatrix} \mathbf{A} - \mathbf{L}_1\mathbf{C} & \mathbf{N} \\ -\mathbf{L}_2\mathbf{C} & 0 \end{bmatrix}$$

Similarly to the proof of Theorem 1, the eigenvalues of the matrix $\mathbf{A}^e$ can be arbitrarily assigned by the matrices $\mathbf{L}_1$ and $\mathbf{L}_2$ if and only if the matrix pair

$$\left(\begin{bmatrix} \mathbf{A} & \mathbf{N} \\ 0 & 0 \end{bmatrix}, [\mathbf{C} \quad 0]\right)$$

is observable, i.e.

$$\operatorname{rank}\left\{\begin{bmatrix} s\mathbf{I} - \mathbf{A} & -\mathbf{N} \\ 0 & s\mathbf{I} \\ \mathbf{C} & \mathbf{0} \end{bmatrix}\right\} = n + r \tag{20}$$

holds for all $s \in \mathbb{C}$. This condition is equivalent to

$$\operatorname{rank}\left\{\begin{bmatrix} \mathbf{A} & \mathbf{N} \\ \mathbf{C} & 0 \end{bmatrix}\right\} = n + r \tag{21}$$

when $s = 0$ and

$$\operatorname{rank}\left\{\begin{bmatrix} s\mathbf{I} - \mathbf{A} \\ \mathbf{C} \end{bmatrix}\right\} = n \tag{22}$$

when $s \neq 0$. This implies that the conditions in Theorem 2 the dynamics of the PI observer (9) for the system (2) can be arbitrarily assigned. Therefore, the eigenvalues of $\mathbf{A}_e$ can be arbitrarily placed at any locations in the left-half complex plane when the conditions in Theorem 2 are satisfied. This means that the dynamics (19) are stabilizable by means of the matrices $\mathbf{L}_1$ and $\mathbf{L}_2$. When the dynamics (19) is asymptotically stable, its solution will converge to the equilibrium. Then from (19) it can be easily seen that

$$\lim_{t \to \infty} \begin{bmatrix} e(t) \\ \hat{f}(t) \end{bmatrix} = \begin{bmatrix} 0 \\ \lim_{t \to \infty} f(x, u, t) \end{bmatrix} \tag{23}$$

Theorem 2 has been proved.

At the same time, the equality (23) also shows that

$$\lim_{t \to \infty} [\hat{f}(t) - f(x, u, t)] = 0$$

This means that $\hat{f}$ is the estimation of f.

In addition, from the proof procedure of Theorem 2 we can see that the convergence rate of the estimation error $e(t)$ to zero is dependent on the eigenvalues of $\mathbf{A}_e$ and the convergence rate of $f(x, u, t)$ to its steady state. However, in the constant case that $\dot{f} = 0$ the convergence speed of the estimation error $e(t)$ to zero is determined only by the eigenvalues of $\mathbf{A}_e$. This may also be seen from the following equation. Let $e_f = \hat{f} - f$. Then when $\dot{f} = 0$, from (19) it follows that

$$\begin{bmatrix} \dot{e} \\ \dot{e}_f \end{bmatrix} = \mathbf{A}_e \begin{bmatrix} e \\ e_f \end{bmatrix} \tag{24}$$

Equation (24) shows that

$$\lim_{t \to \infty} \begin{bmatrix} e(t) \\ e_f(t) \end{bmatrix} = 0$$

for any $e(0)$ and $e_f(0)$ if the eigenvalues of $\mathbf{A}_e$ are in the left-half complex plane.

Theorem 3: *Assume that $f(x, u, t)$ is bounded. Then there exists a high-gain PI observer for system* (2) *such that $\hat{x}(t) - x(t) \to 0$ $(t > 0)$ for any initial states $x(0)$, $\hat{x}(0)$ and $\hat{f}(0)$ if*

(*a*) $(\mathbf{A}, \mathbf{C})$ *is observable*

(*b*) $\operatorname{rank}\left\{\begin{bmatrix} \mathbf{A} & \mathbf{N} \\ \mathbf{C} & 0 \end{bmatrix}\right\} = n + r$

(*c*) $\mathbf{C}\mathbf{A}^i\mathbf{N} = 0$, $i = 0, 1, \ldots, k-2$, *where k is the observability index of* $(\mathbf{A}, \mathbf{C})$, *i.e. the least integer such that*

$$\operatorname{rank}\left\{\begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \vdots \\ \mathbf{CA}^{k-1} \end{bmatrix}\right\} = n \tag{25}$$

Proof: Let $\mathbf{L}_3 = \mathbf{N}$. Then, analogously with the proof of Theorem 2, it is easily verified that the dynamics of PI observer (9) for the system (2) can be arbitrarily assigned by means of the matrices $\mathbf{L}_1$ and $\mathbf{L}_2$ if the conditions (*a*) and (*b*) in Theorem 3 are satisfied.

Under the selection of $\mathbf{L}_3$ the dynamics (11) of the estimation error become (19). When $\mathbf{A}_e$ is stable, the solution to (22) will be also bounded if $f(x, u, t)$ is bounded. Let

$$\begin{bmatrix} \mathbf{L}_1 \\ \mathbf{L}_2 \end{bmatrix} = \rho \begin{bmatrix} \tilde{\mathbf{L}}_1 \\ \tilde{\mathbf{L}}_2 \end{bmatrix}$$

Then (19) may be written as

$$\frac{1}{\rho}\begin{bmatrix} \dot{e} \\ \dot{\hat{f}} \end{bmatrix} = \frac{1}{\rho}\begin{bmatrix} \mathbf{A} & \mathbf{N} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} e \\ \hat{f} \end{bmatrix} - \begin{bmatrix} \tilde{\mathbf{L}}_1 \\ \tilde{\mathbf{L}}_2 \end{bmatrix}\mathbf{C}e - \frac{1}{\rho}\begin{bmatrix} \mathbf{N} \\ 0 \end{bmatrix} f \tag{26}$$

From (26) it follows that

$$\mathbf{C}e = 0 \tag{37}$$

for $\rho \to \infty$. Differentiating (27) and using (11) gives

$$\mathbf{C}\dot{e} = \mathbf{C}(\mathbf{A} - \mathbf{L}_1\mathbf{C})e + \mathbf{C}\mathbf{N}(\hat{f} - f) \tag{28}$$

From the condition (*c*) and (27) we have

$$\mathbf{C}\mathbf{A}e = 0 \tag{29}$$

In the same way under condition (*c*) we can obtain

$$\mathbf{C}\mathbf{A}^i e = 0, \quad i = 0, 1, \ldots, k-1 \tag{30}$$

Then from (27), (29) and (30) it follows that

$$e = 0 \tag{31}$$

owing to the rank condition (25). Substituting (31) into (11) gives

$$\hat{f} - f = 0 \tag{32}$$

because of the full-column rank of $\mathbf{N}$. Equations (31) and (32) mean that the estimates $\hat{x}$ and $\hat{f}$ of the PI observer (9) converge to the states x and the unknown inputs f of the system (2) when ρ tends to infinity. This shows that $\hat{x}$ and $\hat{f}$ may approximate x and f in the case of high gains. This completes the proof of the theorem. The estimation performance of the PI observer has been analysed above. The results show that the PI observer can estimate the states not only of the general linear systems, but also, and more significantly, of systems with unknown inputs. At the same time, the above analysis procedure provides the design method of the PI observer.

□

From the proofs of Theorem 1 and Theorem 2 one can see that it is easy to design the PI observers for general linear systems and systems with convergent unknown inputs by using output-feedback methods, e.g. pole-assignment and LQR methods. Although the design of PI observers for systems with bounded unknown inputs is similar to the design of the above observers, there may be some trouble in the computation of high gains. The way to avoid these troubles is to increase the matrix $\mathbf{L}_3$, i.e. let $\mathbf{L}_3 = p\mathbf{N}$ ($p > 1$). The reason is that the increase of $\mathbf{L}_3$ is equivalent to the increase of the gain matrix $\mathbf{L}_2$. This can be easily seen from the structure of the PI observer depicted in Fig. 1.

4. Examples

In this section two illustrative examples are presented. One is a linear example which illustrates the estimation performance of the PI observer in the case of linear systems. The other example is a single-link manipulator with a flexible joint, which illustrates that the PI observer can also be applied to the nonlinear system with uncertain parameters.

4.1. *Linear system*

Here a simple linear example given by Luenberger himself is used to show the design method and to compare the classical Luenberger observer and PI observer. Consider the system (Luenberger 1971)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \quad y = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{33}$$

The eigenvalues of the Luenberger observer are given as -3, so the observer is built up as

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{bmatrix} = \begin{bmatrix} -5 & 1 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 3 \\ 4 \end{bmatrix} y \tag{34}$$

To design the PI observer, also with all poles at -3, the matrix $\mathbf{L}_3$ is selected as

$$\mathbf{L}_3 = \begin{bmatrix} 2.5 \\ 1 \end{bmatrix} \tag{35}$$

Using the pole-placement technique the observer appears as

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \dot{f} \end{bmatrix} = \begin{bmatrix} -8{\cdot}0011 & 1 & 2{\cdot}5 \\ 0{\cdot}2873 & -1 & 1 \\ -7{\cdot}7171 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ f \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 6{\cdot}0011 \\ -0{\cdot}2873 \\ 7{\cdot}7171 \end{bmatrix} y \tag{36}$$

The initial values for the plant are $x_1(0) = -1$ and $x_2(0) = 0$. The step responses of the Luenberger observer (34) and PI observer (36) are shown in Fig. 2.

It is obvious that the estimation behaviour of a PI observer is near to that of the classical Luenberger observer. This means that the PI observer has no advantages in the case of linear system. However, some simulation examples showed that the PI observer is very robust. This feature of robustness of the PI observer will be investigated in future work of the authors.

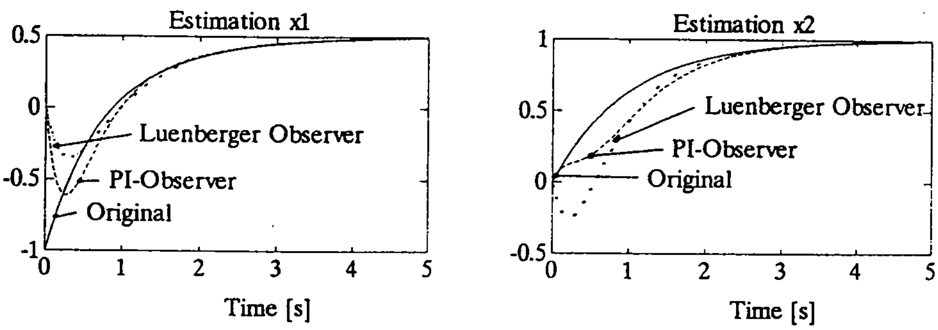

Figure 2. Comparison of (P-) Luenberger and PI observer.

4.2. *Nonlinear system*

Spong and Vidyasagar (1989) introduced the following example of a single-link manipulator with flexible joint. The state space description

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{Mgl}{I}\sin(x_1) - \frac{k}{I}(x_1 - x_3) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{k}{J}(x_1 - x_3) + \frac{1}{J}u \end{aligned}\right\} \tag{37}$$

with the mechanical parameters chosen in the example as $Mgl = 10$, $k = 100$, $I = 1$ and $J = 1$, describes the simplified nonlinear model. With feedback-linearization-control input u the system (37) is linearizable to the new coordinates y and can be rewritten as

$$\left.\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= z_4 \\ \dot{z}_4 &= v \end{aligned}\right\} \tag{38}$$

with v as the desired input. The problems with this typical feedback linearization example are: that without exact knowledge of the plant parameters the system is not exact linearizable; and the states which are needed to calculate the desired input v are not available (Spong and Vidyasargar 1989). Another more promising approach is to construct a dynamic observer to estimate the state variables $z_{3,4}$ using the available measurements

$$y = \begin{bmatrix} z_1 \\ z^2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

which is, at present, an open research problem. Considering only the mechanical system, an observer is designed to estimate all states. Using pole-assignment techniques for the desired poles $\lambda_1 = -50$, $\lambda_{2,3} = -64 \pm j44$ and $\lambda_{4,5} = -34 \pm j54$, and choosing

$$\mathbf{L}_3 = \mathbf{N} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10^5 \end{bmatrix} \tag{39}$$

the observer appears as

$$\dot{\hat{z}} = \begin{bmatrix} \mathbf{A} - \mathbf{L}_1 C & \mathbf{N} \\ -\mathbf{L}_2 \mathbf{C} & 0 \end{bmatrix} \hat{z} + \begin{bmatrix} \mathbf{L}_1 \\ \mathbf{L}_2 \end{bmatrix} y \tag{40}$$

with

$$
\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{L}_1 = \begin{bmatrix} 50{\cdot}0 & 1{\cdot}0 \\ -0{\cdot}1 & 196{\cdot}4 \\ -5{\cdot}6 & 188\,26 \\ -239{\cdot}7 & 933\,320 \end{bmatrix}, \quad \mathbf{L}_2 = [0{\cdot}12 \quad 246{\cdot}4] \qquad (41)
$$

Results for estimating z_3 and z_4 are shown in Fig. 3.

It is obvious that in the case of known parameters the estimation results are good. Here, only to compare with the signal, linearizing transformation information is used. Because of the sensitivity of the problem to the knowledge of the parameters we assumed that $I = I(1 + 0{\cdot}5 \sin(x_1))$, to illustrate the robustness of the developed observer by simulation.

In Fig. 4 it can be seen that the results are also good. This is not surprising, because the observer (40) is built without using any physical information, so nearly the same good results appear. The results can be easily improved if larger gains are selected.

5. Conclusions

In this work an approach to estimating nonlinearities, unmodelled dynamics, or in general unknown inputs has been developed. The unknown inputs are considered

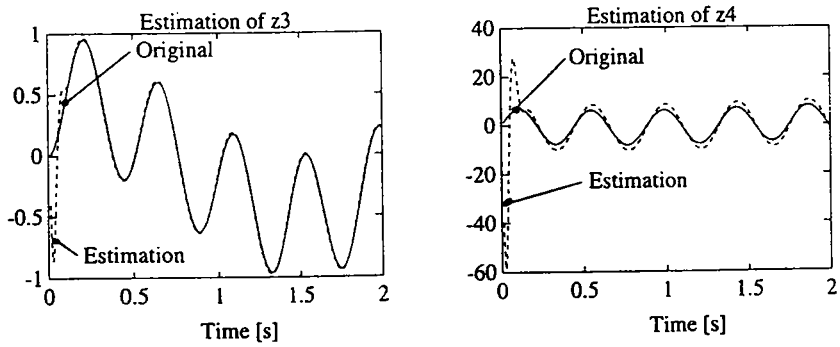

Figure 3. Estimation behaviour for the states z_3 and z_4

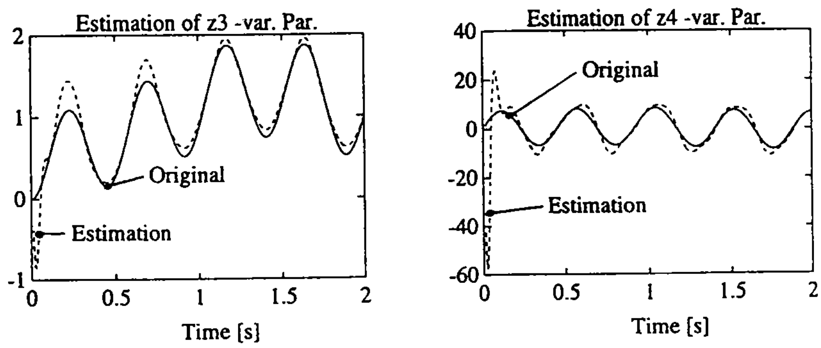

Figure 4. Estimation behaviour for the states z_3 and z_4.

as external disturbances to a linear nominal system. Using an integral of the estimation error, the proposed extended observer scheme estimates, as well as the states, also the nonlinearities. This additional information can improve many technical solutions like diagnosis or observer-based control.

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