

## Evaluation of Pile Diameter Effects on Soil-Pile Stiffness

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### Summary

The applicability of the standard design method for laterally loaded piles (p-y method) to mono pile foundations was investigated. The investigation was performed by analytical methods and three-dimensional finite element analysis reviewing the standard design methods and their assumptions. One of the major results was that the p-y method employs unreasonable high values for the soil stiffness at great depths when determining the necessary embedded length for large diameter piles having a high flexural rigidity beyond common experiences. As a solution a modification of the factor determining the increase of soil stiffness within the p-y method is suggested.

### 1 Introduction

Mono pile foundations transfer moments and lateral forces into the subsoil by horizontal subgrade reaction. A certain embedded length is necessary to ensure that the pile can be considered as rigidly restrained in the soil.

The pile-soil system is modelled as a beam on elastic springs (fig. 1). The material behaviour of the soil is described by non-linear springs. The spring characteristics are defined by a relation between the subgrade reaction per unit length of the pile  $p$  and the horizontal pile deflection  $y$  (p-y curve). These curves were derived from field tests performed with piles of 0.6 m diameter. For cohesionless soil ("sand") Reese et al. [1] calculated the subgrade reaction  $p$  for defined characteristic values of the pile deflection and connected these points by straight lines and a parabola (fig. 1) resulting in the p-y curves for "sand".

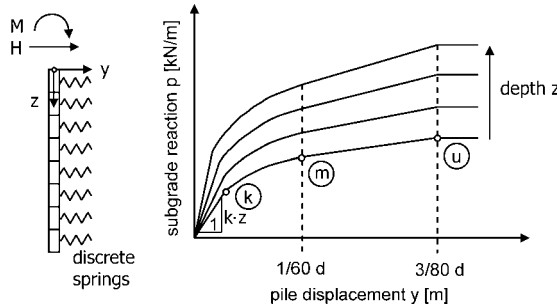


Fig. 1: Beam-on-elastic-springs model and p-y curves

According to API [2] for cohesionless soil the p-y curve is defined by a closed formula:

$$p = A \cdot p_u \tanh((k \cdot z \cdot y)/(A p_u)) \quad (\text{eq. 1})$$

The initial gradient of eq. 1 is the same as the initial gradient of the curve depicted in fig. 1:

$$\left. \frac{dp}{dy} \right|_{y=0} = k \cdot z \quad (\text{eq. 2})$$

Assuming a constant subgrade reaction averaged over the pile diameter  $\sigma = p/d$  the modulus of subgrade reaction  $k_s := d\sigma/dy$  can be calculated from

eq. 2. The modulus of subgrade reaction depends on soil properties and on pile properties as well. Based on the theory of elasticity the relation between the modulus of subgrade reaction  $k_s$  and the oedometric modulus  $E_s$  of the soil  $k_s \approx E_s/d$  was derived (e. g. Kézdi [3], Terzaghi [4]). Together with  $p = \sigma \cdot d$  this relation leads to  $E_s = k \cdot z$ . Thus, in the p-y method a linear increase of the oedometric modulus with depth by a factor of  $k$  is assumed.

In API [2] the factor  $k$  is referred to as "initial modulus of subgrade reaction" (force per cubic length) which is determined depending on the relative density or the effective friction angle of the soil, respectively.

This standard method has been confirmed for pile diameters up to 2 m. However, for the determination of the necessary embedded length for mono piles the applicability of the method to pile diameters beyond common experiences has to be proven.

### 2 Investigation using analytical solutions of the bending differential equation

Generally, the necessary embedded length depends on the pile stiffness  $E \cdot I$ , the pile diameter  $d$  and the modulus of subgrade reaction  $k_s$ . In the case of an elastic subsoil with linear increasing modulus of subgrade reaction from zero at the subsurface to  $k_s(L)$  at the pile foot an analytical solution of the beams bending differential equation was derived by Titze [5]. The necessary embedded length to ensure a rigid restraint at the pile foot is therefore:

$$L = \lambda \cdot L_0, \quad L_0 = \sqrt[5]{\frac{E I L}{d k_s(L)}} \quad (\text{eq. 3})$$

Here,  $\lambda$  is in the range of 4 to 5 [5]. For different pile cross sections the necessary embedded length was first calculated according to the p-y method. A homogeneous soil profile was assumed. Due to the nonlinear spring characteristics in the p-y method the necessary embedded length depends on the loading. Two horizontal forces were applied at the pile head: one forcing a pile head rotation of  $\alpha = 0.2^\circ$  and one forcing a pile head rotation of  $\alpha = 0.7^\circ$ . The resulting pile length  $L$  was used to recalculate the soil

parameter  $E_s$  at the pile foot by use of eq. 3 and  $k_s = E_s/d$ . The soil parameters are listed in tab. 1, the cross sectional values and the calculated embedded length are listed in tab. 2.

effective unit weight	$\gamma' = 10 \text{ kN/m}^3$
effective angle of friction	$\phi' = 40.5^\circ$
relative density	$I_D = 0.55$
factor for increase of $E_s$	$k = 19000 \text{ kN/m}^3$

Tab. 1: Soil parameters

d (m)	t (m)	EI (kNm <sup>2</sup> )	L (m)	
			$\alpha = 0.2^\circ$	$\alpha = 0.7^\circ$
1	0.02	$1553 \cdot 10^3$	11.6	12.0
2	0.03	$18919 \cdot 10^3$	19.0	19.9
4	0.05	$254162 \cdot 10^3$	32.0	33.3
6	0.07	$1203932 \cdot 10^3$	43.6	45.5

Tab. 2: Cross sectional values and necessary embedded length L according to p-y method

For the two load cases the recalculated values of  $E_s$  at the pile foot by use of eq. 3 with  $\lambda = 4$  and  $\lambda = 5$  are listed in tab. 3. In the last column the oedometric modulus according to the p-y method is listed.

d (m)	L (m)	$E_s$ at pile foot (kN/m <sup>2</sup> )		
		eq. 3, $\lambda=4$	eq. 3, $\lambda=5$	$E_s = k L$
$\alpha = 0.2^\circ$				
1	11.6	$88 \cdot 10^3$	$268 \cdot 10^3$	$220 \cdot 10^3$
2	19.0	$149 \cdot 10^3$	$454 \cdot 10^3$	$361 \cdot 10^3$
4	32.0	$248 \cdot 10^3$	$757 \cdot 10^3$	$608 \cdot 10^3$
6	43.6	$341 \cdot 10^3$	$1041 \cdot 10^3$	$828 \cdot 10^3$
$\alpha = 0.7^\circ$				
1	12.0	$77 \cdot 10^3$	$234 \cdot 10^3$	$228 \cdot 10^3$
2	19.9	$124 \cdot 10^3$	$377 \cdot 10^3$	$378 \cdot 10^3$
4	33.3	$212 \cdot 10^3$	$646 \cdot 10^3$	$633 \cdot 10^3$
6	45.5	$288 \cdot 10^3$	$878 \cdot 10^3$	$865 \cdot 10^3$

Tab. 3: Recalculated values of  $E_s$  ( $I_D = 0.55$ )

If a soil with a relative density of  $I_D = 0.7$  is assumed, the value of k acc. to [2] amounts 35000 kN/m<sup>3</sup>. The necessary embedded length according to the p-y method and the recalculated oedometric modulus are listed in tab. 4. The applied horizontal load causes a pile head rotation of  $\alpha = 0.2^\circ$ . Obviously by the use of eq. 3 with  $\lambda \approx 5$  the experienced values of k according to the p-y method can be reconstructed.

d (m)	L (m)	$E_s$ at pile foot (kN/m <sup>2</sup> )		
		eq. 3, $\lambda=4$	eq. 3, $\lambda=5$	$E_s = k L$
1	10.6	$126 \cdot 10^3$	$384 \cdot 10^3$	$371 \cdot 10^3$
2	17.0	$232 \cdot 10^3$	$708 \cdot 10^3$	$595 \cdot 10^3$
4	28.5	$394 \cdot 10^3$	$1204 \cdot 10^3$	$998 \cdot 10^3$
6	38.8	$544 \cdot 10^3$	$1660 \cdot 10^3$	$1358 \cdot 10^3$

Tab. 4: Recalculated values of  $E_s$  ( $I_D = 0.7$ )

However, tab. 3 and 4 show unrealistically high values of  $E_s$  in greater depths in view of the subsoil conditions assumed here. In contrast the recalculated values of  $E_s$  with  $\lambda = 4$  at the pile foot of those piles, which have a diameter and therefore a

stiffness and necessary embedded length that is in the field of experience ( $d = 1$  and  $2$  m), are reasonable.

For the determination of the necessary embedded length the assumed simplification that the oedometric modulus increases linearly with depth should be modified. For this a non-linear variation of  $E_s$  with depth based on the experience values [2] is proposed (fig.2).

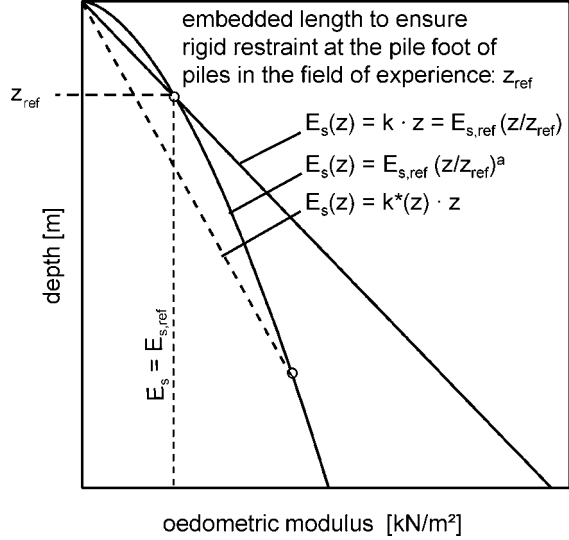


Fig. 2: modified variation of oedometric modulus with depth

The existing values of k acc. to [2] are proven for those depths that affect the behaviour of piles having a diameter up to 1 - 2 m. Thus, the necessary embedded length to ensure a rigid restraint of a pile foot for a pile with  $d = 1$  m was chosen as a reference depth and the corresponding value of  $E_s$  as a reference value. As depicted in fig. 2 the variation of  $E_s$  with depth is described by eq. 4:

$$E_s(z) = E_{s,ref} \left( \frac{z}{z_{ref}} \right)^a = k z_{ref} \left( \frac{z}{z_{ref}} \right)^a \quad (\text{eq. 4})$$

The modified factor  $k^*$  is

$$k^*(z) = E_s(z)/z = k \left( \frac{z_{ref}}{z} \right)^{1-a} \quad (\text{eq. 5})$$

With eq. 3 and use of  $k_s = E_s/d$  the elastic length is calculated to:

$$L_0(EI) = (EI)^{\frac{1}{4+a}} \left( \frac{\lambda^{1-a}}{k z_{ref}^{1-a}} \right)^{\frac{1}{4+a}} \quad (\text{eq. 6})$$

For a pile with flexural rigidity EI and a necessary embedded length of  $L = \lambda L_0$  the modified factor  $k^*$  is:

$$k^*(L) = k \left( \frac{z_{ref}}{L} \right)^{1-a} = k \left( \frac{EI_{ref}}{EI} \right)^{\frac{1-a}{4+a}} \quad (\text{eq. 7})$$

Whereas  $EI_{ref}$  denotes the flexural rigidity of a pile with a necessary embedded length of  $L = z_{ref}$ . In case of  $EI_{ref}/EI = (d_{ref}/d)^4$  the above equation can be formulated to:

$$k^*(d) = k \left( \frac{d_{ref}}{d} \right)^{\frac{4(1-a)}{4+a}} \quad (\text{eq. 8})$$

A more conservative approach is to limit the oedometric modulus to the experience values:  $E_s(z) \leq E_{s,ref}$  (fig. 2). In this case the exponent  $a$  amounts 0, thus  $k^*(d) = k d_{ref}/d$ .

Examples:

(1) pile:  $(d / t / EI) = (4 \text{ m} / 0.05 \text{ m} / 254162 \cdot 10^3 \text{ kNm}^2)$ , relative density of soil:  $I_D = 0.55$

reference pile:  $(d / t / EI / L) = (1 \text{ m} / 0.02 \text{ m} / 1553 \cdot 10^3 \text{ kNm}^2 / 12.0 \text{ m})$

solution:  $(k = 19000 \text{ kN/m}^3 \text{ and an exponent } a = 0.6)$ :  $k^* = 12200 \text{ kN/m}^3$ ,  $L = 36.3 \text{ m}$

Compared to the solution according to the p-y method with a linear increase of the oedometric modulus ( $L = 33.3 \text{ m}$ ) the embedded length is about 10 % larger. The recalculation of the oedometric modulus with eq. 3 and  $\lambda = 4$  leads to a reasonable value of  $E_s(36.3 \text{ m}) = 150 \cdot 10^3 \text{ kN/m}^2$ .

(2) pile:  $(d / t / EI) = (6 \text{ m} / 0.07 \text{ m} / 1203932 \cdot 10^3 \text{ kNm}^2)$ , relative density of soil:  $I_D = 0.7$

reference pile:  $(d / t / EI / L) = (1 \text{ m} / 0.02 \text{ m} / 1553 \cdot 10^3 \text{ kNm}^2 / 10.6 \text{ m})$

solution:  $(k = 35000 \text{ kN/m}^3 \text{ and an exponent } a = 0.5)$ :  $k^* = 16700 \text{ kN/m}^3$ ,  $L = 46.5 \text{ m}$

Compared to the solution according to the p-y method with a linear increase of the oedometric modulus ( $L = 38.8 \text{ m}$ ) the embedded length is about 20 % larger. The recalculation of the oedometric modulus with eq. 3 and  $\lambda = 4$  leads to a value of  $E_s(46.5 \text{ m}) = 264 \cdot 10^3 \text{ kN/m}^2$ . This value is much more reasonable for a dense to very dense sand than the value of  $544 \cdot 10^3 \text{ kN/m}^2$  (tab. 4).

### 3 Numerical simulation

The behaviour of laterally loaded piles was investigated by a three-dimensional model using the finite element method (fig. 3). The piles cross section was modelled as a pipe. The pile and the subsoil are connected by contact pairs that enable the transmission of shear stresses and normal stresses according to the Coulomb friction model. The material behaviour of the soil is modelled with (1) a linear elastic-ideal plastic model with Mohr-Coulomb failure criterion, (2) a hypoplastic material model and (3) an advanced elastoplastic model with Mohr-Coulomb failure criterion that accounts for material nonlinearity in the elastic stress range.

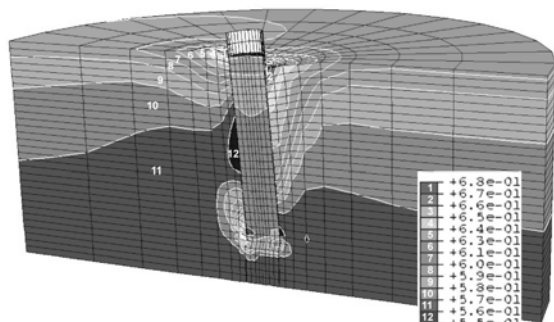


Fig. 3: Laterally loaded pile, distribution of void ratio

The aforementioned examples are investigated by a three-dimensional simulation. The soil is described by the elastoplastic material model with the Mohr-Coulomb failure criterion. For example 1 ( $d = 4 \text{ m}$ ,  $L = 36.3 \text{ m}$ ) the computed pile deflection and the subgrade reaction are depicted in fig. 4. The results according to the p-y method with a modified factor  $k^*$  are shown on the left side, the results obtained by the finite element analysis (abbr. FEA) on the right side. The results are depicted for different load levels. Similarly, the results for example 2 are presented in fig. 5.

As predicted by the analytical procedure, for small lateral loading both piles are fixed at the pile foot. The computed deflection line and the stress variation according to the p-y method with modified factor  $k^*$  and the finite element analysis are similar. In case of the finite element model the soil is in the elastic range of the Mohr-Coulomb model thus the soil stiffness has the initial value of  $E_s$ .

With increasing load in the finite element analysis the soil in the upper strata plastifies and the loading has to be transferred into deeper parts of the soil strata. The zero-crossing of the deflection line and the subgrade stress move downwards. In contrast to the continuum model of the FEA the p-y method employs separate springs that avoid vertical load transfer among the springs. For this the shift in load transfer is not as pronounced as in the FEA. For a vivid demonstration of this difference a very high loading was selected (15.75 MN and 88.00 MN respectively). In case of the p-y method the point of zero-crossing does not change significantly for the computed displacements and stresses even for these high loads. Hence, piles are still restrained whereas according to the FEA the piles lose the rigid restraint at the pile foot (fig. 4 and 5).

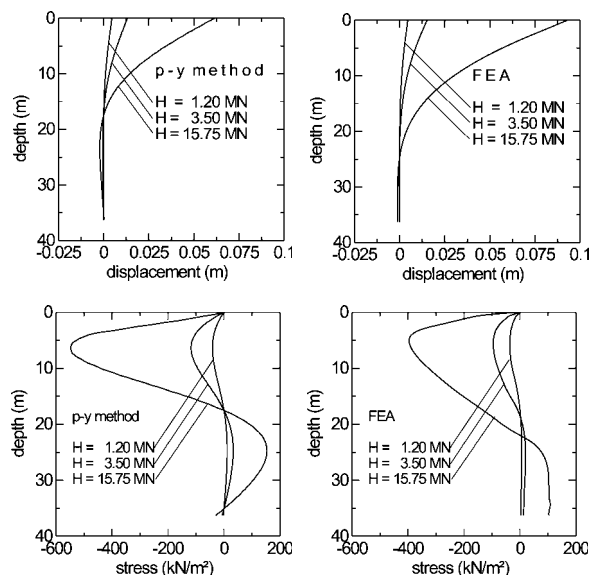


Fig. 4: Computed deflection line and horizontal subgrade stress. Pile:  $d = 4 \text{ m}$ ,  $L = 36.3 \text{ m}$  (ex. 1).

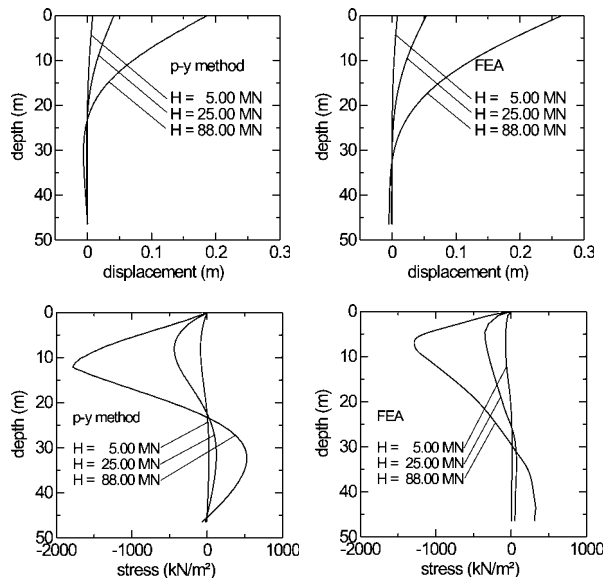


Fig. 5: Computed deflection line and horizontal subgrade stress. Pile:  $d = 6$  m,  $L = 46.5$  m (ex. 2).

#### 4 Conclusion

The suggested modification of  $k^*$  when calculating the necessary embedded length by use of the p-y method is reviewed by finite element analysis. The modification based on the experience values of the p-y method accounts for reasonable values of the soil stiffness. The exponent  $a$  in eq. 4 was approximately chosen to 0.6 in the case of medium dense cohesionless soil and to 0.5 for dense cohesionless soil.

However, due to the simplified assumptions concerning the increase of soil stiffness and the use of experience values the p-y method acc. to [2] with the modified factor  $k^*$  should be used only for preliminary design. To verify the serviceability by calculating pile head deformations an extended investigation is needed.

Generally, in the case of an on-shore site pile load tests are performed to obtain reliable information about pile head deformations. Indeed at off-shore sites pile load tests are not suitable to determine a load-deflection curve. Therefore to verify the serviceability of mono piles only computational methods are available. Since the upper strata is of particular importance for the pile head deformations, the variation of the soil stiffness with depth should be acquired at the specific site and reflected by a preferably accurate numerical model.

#### 5 Notation

$a$	exponent
$A$	empirical adjustment factor within p-y method
$d$	pile diameter (m)
$E_s$	oedometric modulus ( $\text{kN/m}^2$ )
$EI$	flexural rigidity of the pile ( $\text{kNm}^2$ )
$H$	horizontal force (kN)
$\rho$	relative density of soil

$k$	factor for the depth depending increase of the oedometric modulus ( $\text{kN/m}^3$ )
$k^*$	modified factor for the depth depending increase of $E_s$ ( $\text{kN/m}^3$ )
$k_s$	modulus of subgrade reaction ( $\text{kN/m}^3$ )
$L$	embedded length of pile (m)
$L_0$	elastic length (m)
$M$	moment ( $\text{kNm}$ )
$p$	subgrade reaction per unit length of pile ( $\text{kN/m}$ )
$p_u$	ultimate subgrade reaction per unit length of pile ( $\text{kN/m}$ )
$t$	pile wall thickness (m)
$y$	pile displacement (m)
$z$	depth below subgrade surface (m)
$\alpha$	rotation of pile head ( $^\circ$ )
$\gamma'$	effective unit weight of soil ( $\text{kN/m}^3$ )
$\lambda$	ratio of embedded length to elastic length
$\sigma$	horizontal subgrade stress ( $\text{kN/m}^2$ )
$\varphi'$	effective friction angle of soil ( $^\circ$ )
ref	index that denotes a reference value

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