## Reading-up-time

For reviewing purposes of the problem statements, there is a "reading-up-time" of **10 minutes** prior to the official examination time. During this period it is **not** allowed to start solving the problems. This means explicitly that during the entire "reading-up-time" no writing utensils, e.g. pen, pencil, etc. at all are allowed to be kept on the table. Furthermore the use of carried documents, e.g. books, (electronic) translater, (electronic) dictionaries, etc. is strictly forbidden. When the supervisor refers to the end of the "reading-up-time" and thus the beginning of the official examination time, you are allowed to take your utensils. Please **then**, begin with filling in the **complete** information on the titlepage and on page 3.

*Good Luck!*

| | |
|---|---|
| LAST NAME | |
| FIRST NAME | |
| MATRIKEL-NO. | |
| TABLE-NO. | |

# Klausurunterlagen

Ich versichere hiermit, dass ich sämtliche für die Durchführung der Klausur vorgesehenen Unterlagen erhalten, und dass ich meine Arbeit ohne fremde Hilfe und ohne Verwendung unerlaubter Hilfsmittel und sonstiger unlauterer Mittel angefertigt habe. Ich weiß, dass ein Bekanntwerden solcher Umstände auch nachträglich zum Ausschluss von der Prüfung führt. Ich versichere weiter, dass ich sämtliche mir überlassenen Arbeitsunterlagen sowie meine Lösung vollständig zurück gegeben habe. Die Abgabe meiner Arbeit wurde in der Teilnehmerliste von Aufsichtsführenden schriftlich vermerkt.
**Durch die Teilnahme versichere ich, dass ich prüfungsfähig bin. Bei Krankheit werde ich die Klausur vorzeitig beenden und unmittelbar eine Ärztin/einen Arzt aufsuchen.**

THE ABOVE REQUIRED STATEMENTS AS WELL AS THE SIGNATURE ARE MANDATORY AT THE BEGINNING OF THE EXAM.

Duisburg, ______________ ______________________________
(Date) (Student's signature)

Falls Klausurunterlagen vorzeitig abgegeben: ______________Uhr

# Bewertungstabelle

| | |
|---|---|
| Aufgabe 1 | |
| Aufgabe 2 | |
| Die Bewertung gem. PO in Ziffern ist der xls-Tabelle bzw. dem Papierausdruck zu entnehmen. | |

______________________________________________

(Datum und Unterschrift 1. Prüfer, Univ.-Prof. Dr.-Ing. Dirk Söffker)

______________________________________________

(Datum und Unterschrift 2. Prüfer, Dr.-Ing. Sandra Rothe)

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(Datum und Unterschrift des für die Prüfung verantwortlichen Prüfers, Söffker)

Fachnote gemäß Prüfungsordnung: (alternativ: siehe xls-Tabelle bzw. beigefügter Papierausdruck)

| ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1,0 | 1,3 | 1,7 | 2,0 | 2,3 | 2,7 | 3,0 | 3,3 | 3,7 | 4,0 | 5,0 |
| sehr gut | | gut | | | befriedigend | | | ausreichend | | mangelhaft |

Bemerkung: ______________________________________________

**Attention:** Give your answers to ALL problems directly below the questions in the exam question sheet.

You are NOT allowed to use a pencil and also NOT red color (red color is used for corrections).

This exam is taken by me as a
☐ mandatory (Pflichtfach)
☐ elective (Wahlfach)
☐ prerequsite (Auflage)
subject (cross ONE option according to your own situation).

| | |
|---|---|
| Maximum achievable points: | **54** |
| Minimum points for the grade 1,0: | **95%** |
| Minimum points for the grade 4,0: | **50%** |

**General hints:**

1) For the multiple-choice and multiple-choice-similar tasks the following rules are effective:
   i) For tasks with an individual evaluation of subtasks, the following applies: Only correct answers are evaluated with the intended number of points.
   ii) The points achieved in a subtask will be summed up.
   iii) Unless explicitly stated otherwise, only one of the given solution options is correct.
   iv) If subtasks contain more than two answer options and only one solution exists: The marking of multiple answer options is interpreted as a non-response due to the not clear declaration of intention. As a result, no points can be given in this case.

2) If in the exam tasks no information is given for the valid range of numbers for time constants or masses etc.: take for time constants (in sec.), for masses (in kg) positive numbers.

3) If in the exam tasks no information is given for applying negative or positive feedback: use the usual negative feedback.

**Problem 1** (41 Points)

Mark the correct solution in the following statements.

1a) (10 × 1 Point, 10 Points)
The output behaviors of two systems are given.

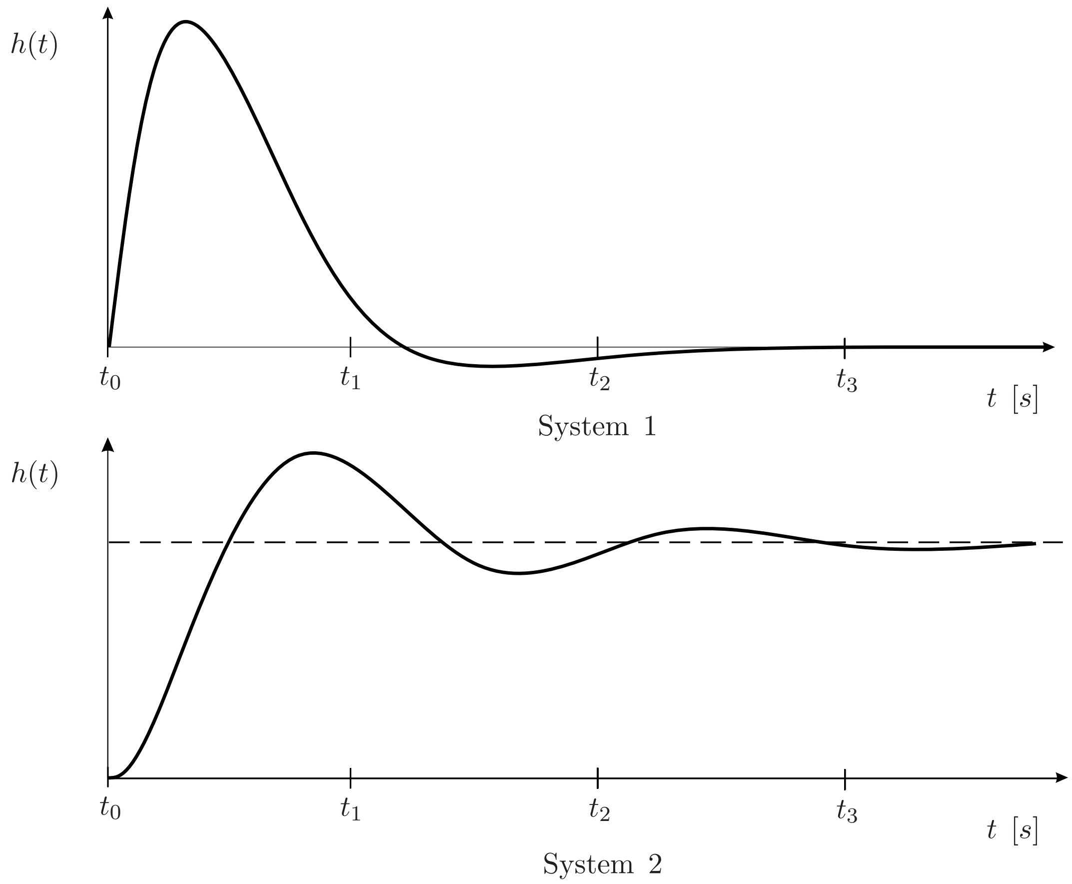



**Figure 1.1**: System outputs

A1) (1 Point)
System 1 shows a

◯ proportional

◯ integral

◯ differential

behavior.

A2) (1 Point)
System 2 shows a

- ◯ proportional
- ◯ integral
- ◯ differential

behavior.

A3) (1 Point)
Which system has a static gain equal to zero?

- ◯ Only System 1
- ◯ Only System 2
- ◯ System 1 and System 2

A4) (1 Point)
The input of System 1 is

- ◯ a step function.
- ◯ an impulse.
- ◯ a sine function.
- ◯ a ramp function.

A5) (1 Point)
The input of System 2

- ◯ is a step function.
- ◯ is an impulse.
- ◯ is a sine function.
- ◯ is a ramp function.
- ◯ can be an arbitrary bounded signal.

A6) (1 Point)
System behaviors are often characterized by time delay. Considering the given output behaviors, this is the case for

- ◯ System 1.
- ◯ System 2.
- ◯ System 1 and System 2.
- ◯ none of the systems.

A7) (1 Point)
The output of System 1 shows stationary behavior at

- ◯ time $t_1$.
- ◯ time $t_2$.
- ◯ time $t_3$.
- ◯ none of the mentioned time instants.

A8) (1 Point)
The output of System 2 shows stationary behavior at

- ◯ time $t_1$.
- ◯ time $t_2$.
- ◯ time $t_3$.
- ◯ none of the mentioned time instants.

A9) (1 Point)
Which system is a zero or first order system?

- ◯ System 1
- ◯ System 2
- ◯ System 1 and System 2
- ◯ None of the systems

A10) (1 Point)
After the time $t_3$, the input of System 1 is zero. This statement is

- ◯ correct because the output approaches zero.
- ◯ wrong because the system has differential behavior.
- ◯ wrong because the output of $PT_1$-systems compensatorily approaches always zero.
- ◯ correct because the behavior is – as it can be seen – only generated by a disturbance, which acts at time $t_0$. An input has never been applied.

☐

1b) (3 × 1 Point, 3 Points)
Numerous control engineering methods and procedures use models or other descriptions based on suitable descriptive means respectively to describe the I/O- or overall system behavior.

B1) (1 Point)
The correctness of models (e.g. in the context of their use for computer-based simulation) is of central relevance. Model validation includes

◯ testing, whether a model is well suited from a programming point of view to deliver the desired output variables based on suitable input variables and model parameters.

◯ testing, whether the model shows the same behavior as a real system at least within the claimed range of validity (e.g. by measurements).

◯ certification of the model description and model behavior by an appropriate authority or testing institution.

B2) (1 Point)
Modeling can be realized in different ways. Theoretical modeling

◯ is based on the use of axioms (first principles) and derives descriptions using specific methods, e.g. mathematics or logic, which are then used within a defined range of validity.

◯ is based on the use of theories on how models are generated, e.g. heuristically or statistically.

◯ is a very new term and describes the use of models that are established with the help of theoretical methods from the field of machine learning. Therefore, theoretical models represent a new class of models as compared to conventional models, which are based on formulas e.g. by Newton, Euler, or Kirchhoff.

B3) (1 Point)

The approximation of $PT_2$ similar behaviors under the use of $PT_1$ and time delay elements based on measurements can be assigned to the field of

- ◯ data-based modeling, e.g. using machine learning.
- ◯ theoretical modeling.
- ◯ experimental modeling.

☐

1c) (5 × 1 Point, 5 Points)
Based on the state space description

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

and the known mthematical relationships

$$\det(\lambda_i I - A) = 0 \quad \rightarrow \lambda_i \quad \text{(Eigenvalue equation)}$$
$$Av_i = \lambda_i v_i \quad \rightarrow v_i \quad \text{(Eigenvector equation)}$$

as well as

$$V = \begin{bmatrix} v_1 & v_2 & \dots & v_n \end{bmatrix} \quad \text{(Modal matrix)}$$
$$AV = V \operatorname{diag}\left[\lambda_i\right] \triangleq \underbrace{V^{-1}AV}_{\tilde{A}} = \operatorname{diag}\left[\lambda_i\right]$$

explain what that means in detail by answering the following subtasks.

C1) (1 Point)
A single Eigenvector $v_i$ describes

- ◯ the frequency of an oscillating/vibrating system.
- ◯ the time-wise constant relationships between the describing state variables $x_i$.
- ◯ the relationships of the modal matrix.
- ◯ the length of an Eigenvalue.
- ◯ the deflection of an oscillating beam.
- ◯ the type of deflection of an oscillating beam.
- ◯ the amplitude of deflection of an oscillating beam.

C2) (1 Point)
A complex Eigenvalue $\lambda_i$ describes

- ◯ the frequency of an oscillating/vibrating system.
- ◯ the frequency of an oscillating/vibrating system if the corresponding damping $D$ is sufficient small.
- ◯ the amplitude of an oscillating/vibrating system if the corresponding damping $D$ is sufficient small.
- ◯ the frequency of an oscillating/vibrating system if the corresponding damping $D$ is sufficient large.
- ◯ the amplitude of an oscillating/vibrating system if the corresponding damping $D$ is sufficient large.
- ◯ the frequency of an oscillating/vibrating system if the real part is sufficient small.
- ◯ the frequency of an oscillating/vibrating system if the real part is sufficient large.
- ◯ the frequency of an oscillating/vibrating system if the imaginary part is sufficient small.
- ◯ the frequency of an oscillating/vibrating system if the imaginary part is sufficient large.

C3) (1 Point)
The modal matrix $V$ is

- ◯ the sum of all Eigenvectors.
- ◯ the product of all Eigenvectors.
- ◯ the diagonalized matrix of the Eigenvalues (in block Jordan form).
- ◯ a matrix composed by the Eigenvectors.
- ◯ the vector representation of the Eigensum.
- ◯ the inverse of a matrix generated by the Eigenvectors.

C4) (1 Point)
The matrix $\tilde{A}$ is achieved by

- ◯ similary transformation using the diagonalized Eigenvalue matrix.
- ◯ multiplying with the modal matrix from the right side.
- ◯ multiplying with the inverse modal matrix from the left side.
- ◯ multiplying with the diagonalized Eigenvalue matrix from the right side.
- ◯ similary transformation using the modal matrix.

C5) (1 Point)
The system matrix $A$ and the corresponding diagonalized system matrix $\tilde{A}$ describe

- ◯ the same physical system in different coordinate systems.
- ◯ the same physical system in the same coordinate systems.
- ◯ two different physical systems in different coordinate systems.
- ◯ two different physical systems in the same coordinate system.

1d) (6 × 1 Point, 6 Points)

D1) (1 Point)
The system described by

$$\begin{bmatrix}\dot{x}_1\\\dot{x}_2\\\dot{x}_3\end{bmatrix}=\begin{bmatrix}0&1&0\\0&0&1\\-d&-e&-f\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}b_3&0&0\\0&b_2&0\\0&0&b_1\end{bmatrix}u;\ y=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix};\ d,e,f,b_1,b_2,b_3>0$$

is considered. The system is a

◯ $PT_2$-system.

◯ $PT_3$-system.

◯ a system of first order with proportional input.

◯ Multiple-Input, Multiple-Output (MIMO) system in state-space representation.

D2) (1 Point)
A controller $u=\begin{bmatrix}0 & 0 & -Kx_2\end{bmatrix}^T$ is used for feedback of the system in task D1). Using a feedback with $K>0$ for $d=4,\ e=3,\ f=3,\ b_3=0,\ b_2=0,$ and $b_1=1$ has

◯ an impact

◯ no impact

on the system behavior.

D3) (1 Point)
The term "step response" denotes the

- ◯ input of the system as a step function.
- ◯ input of the system as a function.
- ◯ output of the system as a step function.
- ◯ output of the system as a function.
- ◯ input of the system if there is a step function on the output side.
- ◯ output of the system if there is a step function on the input side.

D4) (1 Point)
It is $\int\limits_0^\infty \delta(t) = 1$. What does this mean?

- ◯ The amplitude of the impulse $\delta$ is 1.
- ◯ The duration of the impulse $\delta$ is 1.
- ◯ The impulse $\delta$ encloses an area of size 1.
- ◯ The derivative of the impulse $\delta$ is 1 (step function).

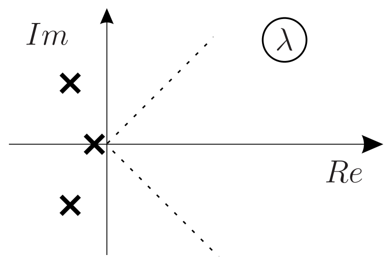
D5) (1 Point)
The I/O-description

$$a_n y^{(n)}(t) + a_{n-1} y^{(n-1)}(t) + ... + y(t) = K[u(t) + \frac{1}{T_\mathrm{I}} u(t) dt];\ a_i \neq 0,\ K,\ T_\mathrm{I} > 0,\ i = 1 \ldots n$$

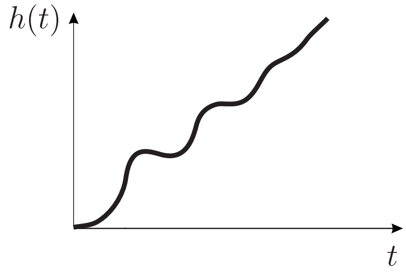
describes a $\mathrm{PIT_n}$-system in standard form for classification. The statement "The system is of $n$-th order" is

- ◯ correct because the coefficient $a_n$ is unequal to zero.
- ◯ correct because $y$ can be replaced by the new output $y_\mathrm{new}$ with $y = \int y_\mathrm{new}\,\mathrm{d}t$.
- ◯ wrong because it is a $\mathrm{PDT_n}$-system.
- ◯ wrong because it is unknown if all coefficients exist.

D6) (1 Point)
A system with the eigenvalue distribution




can show the following behavior.




This statement is

- ◯ correct because according to the eigenvalue distribution integral behavior can be concluded.
- ◯ correct because the conjugate-complex pole pair describes damped vibrations.
- ◯ wrong because the system is unstable.
- ◯ wrong beacuse the pure real-valued Eigenvalue is responsable for the unstable behavior.
- ◯ wrong because the eigenvalue distribution does not describe integral behavior.
- ◯ correct beacuse the pure real-valued Eigenvalue is responsable for the integral behavior.

1e) ($6 \times 1$ Point, 6 Points)
The eigenvalues of four different linear systems without time delay are illustrated in Figure 1.2. Four measured output functions are shown in Figure 1.3.

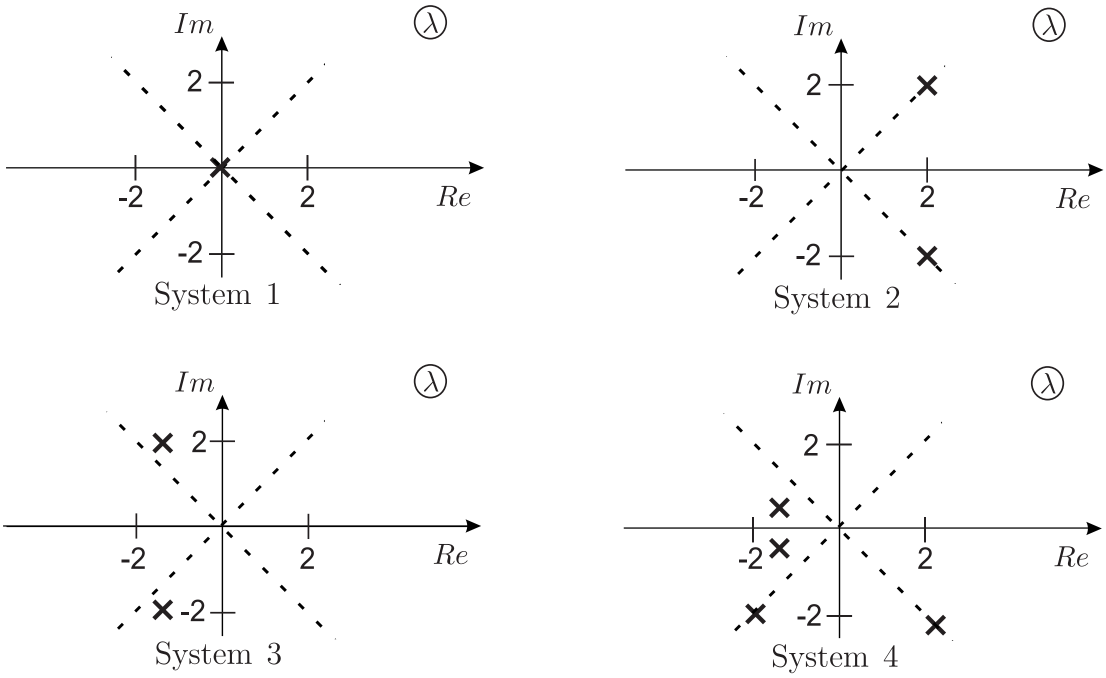



**Figure 1.2**: Eigenvalue distribution of four different systems

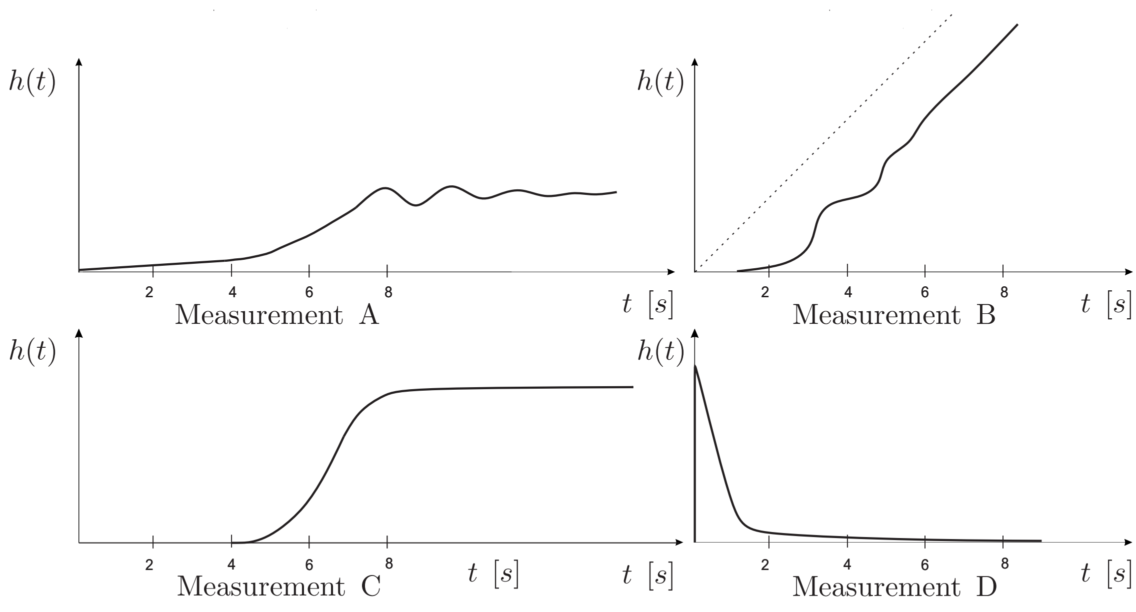



**Figure 1.3**: Output functions

E1) (1 Point)
Measurement B shows a system with

- ◯ proportional
- ◯ integral
- ◯ differential
- ◯ chaotic

behavior.

E2) (1 Point)
Measurement B

- ◯ shows time delay.
- ◯ does not show time delay.
- ◯ shows Bounded-Input, Bounded-Output (BIBO) stable behavior.
- ◯ shows exponentially unstable behavior.

E3) (1 Point)
Measurement C shows a system behavior

- ◯ without dynamics (dynamical properties).
- ◯ with dynamics (dynamical properties).
- ◯ without time delay.
- ◯ without inertia.

E4) (1 Point)
Measurement D shows the step response of a

- ◯ $PDT_1$-system.
- ◯ $DT_1$-system.
- ◯ $PIDT_1$-system.

E5) (1 Point)
Measurement C corresponds to the behavior of a

- ◯ $IT_1T_t$-system with $T_t < 0$.
- ◯ $IT_2T_t$-system with $T_t > 0$.
- ◯ $PIT_2T_t$-system with $T_t < 0$.
- ◯ $PT_2T_t$-system with $T_t > 0$.

E6) (1 Point)
The system behavior shown in Measurement B can be described by

- ◯ System 1.
- ◯ System 3.
- ◯ connecting System 1, System 3, and additional time delay in series.
- ◯ connecting System 1 and System 3 in series.
- ◯ connecting System 1, System 3, and additional time delay in parallel.
- ◯ connecting System 1 and System 3 in parallel.

☐

1f) (4 × 1 Point, 4 Points)

F1) (1 Point)
Which statement is wrong?

- ◯ The location of the eigenvalues in the complex plane allows a statement about the stability of the underlying system.
- ◯ The location of the eigenvalues in the complex plane allows a statement about the damping of the modes of the system.
- ◯ A system is considered as stable if there are no eigenvalues for which $\mathrm{Re}\{\lambda_i\} > 0$.
- ◯ A system is considered as unstable if there are eigenvalues for which $\mathrm{Re}\{\lambda_i\} > 0$.
- ◯ A system is considered as boundary stable if $\mathrm{Re}\{\lambda_i\} \leq 0$ and there is one eigenvalue located at the origin of the $s$-plane.
- ◯ I/O-stable systems are always state stable.

F2) (1 Point)
System descriptions of the type

$$\begin{aligned} y &= \frac{1}{T_\mathrm{I}} \int u \,\mathrm{d}t \\ T_1 \dot{y} + y &= \frac{1}{T_\mathrm{I}} \int u \,\mathrm{d}t \\ \frac{1}{\omega_0^2}\ddot{y} + \frac{2D}{\omega_0}\dot{y} + y &= \frac{1}{T_\mathrm{I}} \int u \,\mathrm{d}t \end{aligned}$$

represent the following behavior:

- ◯ integral I/O behavior with different output dynamics.
- ◯ proportional I/O behavior with different output dynamics.
- ◯ differential I/O behavior with different output dynamics.
- ◯ integral I/O behavior with different input dynamics.
- ◯ proportional I/O behavior with different input dynamics.
- ◯ differential I/O behavior with different input dynamics.

F3) (1 Point)
From a principal point of view, aims regarding reference and disturbance behavior can be distinguished in controller design. Which statement is wrong?

- ◯ Reference and disturbance behavior can be considered separately for linear systems due to superposition of the corresponding outputs.
- ◯ Any stable control loop that meets the requirement for reference control is also suitable for complete disturbance compensation.
- ◯ In case of a proportional system to be controlled, an integral controller is suitable to improve both reference and disturbance behavior.
- ◯ Any asymptotically stable control loop meets the requirement for disturbance compensation for impulse-shaped disturbance signals.

F4) (1 Point)
The Ziegler-Nichols criterion, which is applied to proportional systems that are able to oscillate, is

- ◯ an experimental tuning strategy.
- ◯ a tuning strategy, which is based on analytical considerations to determine the best PID control parameters.
- ◯ a strategy, which always provides the best control parameters according to the ITAE criterion.
- ◯ perfect.
- ◯ complex.
- ◯ a strategy that requires a complex interaction of the automated controller with the controlled system and is therefore sustainable in the course of industry 4.0.
- ◯ a strategy from the year 1942, which will rationalize the jobs of control engineers in the future due to the underlying automation capability.

☐

The block diagram of a system consisting of four transfer elements with $w$ and $u$ as inputs and $y$ as output is given in Figure 1.4.

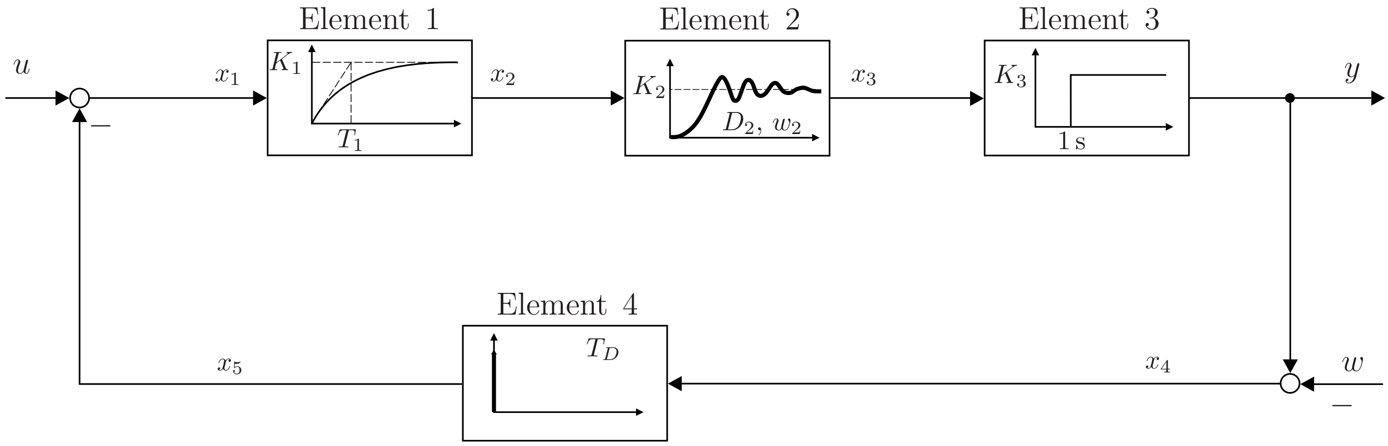



**Figure 1.4**: Block diagram of the system

1g) (4 Points)
Classify the transfer behaviors (type of single transfer element behavior) of the elements 1 to 4 and give the corresponding differential equations using the given notation of inputs and outputs in a suitable form for classification.

1h) (2 Points)
Determine the differential equation of the entire system illustrated in Figure 1.4 with the parameters $T_1 = D_2 = w_2 = T_\mathrm{D} = K_1 = K_2 = K_3 = 1$ using $w$ as input and $y$ as output.

*Hint:* $\mathrm{PT_1T_t} : T_1\dot{y} + y = K_\mathrm{S} \cdot u(t - T_\mathrm{t})$

1i) (1 Point)
Classify the transfer behavior of the entire system in Figure 1.4 neglecting the time delay of Element 3.

$\sum$

**Problem 2** (17 Points)

a) (7 Points)
The block diagram of a nonlinear plant is shown in Fig. 2.1.

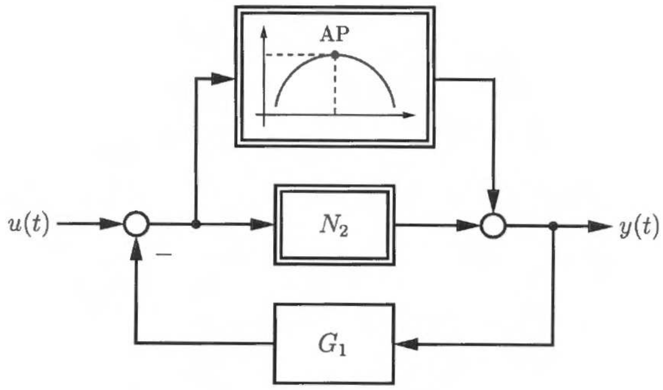



**Figure 2.1**: Nonlinear plant

The system output of the linear subsystem $G_1$ is $y_1(t) = 2 \cdot 1(t)$ for the input signal $u_1(t) = t \cdot 1(t)$. The nonlinear subsystem $N_2$ is described by

$$\ddot{Y}(t)\dot{Y}^2(t) + \dot{Y}(t)\sqrt{Y(t)} - Y(t)U(t) + Y(t) - 6 = 0. \tag{2.1}$$

2a) i) (1 Point)
State the input/output description (equation) of the linear subsystem $G_1$.

2a) ii) (3 Points)

Linearize the equation 2.1 around the general operating point $(\ddot{Y}_0, \dot{Y}_0, Y_0, U_0)$.

iii) (3 Points)
State the input/output description (equation) of the linearized subsystem $N_2$ for the operating point $\ddot{Y}_0 = 0, \dot{Y}_0 = 1, Y_0 = 4, U_0 = 0,25$. Classify the type of system behavior.

2b) (6 Points)
The block diagram shown in Fig. 2.2 is given.

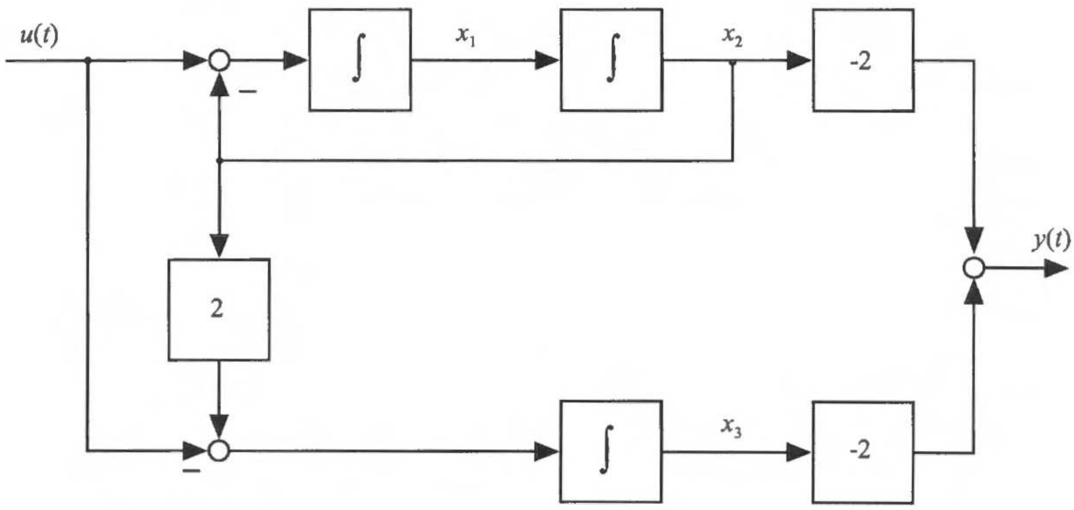



**Figure 2.2**: Block diagram

2b) i) (1 Point)
Decide which of the given variables $u(t), y(t), x_1(t), x_2(t), x_3(t)$ are input, output, or interim quantities.

2b) ii) (4 Points)

Based on the block diagram formulate the corresponding state space description. Assume the state vector to be defined as $x(t) = \begin{bmatrix} x_1(t) & x_2(t) & x_3(t) \end{bmatrix}^T$. Specify the matrices $A$, $B$, $C$.

2b) iii) (1 Point)

Is the considered system a SISO, a MIMO, a SIMO, or a MISO system? Give reasons.

$\sum$