UNIVERSITÄT DUISBURG-ESSEN
Fakultät für Ingenieurwissenschaften, Abt. Maschinenbau und Verfahrenstechnik
Lehrstuhl Steuerung, Regelung und Systemdynamik
Control Theory April 30th, 2021

90 Minutes

Reading-up-time

For reviewing purposes of the problem statements, there is a "reading-up-time" of **10 minutes** prior to the official examination time. During this period it is **not** allowed to start solving the problems. This means explicitly that during the entire "reading-up-time" no writing utensils, e.g. pen, pencil, etc. at all are allowed to be kept on the table. Furthermore the use of carried documents, e.g. books, (electronic) translater, (electronic) dictionaries, etc. is strictly forbidden. When the supervisor refers to the end of the "reading-up-time" and thus the beginning of the official examination time, you are allowed to take your writing utensils. Please **then**, begin with filling in the **complete** information on the titlepage and on page 3.

Good Luck!

LAST NAME	
FIRST NAME	
MATRIKEL-NO.	
TABLE-NO.	

Klausurunterlagen

Ich versichere hiermit, dass ich sämtliche für die Durchführung der Klausur vorgesehenen Unterlagen erhalten, und dass ich meine Arbeit ohne fremde Hilfe und ohne Verwendung unerlaubter Hilfsmittel und sonstiger unlauterer Mittel angefertigt habe. Ich weiß, dass ein Bekanntwerden solcher Umstände auch nachträglich zum Ausschluss von der Prüfung führt. Ich versichere weiter, dass ich sämtliche mir überlassenen Arbeitsunterlagen sowie meine Lösung vollständig zurück gegeben habe. Die Abgabe meiner Arbeit wurde in der Teilnehmerliste von Aufsichtsführenden schriftlich vermerkt.

Durch die Teilnahme versichere ich, dass ich prüfungsfähig bin. Bei Krankheit werde ich die Klausur vorzeitig beenden und unmittelbar eine Ärztin/einen Arzt aufsuchen.

THE ABOVE REQUIRED STATEMENTS AS WELL AS THE SIGNATURE
ARE MANDATORY AT THE BEGINNING OF THE EXAM.

Duisburg, ______________ ______________________________
(Date) (Student's signature)

Falls Klausurunterlagen vorzeitig abgegeben: ____________Uhr

Bewertungstabelle

Aufgabe 1	
Aufgabe 2	
Aufgabe 3	
Die Bewertung gem. PO in Ziffern ist der xls-Tabelle bzw. dem Papierausdruck zu entnehmen.	

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(Datum und Unterschrift 1. Prüfer, Univ.-Prof. Dr.-Ing. Dirk Söffker)

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(Datum und Unterschrift 2. Prüfer, Prof. Dr.-Ing. Mohieddine Jelali, Priv.-Doz.)

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(Datum und Unterschrift des für die Prüfung verantwortlichen Prüfers, Söffker)

Fachnote gemäß Prüfungsordnung: (alternativ: siehe xls-Tabelle bzw. beigefügter Papierausdruck)

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
1,0	1,3	1,7	2,0	2,3	2,7	3,0	3,3	3,7	4,0	5,0
sehr gut		gut			befriedigend			ausreichend		mangelhaft

Bemerkung: ____________________________________

Attention: Give your answers to ALL problems directly below the questions in the exam question sheet.

You are NOT allowed to use a pencil and also NOT red color (red color is used for corrections).

This exam is taken by me as a

☐ mandatory (Pflichtfach)
☐ elective (Wahlfach)
☐ prerequisite (Auflage)

subject (cross ONE option according to your own situation).

Maximum achievable points:	**51**
Minimum points for the grade 1.0:	**95%**
Minimum points for the grade 4.0:	**50%**

General hints:

1) For the multiple-choice and multiple-choice-similar tasks the following rules are effective:
   i) For tasks with an individual evaluation of subtasks, the following applies: Only correct answers are evaluated with the intended number of points.
   ii) The points achieved in a subtask will be summed up.
   iii) Unless explicitly stated otherwise, only one of the given solution options is correct.
   iv) If subtasks contain more than two answer options and only one solution exists: The marking of multiple answer options is interpreted as a non-response due to the not clear declaration of intention. As a result, no points can be given in this case.

2) If in the exam tasks no information is given for the valid range of numbers for time constants or masses etc.: take for time constants (in sec.), for masses (in kg) positive numbers.

3) If in the exam tasks no information is given for applying negative or positive feedback: use the usual negative feedback.

Problem 1 (13 Points)

1a) (3 × 5 × 1 Point)(deleted = 0 Points)
This task is deleted due to Covid-19-related shortening of the examination time of written exams.

1b) (13 Points)
Mark the correct solution in the following statements.

A1) (2 Points)
A typical application of the discrete-time Kalman filter is

- ◯ control of discrete systems.
- ◯ control of continuous systems.
- ◯ time-continuous estimation of the state vector.
- ◯ information fusion of the sensors.
- ☒ the so-called sensor fusion, i.e. the increase in quality of measurement information when other measurement information is omitted/deteriorated.

☐

A2) (11 Points)

The following tasks are based on the scheme below (Fig. 1.1) and the corresponding designations.

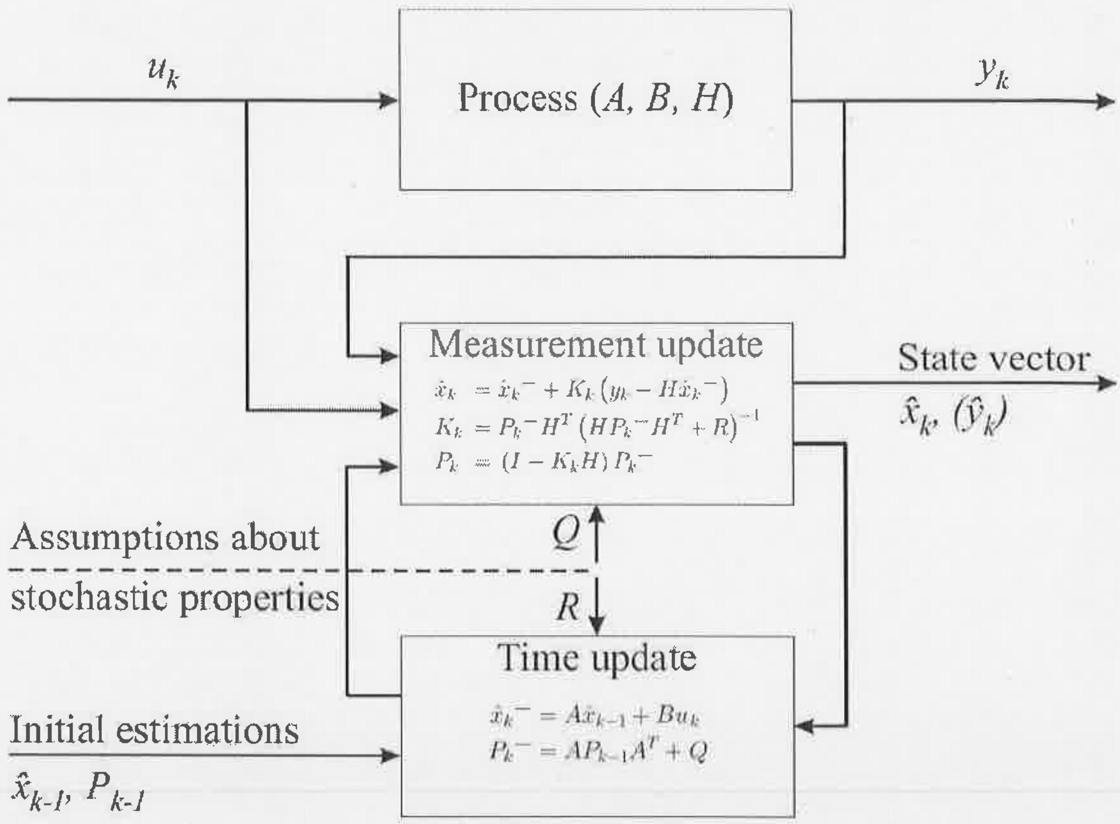

Figure 1.1: Kalman Filter for state reconstruction and process monitoring

A2-1) (2 Points)
The Kalman Filter

- ○ determines the state vector trajectory once in two steps (predictor, corrector).
- ⊗ determines the state vector continuously (time discrete) in two steps (predictor, corrector) .
- ○ determines once the assumptions about the stochastic properties.
- ○ continuously determines the assumptions for the stochastic properties.

A2-2) (2 Points)
The Kalman matrix/gain matrix of the Kalman filter is determined as

$$= \boxed{K_k = P_k^- H^T (H P_k^- H^T + R)^{-1}}$$

A2-3) (2 Points)
The matrices Q and R are

- [x] assumed/guessed or continuously determined.
- [ ] not really needed, one could just assume them as I.
- [ ] needed. If one does not know anything, they are set to 0.
- [ ] determined in advance with the help of further formulas.

A2-4) (2 Points)
The state vector $x(k)$ is algorithmically determined

- [ ] in one step.
- [x] in two steps.
- [ ] iterative by step size control.
- [ ] manipulative by human intervention.

A2-5) (1 Point)
Kalman filters (in comparison to observers) are numerically/computationally

- [x] easier
- [ ] more difficult
- [ ] equivalent

to implement in the concrete realization/application.

A2-6) (2 Points)
In a project, the demand is to minimize the number of sensors by using the Kalman filter, concretely not measuring the output of the system anymore. What does this mean concretely for the state estimation? How does it accordingly behave by the application of Luenberger observers?

Estimation of the state without having measurement is not possible.

None of the approaches can be used in this case.

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Problem 2 (20 Points)
The dynamics of the positioning system shown in Fig. 2.1 should be examined.

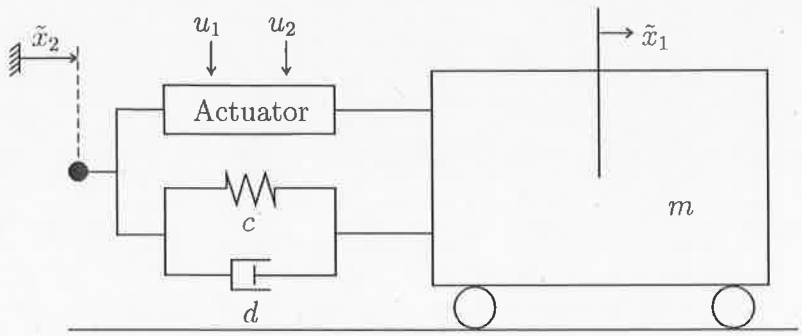


Figure 2.1: Positioning system

The mechanical part is described by

$$m\ddot{\tilde{x}}_1 + d(\dot{\tilde{x}}_1 - \dot{\tilde{x}}_2) + c\tilde{x}_1 = 0$$

with $\tilde{x}_2 = 0$.
After rearranging the description into relative coordinates, it results in

$$\ddot{x}_1 = -\frac{d}{m}\dot{x}_1 - \frac{c}{m}x_1 \, .$$

Considering a novel actuator, whose inputs are voltages, the system can be (approximatively) specified by

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{c}{m} & -\frac{d}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \end{bmatrix} + \begin{bmatrix} b_1 & 0 \\ b_2 & b_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \, .$$

The output is given by

$$y = \begin{bmatrix} c_1 & 0 \\ c_2 & c_1 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \end{bmatrix} \, .$$

2a) (2 Points)
Calculate the eigenvalues of the system (by using the parameters c, d, and $m > 0$).

$$\lambda_{1,2} = -\frac{d}{2m} \pm \sqrt{\frac{d^2}{4m^2} - \frac{c}{m}}$$

2b) (1 Point)
Is the system asymptotically stable (assuming m, d, and $c > 0$)?
Justify your answer.

Using Stodola →
For the given assumption (m, d, c > 0)
the system is always asymptotically Stable.

2c) (6 Points)
The parameters are $m = 2$ kg, $c = 8$ kg/s^2, $d = 8$ kg/s.
For the following tasks use the Hautus criterion:

2c)i) (3 Points)
State the condition for b_1 and b_2 for the full controllability.

For $b_2 + 2b_1 \neq 0$ or $b_2 \neq 0$ ($b_1 \neq 0$ or $b_2 \neq 0$)
the system is fully controllable.

2c)ii) (3 Points)
State the condition for c_1 and c_2 for the full observability.

For $c_1 \neq 0$ or $2c_1 - c_2 \neq 0$ ($c_1 \neq 0$ or $c_2 \neq 0$)
the system is fully observable.

2d) (3 Points)
Considering the parameters given in 2c) use $b_1 = b_2 = 1$. Calculate the state feedback

$$K = \begin{bmatrix} k_1 & k_2 \\ 1 & 1 \end{bmatrix},$$

so that the eigenvalues of the closed loop system are $\lambda_1 = -4$, $\lambda_2 = -7$.

$\det(\lambda I - (A - BK))$, desired characteristic equation

$\rightarrow$ $k_1 = 53/11$

$k_2 = 13/11$

2e) (3 Points)

Use $c_1 = c_2 = 1$. Determine the observer gain

$$L = \begin{bmatrix} 1 & l_1 \\ 1 & l_2 \end{bmatrix},$$

so that the observer eigenvalues $\lambda_1 = -3$, $\lambda_2 = -4$ are obtained.

$\det(\lambda I - (A - LC))$, desired characteristic equation

$\Rightarrow l_1 = 1/3$

$l_2 = 5/3$

2f) (5 Points)
Derive the error equation for the observer dynamics in general and for this special system (with the observer calculated in part 2c).

Is this observer able to compensate the differences in the initial conditions between the state $x(t=0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and the observer state error $e(t=0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ for $t \to \infty$? (Calculate the eigenvalues of the observer dynamics for this purpose).

$$\dot{e} = \begin{bmatrix} -4/3 & 2/3 \\ -29/3 & -17/3 \end{bmatrix} e$$

$\lambda_1 = -3$
$\lambda_2 = -4$

Yes, the observer is able to compensate the estimation error for $t \to \infty$, because the observer is asymptotically stable.

$\sum$

Problem 3 (18 Points)
The system shown in the Figure 3.1 is given.

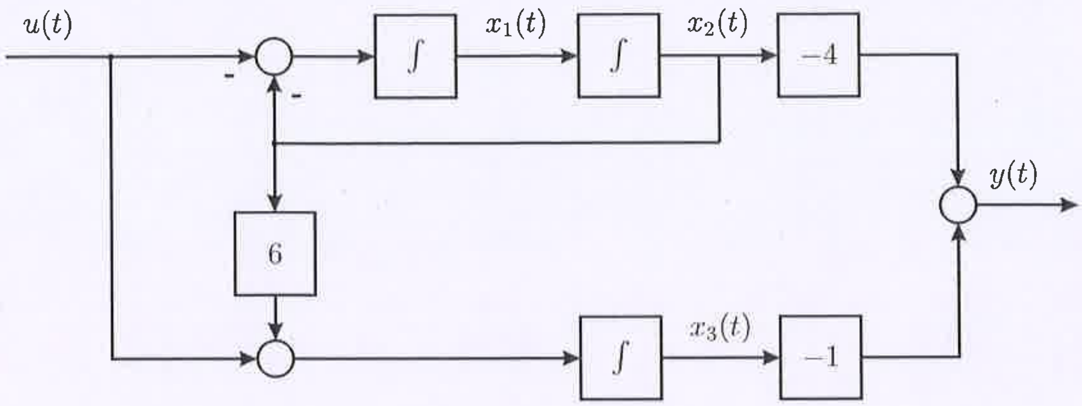


Figure 3.1: Block diagram

3a) (3 Points)
Set up the corresponding state space model from the block diagram with state vector $[x_1 \quad x_2 \quad x_3]^T$.

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 6 & 0 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}}_{B} u$$

$$y = \underbrace{\begin{bmatrix} 0 & -4 & -1 \end{bmatrix}}_{C} x$$

3b) (3 Points)
Determine the transfer function $G(s)$.

$$G(s) = C(sI - A)^{-1}B + D = \frac{-s^2 + 4s + 5}{s(s^2+1)}$$

For the following tasks 3c) nad 3d), a system description is given by

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \text{ and } \quad C = \begin{bmatrix} 0 & -2 & -2 \end{bmatrix}.$$

3c) (2 Points)
Determine the eigenvalues of the system. Is the system asymptotically stable? State reasons.

$\det(\lambda I - A) = 0 \Longrightarrow$ $\lambda_1 = 0$, $\lambda_{2,3} = \pm j$

No, the system is not asymptotically stable because the real part of all eigenvalues are not less than zero.

3d) (1 Point)
Is it possible to conclude from the stability that the system is controllable? State reasons.

No, the stability of system is not related to the contrallability of system and vice - versa.

The state space model

$$\dot{x}(t) = \begin{bmatrix} -7 & -5 \\ 5 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t);\; y(t) = [0 \quad 1]\; x(t),$$

is given.

3e) (4 Points)
Under which conditions is the system fully controllable? Under which conditions is the system fully observable? State reasons.

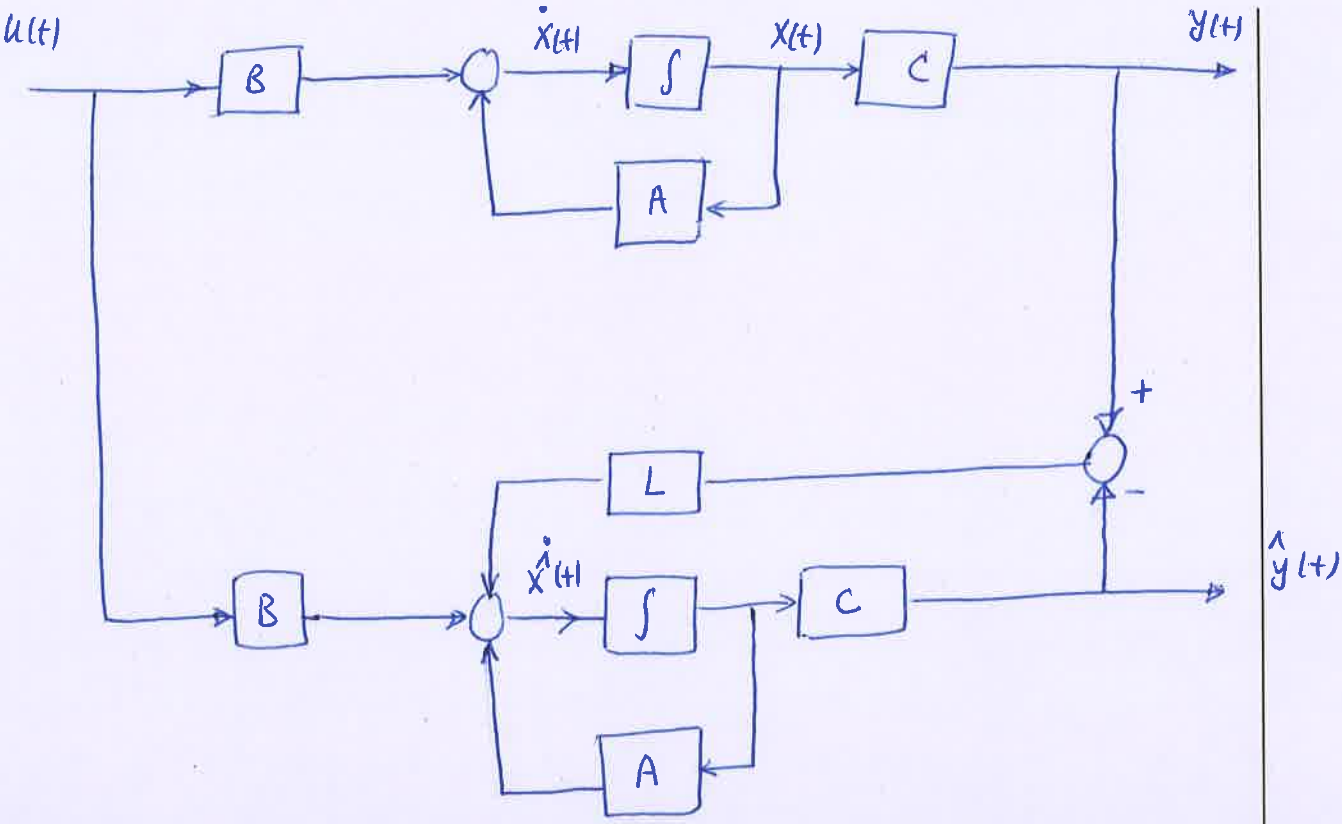
$$Q_S = \begin{bmatrix} 1 & -17 \\ 2 & 5 \end{bmatrix}$$

$\det(Q_S) \neq 0 \rightarrow$ fully controllable

$$Q_B = \begin{bmatrix} 0 & 1 \\ 5 & 0 \end{bmatrix}$$

$\det(Q_B) \neq 0 \Rightarrow$ fully observable

3f) (1 Point)
A state observer should be designed. Draw the block diagram.



3g) (3 Points)
Determine the feedback matrix L of the observer so that the eigenvalues $\lambda_1 = -1$ and $\lambda_2 = -2$ are considered.

$\det(\lambda I - (A - LC))$, desired characteristic equation

$\Rightarrow$ $l_1 = 1$
$l_2 = -4$

3h) (1 Point)
Give the concrete equation (with numerical values) for the error dynamics. How large is the observation error for $t \to \infty$, with the eigenvalues given in task 3g)?

$$\dot{e} = \begin{bmatrix} -7 & -6 \\ 5 & 4 \end{bmatrix} e$$

Based on the eigenvalues the observer is asymptotically stable and therefore

$$\lim_{t \to \infty} e(t) = 0$$

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