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ANALYSIS OF IMPACT RESPONSES USING THE REGULARIZED MODEL APPROACH

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Extended Abstract

The analysis of collisions in multibody systems has been a topic of continuous research in recent years. On the one hand, so-called impact hypotheses are being used which permit to "leap over" the impact time interval, working with an ideal rigid-body model before and after impact. Such approaches offer high numerical efficiency at the cost of often rendering flawed results that might even be in contradiction with energy balance laws in the case of the general oblique and rough impact. Moreover, the rigidity assumption causes problems when multiple contacts with friction are regarded, as in this case the governing system of equations might be over-constrained and no unique solutions exist. On the other hand, there exist so-called regularised models in which the contact area is substituted by a massless spring-damper element and the response of the system during the impact interval is computed by solving the corresponding differential equations numerically. Such models render acceptable results when the relative approach velocity before impact is small as compared to the wave travel velocity within the colliding bodies, a pre-requisite that is fulfilled for most technical impacts. Claimed disadvantages of the regularised model are its poor computational efficiency and the dependency of impact response on the chosen integrator step length. However, the regularised models have the advantage of providing a consistent model even during impact situations in which the number of contacts is high and friction arises. The purpose of this paper is (1) to analyse in more detail, for the standard benchmark example of a colliding oblique rough bar on a plane, the properties of the governing equations of the regularised model assumption and their solutions, (2) to compare the resulting predicted response with that resulting from classical rigid impact hypotheses, and (3) to present some new algorithms for 3D contact problems involving also rolling and partial overlapping.

Consider the impact of a homogeneous bar of length 2ℓ, mass m and radius of gyration ρ, with a rough horizontal surface featuring coefficient of friction μ (Fig. 1). The angular speed of the bar is $\dot{\varphi}$ and the velocity of contact point O is $\underline{v} = [\dot{x}, \dot{y}]^{\mathrm{T}}$. Superscripts '$-$' and '$+$' denote quantities at the start and end of impact, respectively. During impact, all applied and gyroscopic forces, as well as any change of configuration, are neglected. Relevant external effects are the contact forces

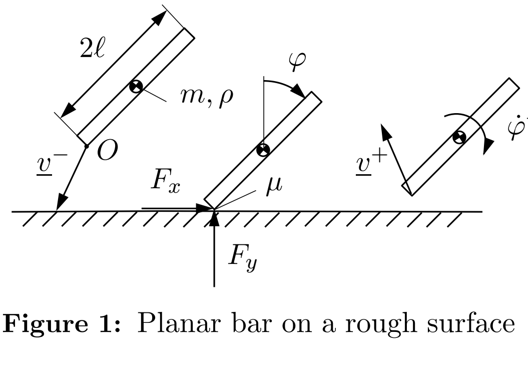

Figure 1: Planar bar on a rough surface

$\underline{F}(t) = [F_x(t), F_y(t)]^{\mathrm{T}}$ and the impulses $\underline{P}(t) = [P_x(t), P_y(t)]^{\mathrm{T}} = \int_{t^-}^{t} \underline{F}(\bar{t})\,\mathrm{d}\bar{t}$. As initial conditions, let the bar approach the contact plane with pure translational velocity $\underline{v}^-$ and angle φ with respect to the surface normal. Newton's second law produces two equations of motion which can be expressed in the acceleration of the contact point. Upon setting $\lambda_x = (\ell/\rho)\sin\varphi$, $\lambda_y = (\ell/\rho)\cos\varphi$; $C_{xx} = 1 + \lambda_x^2$; $C_{yy} = 1 + \lambda_y^2$; $C_{xy} = -\lambda_x\lambda_y$, one obtains the equations of motion and the resulting impact response $\underline{v}^+$ as:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \frac{1}{m} \begin{bmatrix} C_{xx} & C_{xy} \\ C_{xy} & C_{yy} \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix} \doteq \frac{1}{m}\mathbf{C}\,\underline{F} \quad \Rightarrow \quad \underline{v}^+ = \underline{v}^- + \frac{1}{m}\mathbf{C}\,\underline{P}^+ \ . \tag{1}$$

In the regularised model, tangential and normal compliances of the impact zone are modeled by massless springs [4]. Slip-stick effects can be taken into account by introducing a massless 'sledge' which slides for contact forces outside the friction cone. In this model, O represents the contact point, x and y are the deflections of the contact springs, x_S is the location of the center of the sledge (Fig. 2), and $\hat{x} = x_S + x$. Denoting differentiation with respect to dimensionless time $\tau = \sqrt{c_y/m}\,t$ by prime, one obtains the two types of governing equations:

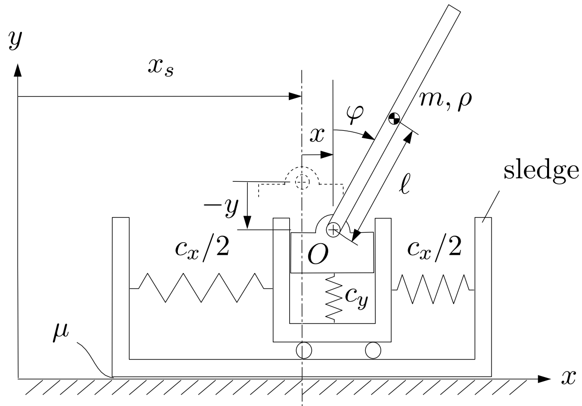

Figure 2: Regularised model

$$\begin{array}{cc} \underline{\text{sticking case}} & \underline{\text{sliding case}} \\ F_x = -c_x x\ ,\ F_y = -c_y y\ ,\ \kappa = c_x/c_y & F_x = \mu^\star F_y\ ,\ F_y = -c_y y\ ,\ \mu^\star = \mu\,\mathrm{sign}(\dot{x}_S) \\ \begin{bmatrix} x'' \\ y'' \end{bmatrix} + \begin{bmatrix} \kappa\,C_{xx} & -C_{xy} \\ -\kappa\,C_{xy} & C_{yy} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0\ , & \begin{bmatrix} \hat{x}'' \\ y'' \end{bmatrix} + \begin{bmatrix} 0 & \mu^\star C_{xx} - C_{xy} \\ 0 & C_{yy} - \mu^\star C_{xy} \end{bmatrix} \begin{bmatrix} \hat{x} \\ y \end{bmatrix} = 0 \end{array} \tag{2}$$

Note that the governing equations now only depend on dimensionless parameters. In particular, the stiffness ratio κ may be bounded even for $c_x, c_y \to \infty$. Indeed, Routh's assumption $\kappa \to \infty$ is quite unrealistic for technical applications, as e.g. for collision of two spheres featuring equal Poisson's ratios $\nu = 0.3$ it holds $\kappa = 0.824$ [4]. Due to the linearity and time-independency of the governing equations, the latter can be solved in terms of the initial conditions $\underline{x}_0 = \underline{x}(0) = [\,x_0\,,\,y_0\,]^{\mathrm{T}}$, $\underline{x}'_0 = \underline{x}'(0) = [\,x'_0\,,\,y'_0\,]^{\mathrm{T}}$ in each case by modal analysis. From these solutions, it is easy to show that, for simple impact cases, such as central or frictionless impact, for which all impact hypotheses coincide, also the regularised model yields ad hoc closed form solutions that match the hypotheses' predictions. However, for the general oblique rough impact case, significant discrepancies may appear. This is shown in Fig. 3 where the regularised model is compared to the rigid-body impact predictions according to Newton's, Stronge's and Poisson's [1, 5, 2, 6] impact hypotheses, respectively, for the case of an inclined planar bar hitting a rough surface. Here κ is the ratio of tangential to normal stiffness, and P_x, P_y are the tangential and normal impulses. The analysis shows that impact hypotheses agree with the regularised model response only for infinite tangential stiffness, while for the more realistic assumptions of $\kappa = O(1)$ significant differences may occur between regularised and rigid-body models response. This confirms the problematic of applying impact hypotheses developed for central impacts to general impact situations.

The above discussed spring-sledge model was extended to (1) impacts with coefficient of restitution $e < 1$, (2) 3D frictional impacts of points on a plane, (3) 3D frictional impacts of circles on a plane, including flat collisions, and (4) 3D frictionless impacts between two disks, with partial overlapping, also including flat collisions. Elastoplastic impacts can be accomplished with the regularised model by introducing massless dampers in parallel to the springs, the damping coefficient being a function of penetration velocity [3]. 3D frictional impacts of points on planes involve the generalisation of the one-dimensional sledge motion of the planar bar example to two-dimensional sledge motion, yielding two nonholonomic constraint equations for the coordinates of the sledge in the sliding case. The sliding sledge model can be also applied to rolling contact situations, where it describes the departure of the disk rolling velocities from the ideal case, the "deflections" of the corresponding spring elements resulting from the time integral of the velocity gap. Transition to surface contact in the flat collision case is accomplished by a smooth transition function. A similar strategy is followed for a disk-disk contact (without friction) with partial overlapping: here, transition between line-line, point-surface, and surface-surface contacts for inclined and collinear axes are realized by appropriate transition functions. An object-oriented multibody implementation allows us to model and simulate a number of examples, of which some shall be shown in the presentation.

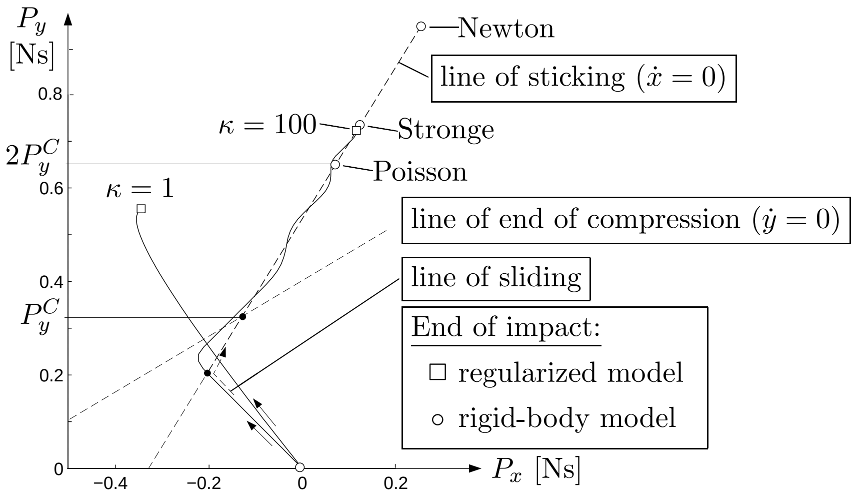

Figure 3: Impact process diagram for planar-bar example

Currently, the team is working on predicting efficiently and accurately the time of impact and of other state transitions (sticking, sliding, lift-off) using a fourth order Runge-Kutta approximation of rigid-body motion and/or approach variable and determining the roots by special polynomial root solvers. Our current experience is that, as opposed to generic root solvers that are integrated in current integrator routines, such as LSODAR, the polynomial root finding algorithms are much more reliable and efficient. Hence, larger step sizes can be chosen without incurring the danger of "missing" contacts or state switches. It is intended to report also on the results of this research at the time of the Colloquium.

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