UNIVERSITÄT DUISBURG ESSEN
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Faculty of Engineering
Department of Civil Engineering
Institute of Mechanics
Prof. Dr.-Ing. habil. J. Schröder

Domain-Decomposition-Based FSI Algorithms for Highly Nonlinear and Anisotropic Elastic Arterial Wall Models in 3D

D-A-CH **Cross-border funding of research projects in Germany, Austria and Switzerland**

In the context of the D-A-CH cooperation the DFG (Deutsche Forschungsgemeinschaft, Germany), the FWF (Fonds für wissenschaftliche Forschung, Austria) and the SNF (Schweizerischer Nationalfonds, Switzerland) arrange a "Lead Agency"-procedure and a "Money follows Cooperation Line" to facilitate the researcher's mobility and the accomplishment of cross-border reserach projects.

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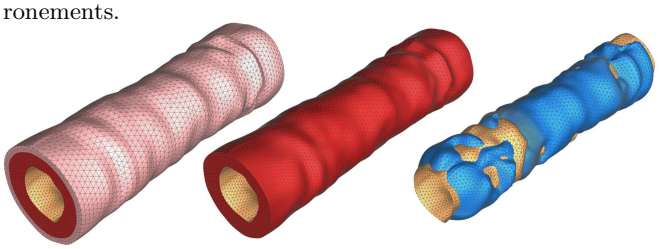
Motivation

Transmural stress distributions of in vivo arteries are a major factor driving, e.g., the processes of arteriosclerosis and arteriogenesis which are well-known to be of a major relevance to the human health. Realistic predictions for transmural stress distributions require a dynamic simulation considering the interaction of the blood flow with the vessel wall (FSI), see also [3]. In this project, algorithms for the fluid-structure interaction are developed based on domain decomposition methods and applied to the computation of realistic transmural stresses in physiological models of arterial walls. In 3D the associated systems of coupled nonlinear partial differential equations have a large number of unknowns and consequently have to be solved using HPC environements.



Biomechanically motivated Fiber-Orientation

The assumption of a constant fiber-orientation in each layer of the artery is not realistic. Therefore, an iterative approach for the calculation of the fiber directions based on the assumption that the fiber-orientation is mainly controlled by the principal stresses, based on [4], is considered.

Modeling of eigenstresses in arterial walls

A new bilogically motivated approach has to be developed incorporating the eigenstresses, which are set free by radial cuts (described by opening angles), but also additional eigenstresses observed for circumferential cuts Therefore, a local initial prestretch $\lambda := \lambda(\gamma)$ in fiber direction at each integration point $\tilde{\boldsymbol{X}}$ has to be incorporated depending on a nonlocal measure γ:

$$\gamma := \psi(\tilde{\boldsymbol{X}}) - \int_{\bar{\Omega}_0^s} \psi(\tilde{\boldsymbol{X}})\, g(\xi)\, d\xi \,,$$

wherein g is a density distribution function along a radial line $\bar{\Omega}_0^s$, parametrized in ξ, through $\tilde{\boldsymbol{X}}$ and the arterial wall.

Viscoelastic material model

For dynamic simulations, where the blood-flow is also taken into account, the visco-elasticity in the material may have a significant impact on the stress distribution inside the arterial wall. Therefore, a visco-elastic material model has to be developed, where the basic framework is provided by Simo [6]. We consider the strain energy function ψ in a decoupled structure with a hyperelastic and polyconvex ψ^{el}, see [2], a volumetric ψ^{vol} and a visco-elastic part ψ^{ve}. Evaluating the Clausius-Duhem inequality, the second Piola-Kirchhoff stresses are calculated by

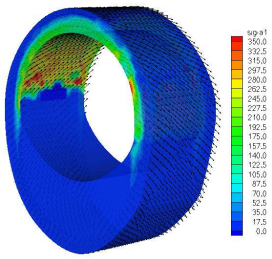
$$\boldsymbol{S} = 2\,\frac{\partial \psi^{vol}}{\partial \boldsymbol{C}} + \sum_{k=1,2}\left[\sum_{i=1,3,6} 2\,\frac{\partial \psi^{el}}{\partial I_i}\,\frac{\partial I_i}{\partial \boldsymbol{C}} + \sum_{i=1,3,6} \boldsymbol{S}_i^{ve}\right]$$

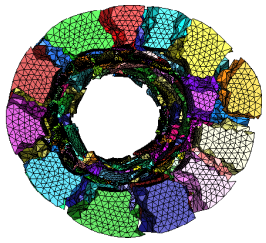
using the invariants $I_1 = \operatorname{tr}\boldsymbol{C}$, $I_3 = \det \boldsymbol{C}$ and $I_6 = \operatorname{tr}[\operatorname{Cof}\boldsymbol{C}(\boldsymbol{1} - \boldsymbol{M}^{(k)})]$ with the structural tensor $\boldsymbol{M}^{(k)} = \boldsymbol{a}^{(k)} \otimes \boldsymbol{a}^{(k)}$ for the fiber family with the direction $\boldsymbol{a}^{(k)}$. The visco-elastic stresses S^{ve} are evaluated by an evolution approach based on a set of linear differential equations, i.e.

$$\dot{\boldsymbol{S}}_i^{ve} + \frac{1}{\tau_i}\boldsymbol{S}_i^{ve} = \beta_i\, \dot{\boldsymbol{S}}_i^{el} \,.$$

Parallel solver environment for FSI

A segregated approach for the implementation of a parallel FSI simulation environment will be chosen. For the solution of the structural problems a FETI domain decomposition method is considered, cf. [5], whose application to arterial walls is stated in [1]. The fluid problems will be solved by a parallel overlapping Schwarz method, developed in the group of A. Quarteroni.





Acknowledgements

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J. Schröder, D. Balzani
S. Fausten
University of Duisburg-Essen
Institute of Mechanics

A. Klawonn[1], O. Rheinbach[2]
A. Heinlein
[1]University of Cologne, [2]TU Freiberg
Mathematical Institute

A. Quarteroni, S. Deparis
EPF Lausanne
Chair of Modeling and Scientific Computing

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